2012 2nd International Conference on Computer Design and Engineering (ICCDE 2012)
IPCSIT vol. 49 (2012) © (2012) IACSIT Press, Singapore
DOI: 10.7763/IPCSIT.2012.V49.17

An Improved Ranking Method for Fuzzy Numbers Using Left and Right Indices

Luu Quoc Dat [+], Vincent F. Yu and Shuo-Yan Chou

Department of Industrial Management, National Taiwan University of Science and Technology
43, Section 4, Keelung Road, Taipei 10607, Taiwan

Abstract. Although many different methods for ranking fuzzy numbers have been proposed in the last few decades, there lack efficient methods that are applicable to a wide variety of situations. Some of the existing methods even have shortcomings or difficulty in calculation. This paper proposes a novel approach based on the left and right indices for ranking fuzzy numbers. The main idea of the proposed approach is to obtain the difference between left and right relative values using different decision levels, which are based on gama-cut level sets. To differentiate symmetric fuzzy numbers efficiency, the proposed method takes into account the decision maker's optimistic attitude of fuzzy numbers. Comparative examples are presented to demonstrate the usages and advantages of the proposed ranking approach. The results show that the proposed method has many comparative advantages, but requires much simpler calculations. Moreover, using the proposed ranking approach, different types of fuzzy numbers can be ranked effectively and efficiently.

Keywords: Ranking fuzzy numbers, Fuzzy numbers, Left and right indices, Decision level.

1. Introduction

Ranking fuzzy numbers plays a very important role in decision analysis under fuzzy environment. Numerous approaches have been investigated for ranking fuzzy numbers in the literature [1-14]. A comparison of some existing approaches can be found in Chou et al. [7]. Chen [5] proposed an approach for ranking fuzzy numbers by using maximizing set and minimizing set. Liou and Wang [11] presented a ranking approach based on integral value index to overcome the shortcomings of Chen's [5] approach. Chu and Tsao [8] proposed an approach for ranking fuzzy numbers with the area between the centroid point and original point. Abbasbandy and Asady [1] introduced an approach to rank fuzzy numbers by sign distance. Wang and Luo [13] proposed an area ranking of fuzzy numbers based on positive and negative ideal points. Kumar et al. [10] proposed new approach for ranking of *L-R* type generalized fuzzy numbers. Ezzati et al. [9] proposed an approach for ranking symmetric fuzzy number based on the new magnitude concepts. Some of these existing approaches, however, have limitations, are difficult in calculation, or they are non-intuitive, can't differentiate symmetric fuzzy numbers, which makes them inefficient in decision process.

To overcome the aforementioned problems, this paper proposes a novel approach for ranking fuzzy numbers based on the concept of left and right indices that is a subtraction of the left relative values from the right relative values using different decision levels gama (γ). In order to rank symmetric fuzzy numbers efficiency, the proposed approach takes into account the decision maker's optimistic attitude (α) of fuzzy numbers. The left and right relative values are used to reflect the decision maker's pessimistic and optimistic viewpoint, respectively. This ranking approach can rank different types of fuzzy numbers effectively and efficiently.

2. Fuzzy Numbers

[+] Corresponding Author. Tel.: +886-2-2737-6333; Fax: +886-2-2737-6344.
E-mail: luuquocdat_84@yahoo.com (L-Q Dat)

A fuzzy number $A=(a,b,c,d;\varpi)$ is a trapezoidal fuzzy number if its membership function has the following form [7, 12]:

$$f_A(x)=\begin{cases}\omega(x-a)/(b-a), & a\le x\le b,\\ \varpi, & b\le x\le c,\\ \omega(x-d)/(c-d), & c\le x\le d,\\ 0, & otherwise,\end{cases} \tag{1}$$

where $\omega(x-a)/(b-a)$ and $\omega(x-d)/(c-d)$ are the left and right membership functions of A, respectively. If $\varpi=1$, then A is a normal trapezoidal fuzzy number and it denoted as $A=(a,b,c,d;1)$. In particular, if $b=c$, then A is a triangular fuzzy number and is denoted as $A=(a,b,\ d;\omega)$ or $A=(a,b,\ d;1)$, if $\omega=1$.

For a fuzzy set A on the real numbers R, the support of A is defined as $S(A)=\{x\in R\mid f_A(x)\succ 0\}$.

3. The Proposed Approach for Ranking Fuzzy Numbers

In this section, firstly, we define the left and right indices of fuzzy numbers. Then, the subtractions of the left relative values from the right relative values are considered based on different decision levels γ associated with the decision maker's optimism attitude α of fuzzy numbers. The proposed ranking approach is described as follows.

Consider n fuzzy numbers $A_i, i=1,2,\ldots,n,$ each with a membership function $f_{A_i}(x)$.

$$f_{A_i}(x)=\begin{cases}f_{A_i}^L(x), & a_i\le x\le b_i,\\ \omega, & b_i\le x\le c_i,\\ f_{A_i}^R(x), & c_i\le x\le d_i,\\ 0, & otherwise,\end{cases} \tag{2}$$

The left and right indices refer to the intersection of the left and the right membership functions of fuzzy numbers A_i with different decision levels γ. A larger γ indicates a higher-level decision. More specifically, if γ is close to one, the pertaining decision is called a "high level decision", in which case only parts of the two fuzzy numbers, with membership values between γ and "1", will be compared [4]. Likewise, if γ is close to zero, the pertaining decision is referred to as "low level decision", since members with membership values lower than both the fuzzy numbers are involve in the comparison. Fig. 1 illustrates the mentioned notion graphically.

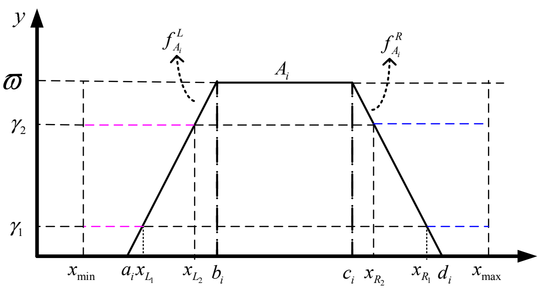

Fig. 1: The left and right indices for fuzzy number A_i.

where x_L and x_R are left and right indices, respectively. Generally, the left and right indices values are defined as

$$x_L(A_i)=f_{A_i^L}(x)\wedge\gamma,\ i=1,2,...,n,\qquad 0\prec\gamma\prec\omega \tag{3}$$

$$x_R(A_i)=f_{A_i^R}(x)\wedge\gamma,\ i=1,2,...,n,\qquad 0\prec\gamma\prec\omega \tag{4}$$

Then, the subtraction of left relative values from right relative values of each fuzzy number A_i with index of optimism α is then defined as follow:

$$D_\alpha^\gamma(A_i)=\alpha\left[x_L(A_i)-x_{\min}\right]-(1-\alpha)\left[x_{\max}-x_R(A_i)\right] \tag{5}$$

where $x_{\min}=\inf S, x_{\max}=\sup S, S=U_{i=1}^n S_i, S_i=\{x/f_{A_i}(x)\succ 0\}$.

Obviously, the fuzzy number A_i is larger if the right relative values, $x_{\max} - x_R(A_i)$, is smaller and the left relative values, $x_L(A_i) - x_{\min}$, is larger. Therefore, for any two fuzzy numbers A_i and A_j, if $D_\alpha^\gamma(A_i) < D_\alpha^\gamma(A_j)$, then $A_i \prec A_j$. If $D_\alpha^\gamma(A_i) > D_\alpha^\gamma(A_j)$, then $A_i > A_j$. Finally, if $D_\alpha^\gamma(A_i) = D_\alpha^\gamma(A_j)$, then $A_i \sim A_j$. It is clear that if $\gamma \geq \omega$, then $D_\alpha^\gamma(A_i) = 0$.

The index of optimism α represents a decision maker's attitude [7], [11]. A larger α indicates a higher degree of optimism. More specifically, the left and right relative values are used to reflect the decision maker's pessimistic, i.e. $\alpha = 0$, and optimistic viewpoint, i.e. $\alpha = 1$, respectively. In addition, when $\alpha = 0.5$, the subtraction values $D_{0.5}^\gamma(A_i)$ representing a moderate decision maker's viewpoint.

For a moderate decision maker, with $\alpha = 0.5$, the subtraction values of each fuzzy number A_i become

$$D_{0.5}^\gamma(A_i) = 0.5\left[x_L(A_i) - x_{\min}\right] - 0.5\left[x_{\max} - x_R(A_i)\right] \tag{6}$$

A decision maker will rank the fuzzy number A_i with larger $D_\alpha^\gamma(A_i)$ higher.

In the case that A_i is a generalized trapezoidal fuzzy number, i.e. $A_i = (a_i, b_i, c_i, d_i; \omega)$, the left and right indices for fuzzy numbers A_i can be determined by the following equations:

$$x_L(A_i) = \gamma(b_i - a_i)/\omega_i + a_i, \qquad 0 \prec \gamma \prec \omega \tag{7}$$

$$x_R(A_i) = \gamma(c_i - d_i)/\omega_i + d_i, \qquad 0 \prec \gamma \prec \omega \tag{8}$$

The subtraction of left relative values from right relative values of each fuzzy number A_i with index of optimism α can be further written as:

$$D_\alpha^\gamma(A_i) = \alpha\left[\gamma(b_i - a_i)/\omega_i + a_i - x_{\min}\right] - (1-\alpha)\left[x_{\max} - d_i - \gamma(c_i - d_i)/\omega_i\right] \tag{9}$$

Since triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers with $b = c$, the difference between left relative values and right relative values of the triangular fuzzy number A_i can be determined by:

$$D_\alpha^\gamma(A_i) = \alpha\left[\gamma(b_i - a_i)/\omega_i + a_i - x_{\min}\right] - (1-\alpha)\left[x_{\max} - d_i - \gamma(b_i - d_i)/\omega_i\right] \tag{10}$$

Further, this paper defines the order $\geq$ and $\leq$ as: $u \geq v$ if only if $u \succ v$ or $u \prec v, u \leq v$ if and only if $u \prec v$ or $u \leq v$. In other words, the proposed method belongs to the first class of Wang and Kerre's categories [12].

Remark 3.1. For two general fuzzy numbers u and v we have:

$$D_\alpha^\gamma(u+v) = D_\alpha^\gamma(u) + D_\alpha^\gamma(v).$$

Proposition. Let E be the set of fuzzy numbers for which the proposed method can be applied, and X and Y are two finite arbitrary finite subsets of E. The proposed ranking approach satisfies the following reasonable axioms for the ordering approaches [12].

A_1: If $u \in X$, then $u \geq u$.

A_2: If $(u,v) \in X^2, u \geq v$ and $v \geq u$, we should have $u \sim v$.

A_3: If $(u,v,w) \in X^3, u \geq v$ and $v \geq w$, we should have $u \geq w$.

A_4: If $(u,v) \in X^2$ and $\inf supp(u) > \sup supp(v)$, we should have $u \geq v$.

A_5: If $(u,v) \in X^2$ and $\inf supp(u) < \sup supp(v)$, we should have $u < v$.

A_6: Let $(u,v) \in (X \cap Y)^2$, we obtain the ranking order $u \succ v$ by $D_\alpha^\gamma(.)$ on Y if and only if $u \succ v$ by $D_\alpha^\gamma(.)$ on X.

A_7: Let $u, v, u+w$ and $v+w$ be element of E. If $u \geq v$, then $u+w \geq v+w$.

A_8: Let $u, v, u+w$ and $v+w$ be element of E. If $u \succ v$, then $u+w \succ v+w$, when $w \neq 0$.

A_9: Let u, v, uw and vw be element of E and $w \geq 0$. If $u \leq v$, then $uw \leq vw$.

Proof. The proofs of A_1 - A_6 and A_9 are clear. For the proof of A_7, let $u \geq v$ then $D_\alpha^\gamma(u) \geq D_\alpha^\gamma(v)$. By adding $D_\alpha^\gamma(w)$ and using remark 3.1, we have $u+w \geq v+w$. Similarly, A_8 holds.

4. Comparative Examples

To demonstrate the usages and advantages of the proposed approach, three numerical examples are presented in this section.

Example 1. Consider the fuzzy numbers $A_1=(3,6,9;1)$ and $A_2=(5,6,7;1)$ shown in Fig. 2 [13]. Using the approach proposed in this paper, the difference between left relative values and right relative values of fuzzy number A_1 and A_2 with index of optimism α can be obtained as $D_\alpha^\gamma(A_1)=\alpha(3\gamma)-(1-\alpha)3\gamma$ and $D_\alpha^\gamma(A_2)=\alpha(\gamma+2)-(1-\alpha)(2+\gamma)$, respectively. It is observed that for a pessimistic decision maker i.e. $\alpha=0$, we have $A_1\succ A_2$ for every $\gamma\in(0,1)$; for an optimistic decision maker, i.e. $\alpha=1$, we have $A_1\prec A_2$ for every $\gamma\in(0,1)$; and for a moderate decision maker, i.e. $\alpha=0.5$, we have $A_1\sim A_2$, for every $\gamma\in(0,1)$. However, using the ranking approaches provided in the literature [5], [8], the ranking order of fuzzy numbers A_1 and A_2 is always the same, i.e. $A_1\sim A_2$. Obviously, decision makers' attitudes towards risks effect on the ranking order of fuzzy numbers.

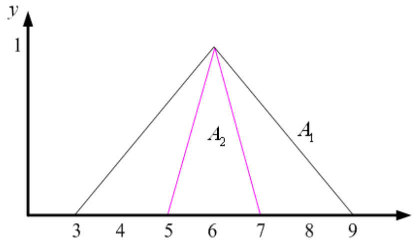

Fig. 2: Fuzzy numbers A_1 and A_2 in example 1

Example 2. Consider three fuzzy numbers $A_1=(5,6,7;1)$, $A_2=(5.9,6,7;1)$ and $A_3=(6,6,7)$, adopted from Chou et al. [8], as shown in Fig. 3. The difference between left relative values and right relative values and the ranking order of A_1, A_2 and A_3 obtained by the proposed approach are shown in Table 1. The results show that when $\alpha=0$, we have $A_1\sim A_2\sim A_3$ for every $\gamma\in(0,1)$. Conversely, when $\alpha=0.5$, or $\alpha=1$, we have $A_3\succ A_2\succ A_1$ for every $\gamma\in(0,1)$ which is similar to the results of other approaches [1-3], [5], [13-14]. However, by Cheng's [6] and Chu and Tsao's [8] approaches the ranking order are $A_1\succ A_2\succ A_3$ and $A_1\succ A_3\succ A_2$, respectively. From Fig. 3, it is easy to verify that the ranking results obtained by Cheng's approach and Chu and Tsao's approach are unreasonable and not consistent with human intuition. Clearly, the proposed ranking approach has consistency result in ranking fuzzy numbers which could not be guaranteed by Cheng's [6] and Chu and Tsao's [8] ranking approaches.

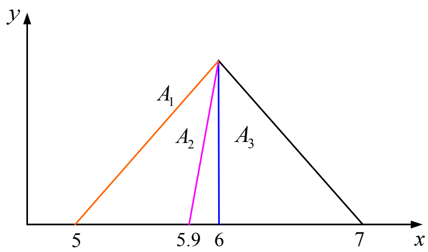

Fig. 3: Fuzzy numbers A_1, A_2 and A_3 in example 2.

Table. 1: Comparative between Fuzzy Numbers A_1, A_2 and A_3 in Example 2.

DM's optimism (α)	$D_\alpha^\gamma(A_1)=\alpha\gamma-(1-\alpha)\gamma$	$D_\alpha^\gamma(A_2)=\alpha(0.1\gamma+0.9)-(1-\alpha)\gamma$	$D_\alpha^\gamma(A_3)=\alpha-(1-\alpha)\gamma$	The proposed method
0	$-\gamma$	$-\gamma$	$-\gamma$	$A_1\sim A_2\sim A_3$
0.5	0	$0.45-0.45\gamma$	$0.5-0.5\gamma$	$A_1\prec A_2\prec A_3$
1	γ	$0.9+0.1\gamma$	1	$A_1\prec A_2\prec A_3$

Example 3. Consider two fuzzy numbers from Liou and Wang [11], i.e., $A_1 = (1,2,5;1)$ and $A_2 = (1,2,2,4;1)$ as shown in Fig. 4. The membership function of A_2 is defined as

$$f_A(x) = \begin{cases} [1-(x-2)^2]^{1/2} & \text{if } x \in [1,2], \\ [1-(x-2)^2/4]^{1/2} & \text{if } x \in [2,4], \\ 0, & \textit{otherwise}, \end{cases}$$

Using our approach, we have

$$D_\alpha^\gamma(A_1) = \alpha(\gamma) - (1-\alpha)(3\gamma), \quad D_\alpha^\gamma(A_2) = \alpha(\sqrt{1-\gamma^2}+2) - (1-\alpha)(2+2\sqrt{1-\gamma^2}).$$

Accordingly, when $\alpha = 0$, we have $A_1 \succ A_2$ for every $\gamma \in (12/13,1)$ and $A_1 \prec A_2$ for every $\gamma \in (0,12/13)$; when $\alpha = 0.5$, we have $A_1 \succ A_2$ for every $\gamma \in (0,1/\sqrt{5})$, $A_1 \prec A_2$ for every $\gamma \in (1/\sqrt{5},1)$, and $A_1 \sim A_2$ for $\gamma = 1/\sqrt{5}$; when $\alpha = 1$, the ranking order is $A_1 \prec A_2$ for every $\gamma \in (0,1)$. Again, decision makers' attitudes towards risks effect on the ranking order of fuzzy numbers. This example reveals that the proposed ranking approach can also rank fuzzy numbers other than triangular and trapezoidal ones.

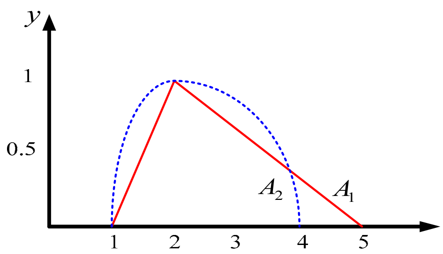

Fig. 4: Fuzzy numbers A_1 and A_2 in example 3.

5. Conclusion

This paper proposed a novel approach for comparing and ranking fuzzy numbers based on left and right indices of fuzzy numbers. To differentiate symmetric fuzzy numbers efficiency, the proposed approach took into account the decision maker's optimistic attitude of fuzzy numbers. Several comparative examples were given to demonstrate the usage, applicability, and advantages of the proposed ranking approach. Comparing with other existing approaches, the proposed approach is more reasonable and efficient. Further, the proposed ranking approach is simple, and can be used for all types of fuzzy numbers, whether they are normal or abnormal, triangular or trapezoidal, and general.

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