

A NSGA-II algorithm to solve a bi-objective optimization of the redundancy allocation problem for series-parallel systems

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Abstract. This paper involves developing a new model for the redundancy allocation problem of series-parallel systems. The system consists of subsystem in series, where components are used in parallel for each subsystem. There are various types of components as candidates for allocation. These components that are chosen from a list that is available in the market are characterized by their cost, weight, and reliability. Besides, they have various levels of performance ranging from functioning perfectly to fail completely. The goal is to find the optimal combination of the components of each subsystem in order to maximize the system reliability and to minimize the cost of purchasing components. A NSGA-II algorithm is developed and proposed to solve the bi-objective problem, where the universal generating function for multi-state components is used to obtain the reliability of a given system. In addition, a penalty function that encourages the solution algorithm to explore within infeasible solutions is proposed. At the end, a numerical example is used to validate the solution and to assess the performance of the proposed methodology under different configurations.

Keywords: Availability; Redundancy allocation problem; Series-parallel systems; Multi-state; Universal Generating Function

1. Introduction

The reliability of a system is directly related to the reliabilities of its components. One of the best and the most efficient methods to improve the system reliability is to use redundant components beside the main components [1]. The redundancy allocation problem (RAP) with many applications in industries is a complex combinational optimization problem that is shown to be NP-hard [2]. Electronic systems, power stations, and manufacturing systems are some examples of the RAP applications [3].

A series-parallel system consists of some subsystems that work in series each containing some components that are arranged in parallel. The RAP of these system involves obtaining an optimal number of components in each subsystem such that the total system availability is maximized while the constraint(s) (usually the system weight) is (are) satisfied.

Component performances of real-world systems are classified into more than the usually assumed two states of "completely working" and "completely failed," resulting in multi-state component systems (MSS), instead of binary systems. In a MSS, both the system and its components are allowed to experience more than two possible states, e.g., completely working, partially working, partially failed, and completely failed [4]. Since the number of MSS states increases very rapidly with increase in the number of its elements, the universal generating function (UGF) was firstly introduced in [5] to reduce the computational complexity involved in evaluating the MSS reliability [6].

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Many researchers using different approaches and techniques studied RAP for series-parallel systems with binary-state components. The approaches were all proposed to solve single-objective (generally maximization of system reliability) RAPs subject to several constraints such as cost, weight, and volume. Tillman et al. [7] presented a review of the current redundancy allocation and reliability literature. Ouzineb et al. [8] developed an efficient Tabu search to solve the binary RAP series-parallel systems to maximize system reliability. However, in many real-world optimization of the RAP, simultaneous optimization of more than one objective function is required. For instance, minimizing total system cost in addition to maximizing the total system reliability might be a case that is investigated in the current research.

In this paper, a new constrained bi-objective model of a multi-state series-parallel redundancy allocation problem is presented and solved. The constraints are the system weight and the number of components used. In the proposed formulation, UGF is employed to evaluate the reliability of each solution. The goal is to find the optimal combination of the components of each subsystem in order to maximize the system reliability and to minimize the cost of purchasing components. Furthermore, a NSGA-II algorithm is developed to solve the problem.

2. Problem Formulation

The problem assumptions, notations, and the model development come in this section as follow.

2.1. Assumptions

The assumptions are:

- Performance state, weight and cost of all components and number of subsystems are determined and known.
- There are several types of component as candidate for allocation in each subsystem.
- All components of a subsystem are selected from a similar type (brand).
- The components are chosen from a list that is available in the market.
- All allocations are active and switching between components is performed perfectly.
- System components are multi-state.
- Minimum and maximum number of allocation for each subsystem is determined and known.

2.2. Notations

The notations that are used in the rest of the paper are as follows:

s :	The number of subsystems in series
n_i :	Total number of component types available for subsystem i
c_{ij} :	Cost of the j -th component type in the i -th subsystem
x_{ij} :	Number of the j -th component type used for the i -th subsystem ($j = 1, 2, \dots, n_i$)
$C(X)$:	Total system cost
$R(X)$:	Total system reliability
w_{ij} :	Weight of the j -th component type in the i -th subsystem
W :	Total system weight
L_i :	Minimum number of components that can be used in parallel for the i -th subsystem
U_i :	Maximum number of components that can be used in parallel for the i -th subsystem

2.3. The mathematical model

Based on the assumptions and notations, the mathematical formulation of the series-parallel system can be expressed as

$$\begin{aligned} \text{Min } C(X) &= \sum_{i=1}^s \sum_{j=1}^{n_i} c_{ij} x_{ij} \\ \text{Max } R(X) & \end{aligned}$$

Subject to:

$$\sum_{i=1}^s \sum_{j=1}^{n_i} w_{ij} x_{ij} \leq W$$

$$L_i \leq x_{ij} \leq U_i \quad , \text{ for } j = k \quad \forall i = 1, 2, \dots, s; j = 1, 2, \dots, n_i$$

$$x_{ij} = 0 \quad , \text{ for } j \neq k \quad \forall i = 1, 2, \dots, s; j = 1, 2, \dots, n_i$$

$$x_{ij} \in \{0, 1, 2, \dots, U_i\}$$

The objective functions are minimization of system cost and maximization of system reliability, where the first constraint implies that the total weight of the designed series-parallel system should be less than W . The second constraint is related to maximum and minimum number of component that is allocated to a subsystem. The third constraint shows this assumption that all components of a subsystem are selected from a similar type. It means when $x_{ij} = 0$, then there is no allocation for the i -th subsystem of the j -th component.

3. UGF and NSGA-II algorithm

Similar to other RAP problems, the bi-objective optimization problem that has been formulated above is NP-hard. In this section a NSGA-II algorithm in which the UGF approach is used to determine the reliability of a system based on its components reliability is developed.

3.1. UGF

In UGF, the probability distribution function of each element is first evaluated using the $u(z)$ function given in equation (1). Then, to evaluate the probability distribution function of MSS, equation (2) is utilized for a pair of components connected in parallel and equation (3) is employed for a pair of components connected in series.

$$u(z) = \sum_{i=1}^I p_i z^{g_i} \quad (1)$$

$$\pi(u_1(z), u_2(z)) = \pi\left(\sum_{i=1}^{k_1} p_i z^{a_i}, \sum_{j=1}^{k_2} q_j z^{b_j}\right) = \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} p_i q_j z^{a_i + b_j} \quad (2)$$

$$\pi(u_1(z), u_2(z)) = \pi\left(\sum_{i=1}^{k_1} p_i z^{a_i}, \sum_{j=1}^{k_2} q_j z^{b_j}\right) = \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} p_i q_j z^{\min\{a_i, b_j\}} \quad (3)$$

where the parameters I , k_1 and k_2 are numbers of possible performance levels for components; g_i , a_i and b_j are physically interpreted as the performance levels of these components; while p_i and q_j are steady-state probabilities of the different performance levels for the components. At the end, the reliability of the entire system is evaluated based on the minimum measure of demand.

3.2. NSGA-II

The steps involved in the developed NSGA-II algorithm are

1. Initialization of parameters and the first population generation of n chromosomes
2. Fitness evaluation of chromosomes
3. Fast non-dominated sorting and crowding distance
4. Selecting parents based on the crowded tournament selection operator
5. Applying continuous uniform crossover and swap mutation
6. Evaluation of offspring and combining parent and offspring population
7. Evaluation of fast non-dominated sort and crowding distance again for solutions
8. Sorting population based on crowding distance and selecting N individual for the new population
9. When a maximum number of generation is reached, stop or return to step 4

In order to achieve better convergence of NSGA-II algorithm to a good solution, a local search heuristic procedure is proposed in this paper. In this procedure, for each type of component, we first generate a integer number between L_i and U_i , randomly. Then, based on the generated numbers, the reliabilities of the

corresponding subsystems are calculated. Next, the j -th component type with the maximum reliability is selected.

An adaptive penalty function $P(x)$ is also used to penalize infeasible solutions for weight constraint. This function is defined as

$$P(x) = \text{Min} \left(1, \frac{W}{W(x)} \right)^\lambda \quad (4)$$

Then, the fitness functions are penalized by

$$F(x) = R(x) \times P(x) \quad (5)$$

$$F(x) = C(x) / P(x) \quad (6)$$

Where $F(x)$ is the fitness of solution x , $R(x)$ and $C(x)$ are the overall reliability and overall cost of the solution x , respectively. Also λ is the penalty factor.

4. Numerical example

One example is considered that consists of three main units connected in series. For each unit, there are several components available. Each component of the system can have different levels of performance, which range from maximum capacity to total failure (Table 1). NSGA-II was employed based on a population size of 100 and 30 generations. Also maximum total weight of system is 700 and minimum measure of demand is 1. The computation time was 162 seconds. Fig. 1 shows 22 solutions found in the Pareto front.

5. Conclusion

In this paper, a series-parallel redundancy allocation problem with multi-state components was considered. The objective functions were the maximization of the total system reliability and the minimization of the total system cost. The components were characterized to have different performance levels, costs, weights, and reliabilities. The UGF approach was employed to evaluate the reliability of each solution to reduce computational complexity of the MSS. In order to achieve better convergence of NSGA-II algorithm to a good solution, a local search heuristic procedure was proposed. The solution to the multi-objective problem is a set of solutions, known as the Pareto-front, from which the analyst may choose one solution for system implementation.

6. References

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Table 1: Input data of the numerical example

Sub System	Component Type	Availability (p _{ij})	Feeding Capacity (%)	Cost	Weight
1	1	0.70	130	65	80
		0.20	100		
		0.10	0		
	2	0.65	100	60	70
		0.25	80		
		0.10	0		
	3	0.60	95	50	75
		0.30	90		
		0.10	0		
	4	0.90	135	80	100
		0.05	80		
		0.05	0		
2	1	0.50	200	120	70
		0.25	140		
		0.20	100		
		0.05	0		
	2	0.60	220	130	100
		0.30	140		
		0.10	0		
	3	0.90	300	200	100
		0.10	0		
	3	1	0.80	160	200
0.15			90		
0.05			0		
2		0.85	140	160	100
		0.15	0		
3		0.90	200	250	90
		0.10	0		
4		0.65	100	100	70
		0.30	80		
		0.05	0		
5		0.50	130	60	50
		0.30	100		
		0.15	50		
		0.05	0		

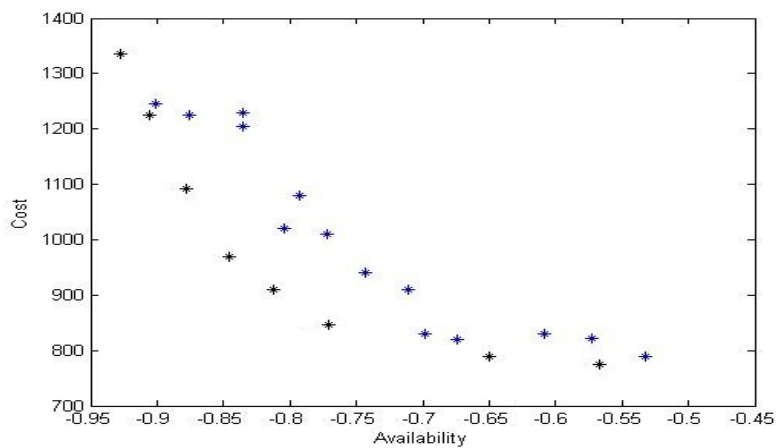


Fig. 1: The NSGA-II Pareto front