

Study on OFDM Adaptive Power Allocation Algorithm

Biying Shi⁺ and Aimin Liu

Information Science and Technology School, Zhuhai Campus, Beijing Institute of Technology
Zhuhai, China

Abstract. OFDM technology will become the key technique in broadband wireless communication system in future due to its advantages in suppressing effectively the fading channels and of high rate of spectrum usage efficiency. The adaptive modulation technology in OFDM system can choose proper modulation methods and signal power for every sub-channel, which improves greatly the system performance. To study on the adaptive modulation technology in OFDM system is significant for the development and applications of the broadband wireless communication system in the future. This essay proposes an adaptive modulation technology suitable for the OFDM system with fixed rate. The algorithm combines the sub-band division with the greedy allocation algorithm to simplify the algorithm complexity. Through gathering the adjacent sub-channels together and allocating the same number of bits to them, the signaling overhead in practical application can be reduced effectively. Through choosing different sub-band width, the complexity of the algorithm can be adjusted flexibly. And good compromise can be made between the algorithm complexity and the performance. Compared to the existing molecular band adaptive allocation algorithm, in the condition of the same performance, new algorithm has lower complexity. The new algorithm proposed in this essay has high application flexibility and is suitable for practical applications.

Keywords: OFDM; adaptive; power allocation algorithm

1. Introduction

From the development of the first generation to the third generation mobile communication technology, new transmission technologies are coming up to meet people's need for communication system with higher transmission rate, higher rate of spectrum usage efficiency, such as CDMA technology, Rake reception technology, multiuser detection, diversity combining technology, smart antenna technology, Turbo encoding and decoding technology, MIMO technology and so on. These technologies can to certain degree reduce the signal fading caused by the wireless environment channel. But, for the broadband signal, they can not solve basically signal fading through the wireless channel. The system may still need the complex balancing technique to suppress the fading effect, namely, frequency selective fading and time selective fading. Especially for the broadband signal, due to the short symbol period circle, inter-symbol interference (ISI) caused by multipath propagation is very serious. OFDM (Orthogonal Frequency Division Multiplexing) technology can effectively suppress the frequency selective fading of the wireless channel without complex balancing technique. It can eliminate ISI effect through introducing symbol prefix and high rate of the spectrum usage efficiency can be realized. These advantages make OFDM technology considered as the most important technology of the fourth generation mobile communication system has become the research hotspot widely at present.

2. Brief Introduction to OFDM Technology

⁺ Email: biying.shi@gmail.com

OFDM is short for Orthogonal Frequency Division Multiplexing. In many occasions, it is also called Multi-carrier Modulation, MCM for short. OFDM is not only a modulation technology but also a multiplexing technology. OFDM is a high rate transmission technology in wireless environment, it can satisfy the demands of high speed and anti-interference at the same time.

In traditional frequency division multiplexing methods, the spectrum is not overlapped between sub-carriers. It is necessary to use lot of sending filters and reception filters. In this way, the complexity and cost of the system will greatly increase. Meanwhile, in order to reduce inter-carrier interference, enough frequency space shall be kept between sub-carriers, which can reduce the frequency usage efficiency of the system. The main though of the OFDM technology is to divide the whole channel into many sub-channels and perform signal modulation and transmission in each sub-channel. The carrier of each sub-channel is orthogonal and the spectrum of the adjacent sub-channels is overlapped.

Compared to the single carrier and the traditional frequency division multiplexing transmission technology, OFDM technology has many advantages as follows: (1) signals are transmitted in parallel through multi-carriers. Each sub-channel is nearly flat fading so that the frequency selective fading can be effectively suppressed. The original signals can be better converted by the reception end without balancing or with simple balancing. (2) the ISI and ICI can be eliminated effectively by using cyclic prefix technology. (3) the spectrum of the adjacent sub-channels is overlapped so that high rate of spectrum usage efficiency can be realized. When there are a large quantity of sub-channels, the rate of the spectrum usage efficiency is nearly 2Baud/Hz. (4) the digital processing chip can be changed through IFFT/FFT to realize the modulation and demodulation of the multi-carriers to be convenient for practical applications. (5) adaptive modulation technology can be applied according to the various transmission features of the sub-channels to choose various modulation methods sending power for each sub-channel to improve the system performance. (6) asymmetrical transmission in uplink and downlink can be realized easily.

3. System Model and Mathematical Description of OFDM

DSP technology has not been applied in the initial OFDM technology. Suppose there are N sub-carriers in the system; the total occupied bandwidth is W Hz; symbol length is T second, in which the CP length is T_{cp} second (namely, the actual signal length is $T - T_{cp}$ second),

$$T = \frac{N}{W} + T_{cp}$$

(therefore, the channel usage efficiency is

$$\frac{1/T}{1/(T - T_{cp})} = \frac{T - T_{cp}}{T}.$$

The signal of the sending end can be described as the following mathematical form:

$$\varphi_k(t) = \begin{cases} \frac{1}{\sqrt{T - T_{cp}}} \exp\left\{j2\pi \frac{W}{N} k(t - T_{cp})\right\} & t \in [0, T] \\ 0 & \text{else} \end{cases} \quad (1)$$

Otherwise:

$$\varphi_k(t) = 0 \quad k=0,1,2, \dots, N-1$$

Notice: the product form has been used in the formula above to make the value of the signals between $[0, T_{cp}]$ and $[T - T_{cp}, T]$ are the same, embodying the definition of CP; additionally, different sub-carriers are embodied by using the various values k takes.

The L^{th} OFDM signal is

$$S_l(t) = \sum_{k=0}^{N-1} x_{k,l} \varphi_k(t - lT)$$

In which, $x_{k,l}$ ($k=0,1,\dots,N-1$) are a cluster of signal points, which are transmitted respectively in the k^{th} ($k=0,1,\dots,N-1$) sub-carrier of the L^{th} OFDM.

If there is an unlimited long OFDM symbol sequence to be transmitted, this sequence can be expressed as:

$$S(t) = \sum_{l=-\infty}^{+\infty} S_l(t) = \sum_{l=-\infty}^{+\infty} \sum x_{k,l} \varphi_k(t-lT) \quad (2)$$

Suppose the channel impulse response is $g(\tau; t)$, τ is limited to $[0, T_{cp}]$, namely, limited to within the length of CP, the receiving signal is:

$$r(t) = (g * s)(t) + n(t) = \int_0^{T_{cp}} g(\tau; t) s(t-\tau) d\tau + n(t) \quad (3)$$

In which, $n(t)$ is AWGN (Additive White Gaussian Noise); the “*” in $(g*s)$ refers to the convolution. If the matched filter is used as the receiver,

$$\Psi_k(t) = \begin{cases} \varphi_k^*(T-t) & t \in [0, T_{cp}] \\ 0 & \text{else} \end{cases} \quad (4)$$

In this way, the signal from $\Psi_k(t)$ shall be $\varphi_k^*(t), t \in [T_{cp}, T]$. Therefore, CP is eliminated in the reception end. CP includes all the ISI; there is no ISI in the signal in which CP is eliminated coming out from the receiving end. Therefore, calculating the output of the k^{th} sub-carrier, the subscript L can be ignored, namely, the receiving signal y_k can be expressed as:

$$\begin{aligned} y_k &= (r * \Psi_k)(t) |_{t=T} = \int_{-\infty}^{+\infty} r(t) \Psi_k(T-t) dt \\ &= \int_{T_{cp}}^T \left(\int_0^{T_{cp}} [g(\tau; t) \sum_{k'=0}^{N-1} x_{k'} \varphi_{k'}(t-\tau)] d\tau \right) \varphi_k^*(t) dt + \int_{T_{cp}}^T \tilde{n}(T-t) \varphi_k^*(t) dt \end{aligned} \quad (5)$$

Again suppose the channel multipath time delay is limited to CP, namely, $\tau \in [0, T_{cp}]$, use $g(\tau)$ to express the channel features,

$$y_k = \sum_{k'=0}^{N-1} x_{k'} \int_{T_{cp}}^T \left[\int_0^{T_{cp}} g(\tau) \varphi_{k'}(t-\tau) d\tau \right] \varphi_k^*(t) dt + \int_{T_{cp}}^T \tilde{n}(T-t) \varphi_k^*(t) dt \quad (6)$$

While

$$\begin{aligned} \int_0^{T_{cp}} g(\tau) \varphi_{k'}(t-\tau) d\tau &= \int_0^{T_{cp}} g(\tau) \frac{e^{j2\pi k'(t-\tau-T_{cp})W/N}}{\sqrt{T-T_{cp}}} d\tau \\ &= \frac{e^{j2\pi k'(t-T_{cp})W/N}}{\sqrt{T-T_{cp}}} \int_0^{T_{cp}} g(\tau) e^{-j2\pi k' \tau W/N} d\tau \end{aligned} \quad (T_{cp} < t < T) \quad (7)$$

Suppose the Fourier transform of $g(\tau)$ is $G(f)$, let

$$h_{k'} = G(k' \frac{W}{N}) = \int_0^{T_{cp}} g(\tau) e^{-j2\pi k' \tau W/N} d\tau$$

$h_{k'}$ is the impulse response of the channel features $g(\tau)$ in the k' sub-carrier. Notice that:

$$\varphi_{k'}(t) = \frac{e^{j2\pi k'(t-T_{cp})W/N}}{\sqrt{T-T_{cp}}} \quad (8)$$

The receiving signal y_k can be simplified as:

$$\begin{aligned} y_k &= \sum_{k'=0}^{N-1} x_{k'} \int_{T_{cp}}^T \frac{e^{j2\pi k'(t-T_{cp})W/N}}{\sqrt{T-T_{cp}}} h_{k'} \varphi_k^*(t) dt + \int_{T_{cp}}^T \tilde{n}(T-t) \varphi_k^*(t) dt \\ &= \sum_{k'=0}^{N-1} x_{k'} h_{k'} \int_{T_{cp}}^T \varphi_{k'}(t) \varphi_k^*(t) dt + n_k \end{aligned} \quad (9)$$

In which,

$$n_k = \int_{T_{cp}}^T \tilde{n}(T-t) \varphi_k^*(t) dt \quad (10)$$

Because $\varphi_k(t)$ ($k=0, 1, \dots, N-1$) is orthogonal, then

$$\int_{T_{cp}}^T \varphi_{k'}(t)\varphi_k^*(t)dt = \int_{T_{cp}}^T \frac{e^{j2\pi k'(t-T_{cp})W/N}}{\sqrt{T-T_{cp}}} \frac{e^{-j2\pi k(t-T_{cp})W/N}}{\sqrt{T-T_{cp}}} dt = \delta(k-k') \quad (11)$$

Therefore,

$$y_k = \sum_{k'=0}^{N-1} x_{k'} h_k \delta(k-k') + n_k = x_k h_k + n_k \quad (12)$$

Here, n_k is still AWGN. In light of this, the impact of the channels on the signals is product relation but not convolution. So the code elements both in front and at back can not interfere the current code elements, namely, there is no interference between symbols.

4. OFDM Adaptive Power Allocation Algorithm

In OFDM system, the adaptive modulation is being performed for choosing proper modulation methods and sending power for each sub-channel according to the transmission features of the sub-channels. This process is completed through adaptive bit and power algorithm. The adaptive bit and power allocation algorithm is the core and basis of the adaptive modulation technology in OFDM system. The adaptive modulation bit and power allocation algorithm allocates proper bit number (namely, determines the modulation methods) and signal power for each sub-channel according to the channel state information. According to the allocation result, the sending end performs signal modulation and the receiving end performs signal demodulation.

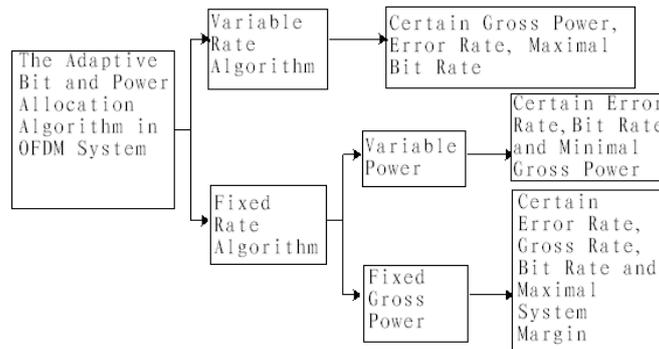


Fig. 1 Classifications of adaptive bit and power allocation algorithm in OFDM system

The bit and power allocation algorithm in OFDM system can be divided into two categories of variable rate and fixed rate according to the constraint condition of systematical bit rate and power rate, in which, the fixed rate algorithm can also be divided into variable power and fixed power algorithms. The difference between these algorithms of different categories lies in the different optimized targets of the algorithm. But, in essence, they are equivalent. They are all for using the power most effectively. For the fixed rate algorithm, in the condition of the unfixed gross power, the optimized target is to make the gross power minimal; in the condition of the fixed gross power, the optimized target is to make the system margin maximal. When the system margin is maximal, the power the system needs to meet certain rate and error rate is minimal. Therefore, the two are equivalent. For the variable algorithm, generally, the gross power of the system is fixed. The optimized target of the system is to make the rate of the system maximal in the condition of certain error rate. This optimized target can also be realized through the adjusted fixed rate algorithm and variable power algorithm. What needs to do is to eliminate the constraint condition of the rate and introduce the constraint condition of the maximal power. Thus, the terminal condition of the algorithm will be changed and each allocation process of the original algorithm can keep unchanged. Therefore, in essence, the fixed rate algorithm and variable rate algorithm are equivalent.

We use the thought of dividing the sub-bands to simplify the existing OFDM system algorithm. Use the equivalent sub-channel as the basis unit to implement the greedy allocation algorithm. The optimization objective of this algorithm is to determine the optimized bit number for the spread frequency branch to make the sending power the system needs lowest in the condition of certain system data rate and error rate. This optimization target and constraint condition can be described as follows:

Optimization object:

$$\min_{b_j \in D} \sum_{j=1}^{N_s} P_j(b_j) \quad (13)$$

Constraint condition:

$$\sum_{j=1}^{N_s} b_j = R'_b, P_j(b_j) \geq 0, b_j \geq 0 \quad (14)$$

$$BER_i = BER_{\text{target}}, i \in U \quad (15)$$

In which, N_s is the number of the equivalent sub-bands; b_j is the allocated bit number of the j^{th} “equivalent sub-band”; $P_j(b_j)$ is the launching power needed to transmit b_j bit; R_b is the bit number included in each sending symbol (namely, the bit number to be allocated in each allocation process); set $D=\{0,1,2,\dots,l\}$ is the value which b_j may takes; \min refers to the minimal value; BER_{target} is the system targeted error rate (the error rate when coding is not performed). When b_i bits are transmitted in the i^{th} sub-channel and the i^{th} channel’s transmission signal power is P_i , BER_i refers to the mean error rate of this channel. U means the sequence set of all the usable equivalent channels and it is a subset of the sequence set of all the equivalent sub-channels in the system, namely, $U \subseteq \{i|i=1,2,\dots,N_s\}$. For the fixed rate OFDM system, if the system rate is R_b bit/OFDM symbol, the actual bits number to be allocated as needed in each allocation process is:

$$R'_b = \frac{R_b}{V} \quad (16)$$

The algorithm mainly includes the following steps:

Step 1: determine the sub-band width V and the sub-band equivalent gain plan. Calculate the actual bit number R'_b to be allocated as needed.

Step 2: divide continuously all the sub-channels into N_s sub-bands and each sub-band includes V sub-channel according to their location in the spectrum. Calculate each sub-band’s equivalent gain.

Step 3: in all the sub-bands, allocate proper bit number for each sub-band according to greedy allocation algorithm.

Step 4: copy the bit number allocated to each sub-band to each sub-channel in the sub-band. Calculate the sending power each sub-channel needs (suppose each sub-channel’s noise power equals).

$$P_{i,j} = \frac{\Gamma \sigma_0^2}{|H_{i,j}|^2} (2^{b_i} - 1) \quad (17)$$

In which, $i=1,2,\dots,N_s, j=1,2,\dots,V$. b_i is the bit number allocated to the i^{th} sub-band. σ_0^2 is the variance of the noise in the sub-channel.

If this algorithm is used in the system whose optimization objective is “maximal system margin”, after calculating the power according to (17) formulas, another step to adjust the power shall be needed to make the sending power of OFDM symbol equal to the gross power rating. Because in the algorithm, the error rate of each sub-channel is still designed to be the same. Therefore, the best adjustment method is to multiply the power of each sub-channel by the same coefficient. The power adjustment of each sub-channel is performed as follows:

$$P'_{i,j} = P_{i,j} \cdot \frac{P_T}{\sum_{i=1}^{N_s} \sum_{j=1}^V P_{i,j}} \quad (18)$$

In which, $i=1,2,\dots,N_s, j=1,2,\dots,V$. P_T is the gross sending power rating of each OFDM symbol.

The algorithm (SEGL) is shown as figure 2.

5. Conclusion

OFDM technology will become the key technique in broadband wireless communication system in future due to its advantages in suppressing effectively the fading channels and of high rate of spectrum usage

efficiency. The adaptive modulation technology in OFDM system can choose proper modulation methods and signal power for every sub-channel, which improves greatly the system performance. To study on the adaptive modulation technology in OFDM system is significant for the development and applications of the broadband wireless communication system in the future. This essay proposes an adaptive modulation technology suitable for the OFDM system with fixed rate. The algorithm combines the sub-band division with the greedy allocation algorithm to simplify the algorithm complexity. Through gathering the adjacent sub-channels together and allocating the same number of bits to them, the signaling overhead in practical application can be reduced effectively. Through choosing different sub-band width, the complexity of the algorithm can be adjusted flexibly. And good compromise can be made between the algorithm complexity and the performance. Compared to the existing molecular band adaptive allocation algorithm, in the condition of the same performance, new algorithm has lower complexity. The new algorithm proposed in this essay has high application flexibility and is suitable for practical applications

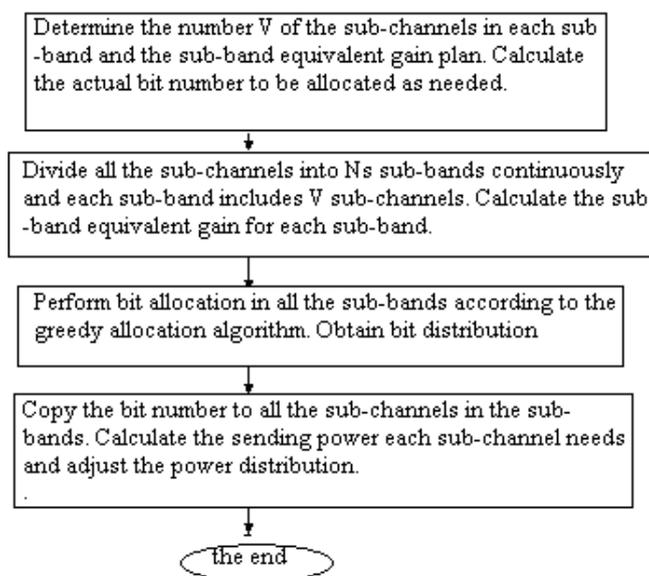


Fig. 2 Flow figure of the algorithm

6. References

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