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Maximum Entropy Analysis of Priority Censored Loss System

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Abstract. Cellular communication demands for seamless mobility, mobile broadband and multimedia services, LTE is the technology that addresses all these demands. Data rates offered by this technology are up to 1Gbits/sec and offers improved performance at the edge of cell i.e. fast handoff. Due to the high intensity of packets the traffic behavior is bursty; packets/calls arrive in bulks and after getting/without getting service depart in bulks. In censored cell the arrival process is not halted when the system reaches its full capacity i.e. continuous arrival process and the packets/calls that arrive when the system is full are blocked. In this paperLagrange coefficients are determined analytical to carry out Maximum Entropy (ME)analysis of priority censored system for LTE cell, priority is given to handoff calls by assigning dedicated channels. Traffic arrival and departure process follow Generalized exponential distribution i.e. calls arrive for service in burst/s and after getting service/without getting service departs in burst/s. Originating calls are blocked when all shared channels are busy and handoff voice calls are blocked when all channels are busy processing calls. A new exact closed form steady state probability distribution is devise and numerical results are generated to analyze impact of traffic patterns on QoS.

Keywords: Queueing theory, GE distribution, Maximum Entropy, Performance Evaluation, Cellular Systems, Loss System, LTE.

1. Introduction

Queueing network models (QNMs) with finite capacity are of paramount importance towards effective QoS (Quality of Service) and congestion control in discrete traffic flow systems. Blocking in these networks arises when the system reaches its full capacity. In some systems priority is assigned to jobs/calls/packets, so that the jobs/calls/packets with highest priority get service first. To devise exact closed-form solutions for these models with blocking are not generally possible apart from some special cases such as reversible queueing networks (i.e. Finite capacity networks with corresponding infinite capacity networks that are reversible or Two Station Cyclic queues) [1, 2]. Kouvatsos in a series of publications [3, 4, 5, 6, 7], laid the foundation of a new analytical frame work which can be used to derive minimally biased approximations of performance distributions for queues, and queueing networks with General / Generalized exponential traffic with infinite buffers, subject to mean rate and squared coefficient of variance (SCV) of, inter arrival and service time distribution, represented byλ, C^2_a, μ and C^2_s respectively. The minimal biased approximations of performance distributions for finite queue with General/Generalized exponential distribution with finite/No buffer derived are hypothetical deduced from queues with infinite buffer [9, 10, 11].

2. Maximum Entropy Formalism

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The ME formalism provides an analytical solution approximated to that of queueing system and networks with stochastic and operational analysis is subject to the following constraints

2.1. Normalization

Sum of all probabilities is equal to 1, i.e. $\sum_{n=0}^{N} P_N(n) = 1$

2.2. Utilization, U: 0 < U < 1

$$\sum_{n=0}^{N} h(n)\, P_N(n) = U where\ \ h(n)\begin{cases} 0 & n=0 \\ 1 & n \neq 0 \end{cases}$$

2.3. Mean queue length

$$\sum_{n=1}^{N} nP_N(n) = < n >$$

Which is zero in this particular case as it is a loss system.

2.4. Full buffer state probability,

$$P_N = \emptyset, 0 < \ \emptyset < 1$$

is written as

$$\sum_{n=0}^{N} f(n) P_N(n) = \ \emptyset \quad where \quad f(n) = \begin{cases} 0, & n < N \\ 1, & n = \ N \end{cases}$$

General ME solution that maximizes system entropy function

$$H(p) \ = \ -\sum_{n=0}^{N} P(n) \ln(P(n))$$

Is given by

$$P(n) = \frac{1}{z} g^{h(n)} x^n y^{f(n)} \qquad where\ h(n) = \begin{cases} 1\, for\ n = 0 \\ else\ 0 \end{cases} \quad ; \quad f(n) = \ \begin{cases} 1\, for\ n = N \\ else\ 0 \end{cases}\ and\ z = \frac{1}{p_0}$$

where g, x and y are Lagrange multipliers.

In this paper, Lagrange coefficients are derived from two stage hyper exponential model to capture the burstiness of the traffic of a censored priority loss system, the exponential distributions does not fully capture the traffic behavior of modern telecommunication systems due to higher data rates. These coefficients are than substituted in general ME solution that maximize system entropy function to calculate entropy. ρ(rhoa) and squared coefficient of variance of inter service and inter arrival time is varied and behavior of the traffic is analyzed.

Rest of the paper is organized as follow; Section 3 presents loss system model and its description. Section 4 is flow balance equations, Lagrange coefficients from the flow balance equations are derived in section 5. Some typical numerical experiments are carried out in Section 6. Conclusions and remarks on future work follow in Sections 7 and 8, respectively.

3. Analytical Model

Traffic arriving at receiver centre consists of originating calls and Handoff Calls. let h_h and h_o be two random variables of inter arrival time of originating calls and Handoff Calls, and are exponentially distributed with parameter λ_o and λ_h respectively, therefore by memory less property of exponential distribution remaining inter arrival time Δh_h and Δh_o is also exponentially distributed, with parameter $\Delta\lambda_o$

and $\Delta\lambda_h$. Next arrival will be min (Δh_h, Δh_o) [2]. Hence next arrival will also be exponentially distributed with parameter $1/\lambda$ i.e. sum of λo and λ_h i.e. $\lambda = \lambda_o + \lambda_h$

Channel holding time has exponential distribution with mean rate μ.There are C_d shared channels. Originating calls that arrive and find shared channels busy processing calls are lost while Handoff Calls are lost when all C channels are busy processing calls; lost calls depart immediately and never return to system.

Calls arrive in bulks; mean number of calls arriving in bulk per unit of time is by $\delta\lambda$, where δ is mean number of bulks arriving per unit of time. Also $\gamma\mu$ is mean number of calls departing in bulk per unit of time, where γ is mean number of departing bulks. Model diagram of system is shown in figure 1.

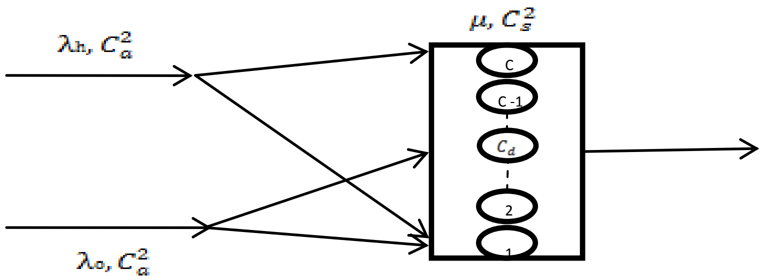

Fig. 1 Priority loss system Model diagram

State transition diagram is shown in figure 2.

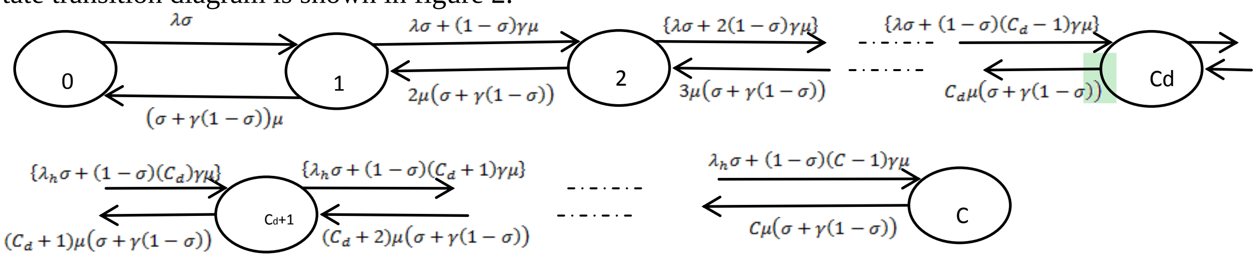

Fig 2 Priority loss system State Transition Diagram

4. Flow Balance Equations

Mean flow in = Mean flow out

$$\lambda\sigma\gamma p(0) = \gamma\mu\big(\sigma + \gamma(1-\sigma)\big)p(1)$$

$$p(1) = \frac{\lambda\sigma}{\mu\big(\sigma+\gamma(1-\sigma)\big)}p(0) \tag{1}$$

$$\{\lambda\sigma + (1-\sigma)\gamma\mu\}\gamma p(1) = 2\gamma\mu\big(\sigma + \gamma(1-\sigma)\big)p(2)$$

$$p(2) = \frac{\{\lambda\sigma+(1-\sigma)\gamma\mu\}}{2\mu\big(\sigma+\gamma(1-\sigma)\big)}p(1) \tag{2}$$

..

$$p(C_d) = \frac{\{\lambda\sigma+(1-\sigma)(C_d-1)\gamma\mu\}}{C_d\mu\big(\sigma+\gamma(1-\sigma)\big)}p(C_d - 1) \tag{3}$$

$$p(C_d + 1) = \frac{\{\lambda_h\sigma + (1-\sigma)(C_d)\gamma\mu\}}{(C_d + 1)\mu\big(\sigma + \gamma(1-\sigma)\big)}p(C_d)$$

.

$$. p(C_d + 2) = \frac{\{\lambda_h\sigma+(1-\sigma)(C_d+1)\gamma\mu\}}{(C_d+2)\mu\big(\sigma+\gamma(1-\sigma)\big)}p(C_d + 1) p(C-1) = \frac{\{\lambda_h\sigma+(1-\sigma)(C-2)\gamma\mu\}}{(C-1)\mu\big(\sigma+\gamma(1-\sigma)\big)}p(C-2)$$

$$p(C) = \frac{\{\lambda_h\sigma+(1-\sigma)(C-1)\gamma\mu\}}{C\mu\big(\sigma+\gamma(1-\sigma)\big)}p(C-1) \tag{4}$$

$p(n) = p(0)\ \ for\ n = 0$

$$p(n) = \frac{\prod_{i=1}^{n}\{\lambda\sigma + (1-\sigma)(i-1)\gamma\mu\}}{n!\left[\mu\left(\sigma + \gamma(1-\sigma)\right)\right]^{n}} p(0) \quad for\ 1 \le n \le C_d$$

$$p(n) = \frac{\left[\prod_{i=1}^{C_d}\{\lambda\sigma + (1-\sigma)(i-1)\gamma\mu\}\right] * \prod_{j=1}^{n-C_d}\{\lambda_h\sigma + (1-\sigma)(j + C_d - 1)\gamma\mu\}}{n!\left[\mu\left(\sigma + \gamma(1-\sigma)\right)\right]^{n}} p(0) \quad for\ \ C_d < n \le C$$

Since sum of all probabilities is equal to 1, therefore we can write

$$\sum_{n=0}^{C} p(n) = 1$$

$$p(0) + p(1) + p(2) + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots + p(c)$$

$$= 1p(0) + \sum_{n=1}^{C_d} \frac{\prod_{i=1}^{n}\{\lambda\sigma + (1-\sigma)(i-1)\gamma\mu\}}{n!\left[\mu\left(\sigma + \gamma(1-\sigma)\right)\right]^{n}} p(0)$$

$$+ \sum_{n=C_d+1}^{C} \frac{\left[\prod_{i=1}^{C_d}\{\lambda\sigma + (1-\sigma)(i-1)\mu\gamma\}\right] * \prod_{j=1}^{n-C_d}\{\lambda_h\sigma + (1-\sigma)(j + C_d - 1)\gamma\mu\}}{n!\left[\mu\left(\sigma + \gamma(1-\sigma)\right)\right]^{n}} p(0) = 1$$

$$p(0) = \{1 + \sum_{n=1}^{C_d} \frac{\prod_{i=1}^{n}\{\lambda\sigma + (1-\sigma)(i-1)\gamma\mu\}}{n!\left[\mu\left(\sigma + \gamma(1-\sigma)\right)\right]^{n}} + \sum_{n=C_d+1}^{C} \frac{\left[\prod_{i=1}^{C_d}\{\lambda\sigma + (1-\sigma)(i-1)\mu\gamma\}\right] * \prod_{j=1}^{n-C_d}\{\lambda_h\sigma + (1-\sigma)(j + C_d - 1)\gamma\mu\}}{n!\left[\mu\left(\sigma + \gamma(1-\sigma)\right)\right]^{n}}\}^{-1}$$

5. Derivation of Lagrange Coefficients

To get expressions in terms of inter arrival time and squared coefficient of variance λ, C^2_a and inter service time and squared coefficient of variance μ and C^2_s;$\delta = 2/ C^2_a+1$ and $\gamma=2/ C^2_s+1$
is substituted in eq. (1), (2), (3) and (4), and Lagrange coefficients are deduced, the equations can be than extended to C servers as above.

$$g = \frac{\rho C_s^2 + \rho}{C_s^2 + C_a^2}$$

$$x = \frac{\rho C_s^2 + \rho + (n-1)C_a^2 - n + 1}{n * (C_s^2 + C_a^2)}\ where\ 1 \le n \le C_d$$

$$\mathrm{x} = \frac{(\frac{\lambda_h}{\mu})C_s^2 + (\frac{\lambda_h}{\mu}) + (n-1)C_a^2 - n + 1}{n*(C_s^2 + C_a^2)}\ \text{where}\ \mathrm{C_d} < n < C$$

$$y = \frac{(\frac{\lambda_h}{\mu})C_s^2 + \rho + (c-1)C_a^2 - c + 1}{c * (C_s^2 + C_a^2)}$$

Hence we can write that

$$p(n) = gp(0) for\ n = 1$$
$$p(n) = gx^{n-1}p(0) for\ 1 < n < c$$
$$p(n) = gx^{c-1}yp(0) for\ n = c$$

Since sum of all probabilities equal to 1, therefore p(0) can be written as

$$p(0) = [1 + \left(g * \sum_{n=1}^{c-1} x^n\right) + g * x^{c-1} * y]^{-1}$$

6. Numerical Results

In this section numerical experiments are carried out. These experiments illustrate the credibility of solution presented and also assess the impact of different traffic patterns on QoS.

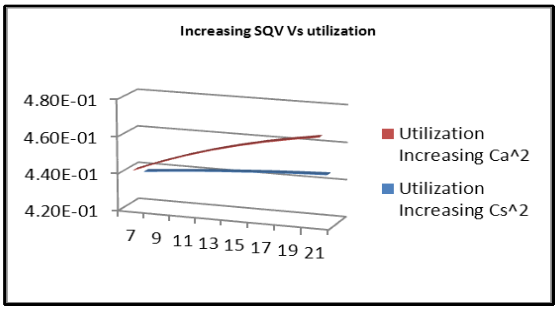

Fig. 3

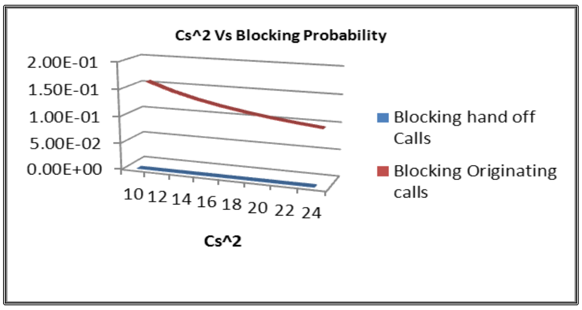

Fig. 4

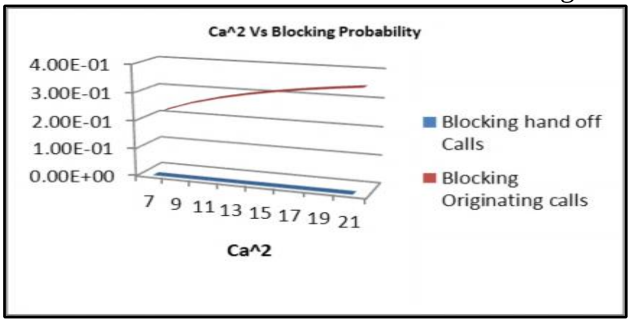

Fig. 5

Blocking of packets/calls decreases with the increase in Squared Coefficient of variance of Inter Service time while increases with increase in Squared Coefficient of variance of Inter Service time, which is obvious from fig 4 and 5. The system is a -priority loss system, where priority is given to handoff voice calls. The blocking probability of originating calls is higher than handoff voice calls as there are dedicated channels for handoff voice calls.

Utilization is the time server is busy, given by 1-p(0) where p(0) is the probability of system being idle. For higher SCV for inter arrival time, utilization is higher and vice versa for SCV of inter service time as shown in fig 3.

7. Conclusions

In this paper maximum entropy formalism of censored priority loss system is carried out, where the main objective was to determine analytically Lagrange coefficients for entropy formalism, previously deduce for single server systems by asymptotic connections with queues of infinite capacity. This method can be used to model traffic behavior of systems where the arrival and departure process follows Batch Poisson process, systems with general inter arrival and inter service time distribution etc.

8. Future Work

The methodology will be used to investigate priority systems, systems with heterogeneous servers; Servers where the traffic follows batch Poisson arrivals and departures. In [12, P249-260] principle of Maximum Relative Entropy (MRE) is used as a method of inference to the problem of estimating the finite buffer stationary queue length distribution (qld), given, as a prior estimate, the qld of corresponding infinite buffer queue. The MRE method in [12] has shown to yield exact results for the $M^G/G/1/N$, $Geo^G/G/1/N$ and $GI^G/D/1/N$, the approach adopted in this paper can be used to investigate GE, BPP and other queues.

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