

# An Improved Proportionate Normalized Least Mean Square Algorithm with Orthogonal Correction Factors for Echo Cancellation

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**Abstract.** Recently, the proportionate normalized least mean square (PNLMS) algorithm was developed in the context of network echo cancellation to get fast convergence rate for the sparse impulse response. Unfortunately, the PNLMS converges much slower than the normalized least mean square (NLMS) algorithm when the impulse response becomes dispersive. Thus, the improved PNLMS (IPNLMS) algorithm was developed which is less sensitive to the sparseness of the echo path and outperforms both the NLMS and PNLMS. In this paper, we propose an improved proportionate NLMS algorithm with orthogonal correction factors. The proposed algorithm extends the concept of the IPNLMS algorithm to NLMS with orthogonal correction factors (NLMS-OCF) algorithm and it realizes the generalized IPNLMS algorithm. Experimental results show that the proposed algorithm performs better than the NLMS-OCF and proportionate NLMS-OCF algorithm whatever the nature of the impulse response is.

**Keywords:** Adaptive filter, improved proportionate, NLMS-OCF, echo cancellation.

## 1. Introduction

In the network echo cancellation scheme, an adaptive filter plays an important role on identifying the echo path, and many adaptive filtering algorithms are developed to improve the performance of the filter [1]. Recently, proportionate normalized least mean square (PNLMS) algorithm has been proposed to achieve fast convergence rate when the impulse response is sparse [2]. The idea of PNLMS is to update each coefficient of the filter by adjusting the step sizes in proportion to the estimated filter coefficients. From this concept, the generalization of PNLMS algorithm is realized by applying the structure of PNLMS algorithm to NLMS-OCF [3] which is proposed to solve the problem of slow convergence rate of the NLMS, and it is referred as proportionate NLMS-OCF (PNLMS-OCF) algorithm [4]. The PNLMS and PNLMS-OCF algorithms are designed to perform well in the sparse impulse response. However, when the impulse response is dispersive, they converge slower than the NLMS and NLMS-OCF, respectively. It means that the rule of adjusting the step sizes of proportionate-type algorithms is far from optimal. To overcome this problem, an improved PNLMS (IPNLMS) algorithm is suggested [5]. It presents more optimal way of determining the step sizes and shows better performance than the NLMS and PNLMS regardless of the nature of the impulse response.

In this paper, the improved proportionate NLMS-OCF algorithm (IPNLMS-OCF) is proposed. The proposed algorithm adopts the idea of the IPNLMS to the NLMS-OCF algorithm, so the generalization of IPNLMS algorithm can be established. The method of developing IPNLMS-OCF is similar to that of the PNLMS-OCF, so the PNLMS-OCF is introduced in section 2 to describe the structure of NLMS-OCF algorithm in the case of the “proportionate” idea is applied. Experimental results show that the proposed algorithm achieves fast convergence rate than other existing algorithms whatever the nature of the impulse response is.

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## 2. Proportionate Normalized Least Mean Square Algorithm with Orthogonal Correction Factors

In this section, the proportionate NLMS-OCF (PNLMS-OCF) algorithm used for echo cancellation is briefly explained. Fig. 1 shows structure of echo canceller, and the following notations are used.

- $x_n$ : Far-end signal (input signal)
- $y_n$ : Echo and background noise (desired signal)
- $\mathbf{x}_n = [x_n, x_{n-1}, \dots, x_{n-L+1}]^T$ : Excitation vector (input vector)
- $\mathbf{h} = [h_0, h_1, \dots, h_{L-1}]^T$ : True echo path (impulse response)
- $\hat{\mathbf{h}}_n = [\hat{h}_{0,n}, \hat{h}_{1,n}, \dots, \hat{h}_{L-1,n}]^T$ : Estimated echo path (estimated weight vector)

where  $L$  is the length of the echo path, and  $n$  is the time index.

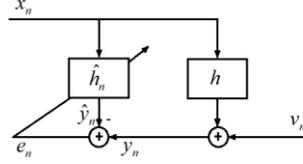


Fig. 1: Structure of echo canceller.

The echo canceller filters  $x_n$  by an echo-path estimate  $\hat{h}_n$ , to obtain an echo estimate  $\hat{y}_n$ . If  $\hat{h}_n$  estimates  $h$  well, then  $\hat{y}_n$  can cancel the echo portion of  $y_n$  so that the return signal (or error signal)  $e_n$  is about equal to the near-end signal  $v_n$ .

Proportionate-type algorithms such as PNLMS and PNLMS-OCF have been proposed [2],[4]. These algorithms are designed to apply an adaptive individual step size to each coefficients of the filter. The step sizes are calculated from the last estimate of the filter coefficients in such a way that a larger coefficient receives a larger increment, thus increasing the convergence rate of that coefficient. It means that active coefficients are adjusted faster than non-active coefficients, so the algorithms show fast convergence rate for sparse impulse responses (i.e., responses for which only a small percentage of coefficients is significant). Most impulse responses in the telephone network have this characteristic, so it is very useful in echo cancellation scheme.

The weight update equation of the PNLMS-OCF is as follows:

$$\hat{\mathbf{h}}_n = \hat{\mathbf{h}}_{n-1} + \mu_0 \mathbf{x}_n + \mu_1 \mathbf{x}_n^1 + \dots + \mu_M \mathbf{x}_n^M \quad (1)$$

where  $\mathbf{x}_n^k$  ( $k = 1, 2, \dots, M$ ) are the components of  $\mathbf{x}_{n-kD}$  which is called orthogonal correction factors (OCFs) because they are chosen to be orthogonal to each other,  $D$  is the delay between input vectors used for updates. The corresponding step  $\mu_k$  ( $k = 0, 1, \dots, M$ ) is calculated according to

$$\mu_k = \begin{cases} \frac{\bar{\mu} \mathbf{G}_{n-1} e_n}{\mathbf{x}_n^T \mathbf{G}_{n-1} \mathbf{x}_n + \delta_{PN}} & \text{for } k = 0, \\ \frac{\bar{\mu} \mathbf{G}_{n-1} e_n^k}{\mathbf{x}_n^{kT} \mathbf{G}_{n-1} \mathbf{x}_n^k + \delta_{PN}} & \text{for } k = 1, 2, \dots, M, \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where  $\bar{\mu}$  ( $0 < \bar{\mu} < 2$ ) is the adaptation step,  $\delta_{PN}$  is a regularization parameter, and

$$e_n = y_n - \mathbf{x}_n^T \hat{\mathbf{h}}_{n-1} \quad (3)$$

$$e_n^k = y_{n-kD} - \mathbf{x}_{n-kD}^T \hat{\mathbf{h}}_{n-1}^k \quad (4)$$

$$\hat{\mathbf{h}}_{n-1}^k = \hat{\mathbf{h}}_{n-1} + \mu_0 \mathbf{x}_n + \mu_1 \mathbf{x}_n^1 + \dots + \mu_{k-1} \mathbf{x}_n^{k-1} \quad (5)$$

$$\mathbf{G}_{n-1} = \text{diag}\{g_{0,n-1}, \dots, g_{L-1,n-1}\}. \quad (6)$$

The components of a diagonal matrix  $\mathbf{G}_{n-1}$  adjust the step sizes of the individual taps of the filter and they are determined as follows:

$$\gamma_{l,n} = \max\{\rho \max[\delta, |\hat{h}_{0,n}|, \dots, |\hat{h}_{L-1,n}|], |\hat{h}_{l,n}|\}, \quad (7)$$

$$g_{l,n} = \frac{\gamma_{l,n}}{\sum_{i=0}^{L-1} \gamma_{i,n}}, \quad 0 \leq l \leq L-1, \quad (8)$$

where parameter  $\delta$  and  $\rho$  are positive numbers with typical values  $\delta = 0.01$  and  $\rho = 5/L$ . The parameter  $\rho$  prevents  $\hat{h}_{l,n}$  from stalling when it is much smaller than the largest coefficient and  $\delta$  regularizes the updating when all coefficients are zero at initialization.

### 3. Improved Proportionate Normalized Least Mean Square Algorithm with Orthogonal Correction Factors

In this section, an IPNLMS-OCF algorithm is proposed. For dispersive impulse response, the convergence rate of the PNLMS-OCF algorithm is slower than that of the NLMS-OCF algorithm. Thus, equation (8) has to be modified to overcome this problem.

Recently, an IPNLMS algorithm has been proposed which uses modified rule of adjusting the step sizes of the individual taps of the filter. This algorithm has similar weight update equation as the PNLMS algorithm, but adopts the modified rule of adjusting the step sizes. Based on this concept, the objective of the IPNLMS-OCF algorithm is applying the idea that was firstly introduced in the IPNLMS algorithm to NLMS-OCF algorithm.

The weight update equation of the IPNLMS-OCF is given by

$$\hat{\mathbf{h}}_n = \hat{\mathbf{h}}_{n-1} + \mu_0 \mathbf{x}_n + \mu_1 \mathbf{x}_n^1 + \dots + \mu_M \mathbf{x}_n^M \quad (9)$$

where  $\mathbf{x}_n^k$  ( $k = 1, 2, \dots, M$ ) are the OCFs. The corresponding step  $\mu_k$  ( $k = 0, 1, \dots, M$ ) is calculated according to

$$\mu_k = \begin{cases} \frac{\bar{\mu} \mathbf{G}_{n-1} \mathbf{e}_n}{\mathbf{x}_n^T \mathbf{G}_{n-1} \mathbf{x}_n + \delta_{IPN}} & \text{for } k = 0, \\ \frac{\bar{\mu} \mathbf{G}_{n-1} \mathbf{e}_n^k}{\mathbf{x}_n^{kT} \mathbf{G}_{n-1} \mathbf{x}_n^k + \delta_{IPN}} & \text{for } k = 1, 2, \dots, M, \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

where  $\bar{\mu}$  ( $0 < \bar{\mu} < 2$ ) is the adaptation step,  $\delta_{IPN}$  is a regularization parameter of the IPNLMS-OCF, and the other parameters used in equation (10) is according to equation (3)-(6). When the number of OCFs,  $M$ , is set to 0, equation (9) is identical to the weight update equation of the IPNLMS, so the IPNLMS-OCF algorithm can be said to be a generalized IPNLMS algorithm. The components of a diagonal matrix  $\mathbf{G}_{n-1}$  are determined as follows [5]:

$$g_{l,n} = \frac{1-\alpha}{2L} + (1+\alpha) \frac{|\hat{h}_{l,n}|}{2 \|\hat{\mathbf{h}}_n\|_1 + \epsilon}, \quad 0 \leq l \leq L-1, \quad (11)$$

where  $\epsilon$  is a small positive number, and

$$\|\hat{\mathbf{h}}_n\|_1 = \sum_{l=0}^{L-1} |\hat{h}_{l,n}|. \quad (12)$$

The parameter  $\alpha$  is a positive number with range between -1 and 1. For  $\alpha = -1$ , it can be easily noticed that the IPNLMS-OCF algorithm and NLMS-OCF algorithm are identical. For  $\alpha$  goes close to 1, the IPNLMS-OCF algorithm behaves like the PNLMS-OCF algorithm. The good choices for  $\alpha$  are 0 or -0.5 in practice. With proper choice on parameter  $\alpha$ , IPNLMS-OCF algorithm always behaves better than NLMS-OCF and PNLMS-OCF, whatever the impulse response is.

## 4. Experimental Results

In this section, we compare the NLMS-OCF, PNLMS-OCF, IPNLMS, and proposed IPNLMS-OCF algorithm in the context of a network echo canceller. The echo path  $\mathbf{h}$  and its estimate  $\hat{\mathbf{h}}_n$  are assumed to have the same length  $L=1024$ . The input signal  $x_n$  is either a white Gaussian random sequence through the system

$$G(z) = \frac{1}{1 - 0.9z^{-1}}$$

or a speech signal that is sampled at 8kHz, and the signal-to-noise ratio (SNR) is set to 30dB. The parameter values for each algorithm are set to  $\delta = 0.01$ ,  $\rho = 5/L$  (PNLMS-OCF),  $\alpha = -0.5$ ,  $\epsilon = 0.001$  (IPNLMS, IPNLMS-OCF), and the regularization parameter of each algorithm is set to  $\delta_N = \sigma_x^2$ ,  $\delta_{PN} = \delta_N / L$ , and  $\delta_{IPN} = \delta_N / 2L$  where  $\delta_N$  is a regularization parameter used for the NLMS-OCF and  $\delta_{IPN}$  is also used for both IPNLMS and IPNLMS-OCF. The mean square deviation (MSD) is used as the performance indicator.

Figure 2 shows the MSD learning curves of the IPNLMS and IPNLMS-OCF algorithm for dispersive impulse response. We choose  $\bar{\mu} = 1$  for two algorithms,  $M = 0, 1, 2, 3$  and  $D = 20$  for IPNLMS-OCF. The value of  $D$  is set to clearly show the improvement of the convergence rate. The input signal is a white Gaussian through channel  $G(z)$ . We can notice that convergence rate of the IPNLMS-OCF algorithm becomes faster as the number of OCFs and the delay increase, and the IPNLMS-OCF with  $M = 0$  is identical to the IPNLMS. Thus, the IPNLMS-OCF algorithm generalizes the IPNLMS, and it can control the convergence rate by adjusting the number of OCFs or delay.

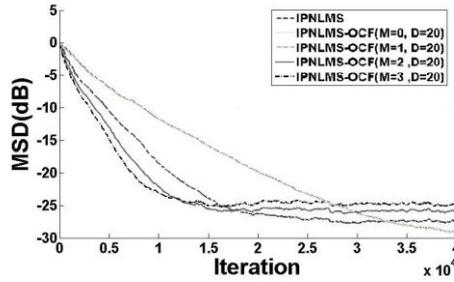


Fig. 3: MSD learning curves of the IPNLMS and IPNLMS-OCF algorithm (dispersive impulse response, input sequence: white noise through a system  $G(z)$ )

Figures 3-4 show the MSD learning curves of the NLMS-OCF, PNLMS-OCF, and IPNLMS-OCF for sparse impulse response and dispersive impulse response, respectively. We choose  $\bar{\mu} = 1$ ,  $M = 3$ , and  $D = 1$  for all algorithms. The input signal is a white Gaussian through channel  $G(z)$  in these cases. We can see that the convergence rate of IPNLMS-OCF is similar to that of the PNLMS-OCF, and faster than that of the conventional NLMS-OCF for sparse impulse response. For dispersive impulse response, the convergence rate of the IPNLMS-OCF is the fastest among three algorithms, where the PNLMS-OCF converges slowly than the NLMS-OCF. The IPNLMS-OCF algorithm achieves fast convergence rate whatever the impulse response is.

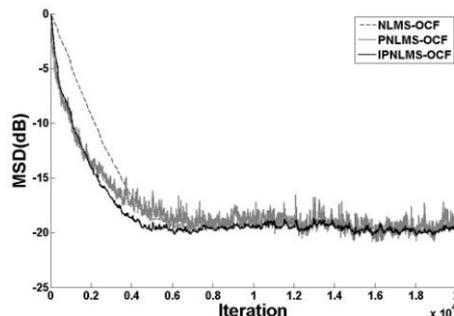


Fig. 3: MSD learning curves of the NLMS-OCF, PNLMS-OCF, and IPNLMS-OCF algorithm (sparse impulse response, input sequence: white noise through a system  $G(z)$ )

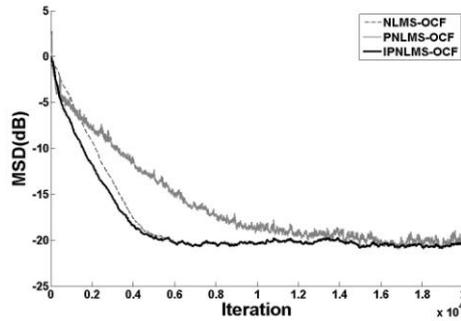


Fig. 4: MSD learning curves of the NLMS-OCF, PNLMS-OCF, and IPNLMS-OCF algorithm (dispersive impulse response, input sequence: white noise through a system  $G(z)$ )

Figure 5 shows the MSD learning curves of NLMS-OCF, PNLMS-OCF, and IPNLMS-OCF for dispersive impulse response when the input signal is a speech. We choose  $\bar{\mu} = 1$ ,  $M = 23$ , and  $D = 24$  for all algorithms. The convergence rate of the IPNLMS-OCF is better than that of the PNLMS-OCF and the NLMS-OCF.

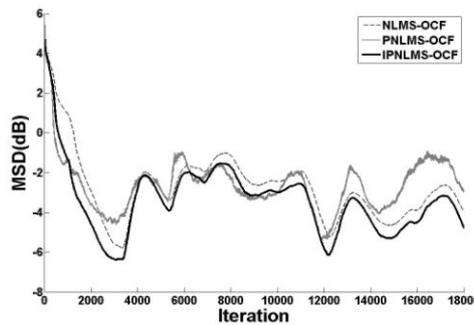


Fig. 5: MSD learning curves of the NLMS-OCF, PNLMS-OCF, and IPNLMS-OCF algorithm (dispersive impulse response, speech input signal)

## 5. Conclusions

In this paper, we proposed IPNLMS-OCF algorithm which adopted the idea of the IPNLMS to the NLMS-OCF. The experimental results showed that the IPNLMS-OCF algorithm was established as a generalized IPNLMS algorithm and it outperformed the conventional NLMS-OCF algorithm and PNLMS-OCF algorithm regardless of the nature of the impulse response.

## 6. Acknowledgements

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