

Proposed a new Fuzzy C-Means Algorithm based on Electrical Rules

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Abstract. Every day, people deal with different types of data and different types of measurements and observations. Data describe the characteristics of data and summarize the result of experiment. Clustering or classification of data is an important activity to partitioning the data point in to similarity classes. One of the problems with Fuzzy C-Means (FCM) is that this algorithm cannot produce a good partitioning where the objective function is minimizing. The proposed fuzzy clustering algorithm uses electrical rules in FCM to obtain a low degree of variation and a large separation distance between clusters with respect to minimizing the objective function of FCM algorithm. Our algorithm optimized the validity indexes of Bezdek when the number of clusters is in the best state. Experiment results show the effectiveness of our proposed clustering model.

Keywords: Fuzzy c-mean, validity index, Coulomb's law

1. Introduction

One of the most important activities for analysis of data is to classify or clustering data into a set of clusters. People usually try to identify features of data and compare these features with those of known objects based on their similarity or distance according to sum rules. Clustering algorithms separation data objects certain number of categories. Clustering has a long history [1], dating back to Aristotle. Clustering algorithm allocated object to a cluster that this is a most popular problems for crisp clustering. So fuzzy logic (Zade, 1965)[2] create extreme clustering, rather than binary or crisp ones for fuzzy clustering, this problem is solved and the object can allocate to all of the clusters with a certain degree of membership (Bezdek, 1981)[3].

FCM is a most popular algorithm of fuzzy clustering that try to find a partition for a set of objects while minimizing the objective function [3]. A large number of heuristic methods have also been proposed to clustering data based on FCM. Until now, proposed optimization approaches for fuzzy partitioning, speeding up of FCM and the improvements of the drawbacks of FCM as shown as table 1.

In this paper we present a method of fuzzy clustering based on coulomb's law in section 2 and then we tested our method on data set from the machine learning library at UC Irvine and compare this proposed method with FCM algorithm that result of this comparing explore in section 3. Finally, in section 4 we explore conclusion of our research.

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Table1. Literature review of proposed method for improving FCM

References ID	researchers	Approches	Published year
[4]	Gath and Geva	Initialization problem by dynamically adding cluster centers	1989
[5]	Kersten	Use of fuzzy medians with a city block distance	1997
[6]	wu and Yang	New possibility algorithm for cost function	2006
[7]	Ozdemir and Akarun	Proposed a partition index maximization (PIM) algorithm	2002
[8]	Wu et al	proofing the volumes of each cluster core in PIM are equal	2005
[9]	Cheng and zhang	Entropy based subspace clustering mining numerical data	1999

2. A Fuzzy C-Mean Model Based on Coulomb's Low

The notations used in the paper are introduced follows in table 2.

Table2. Notation of our article

Row	symbols	Description
1	$X = \{X_1, \dots, X_n\}$	Data vector of j^{th} object
2	u_{ij}	Membership coefficient of the j^{th} object in the i^{th} cluster
3	$U=[u_{ij}]_{c \times n}$	Fuzzy partition matrix
4	$C \in \{1, \dots, c\}$	Cluster number
5	$V=[v_1, \dots, v_c]$	Cluster center matrix
6	$m \in [1, \dots, \infty)$	Fuzzification parameter usually set to 2
7	$D_{ij}=D(x_j, m_i)$	Distance between x_j and m_i
8	D_{ij}^2	Euclidean distance function
9	$J(p)$	Objective function
10	q_i	Value of data point in data set X

2.1. Fuzzy C-Mean Algorithm

FCM is one of the common algorithms of fuzzy clustering methods that proposed withBezdek in 1981[10].

The objective function of FCM method in Eq.1 can be minimizingwith an iterative procedure that leads to the FCM algorithm. The memberships function and center matrix update equations all obtained through alternating optimization. The steps of the FCM method are summarizing as follows:

Step1. Select appropriate values for m , c , and a small positive number ϵ . initialize the center matrix V randomly. Set step variable $t=0$;

Step2. Update the membership matrix U by

$$U_{ij}^{(t+1)} = \frac{1}{\left(\sum_{l=1}^c \left(\frac{D_{lj}}{D_{ij}}\right)^{2/(1-m)}\right)} \quad (2)$$

Step3. Update the center matrix V by

$$V_i = \frac{\sum_{k=1}^n U_{ik}^m X_k}{\sum_{k=1}^n U_{ik}^m}, \text{ for } i=1, \dots, c; \quad (3)$$

Step4. Repeat steps 2 and 3 until $\max \{\|U_{t+1} - U_t\| \leq \epsilon\}$.

Calculate objective function $J(p)$ as follows:

$$J = \sum_{i=1}^c \sum_{k=1}^n U_{ik}^m d_{ik}^2 = \sum_{i=1}^c \sum_{k=1}^n U_{ik}^m \|X_k - V_i\|^2 \quad (4)$$

2.2. Coulomb's law

We used electrical rules that described electrostatic actions between electrical points. Firstly, this rule proposed with Charls Coulomb[11]. Ration between electrical power and distance lastly discovered with Henry Cowndish[12]. Point charges that collected generate anelectric filedwith respect to characterize follows:

1. Represents the electric charge density of the electric field is create.
2. Due to the increased distance between the point charges, the electric field strength decreases.

2.3. Validity indexes

In generally clustering validity problem can be change to determination of optimum number clusters. Most studies on cluster validity design for FCM algorithm that reviewed withdubes and jane in 1979[13], 1988. The best known validity indexes were proposed for evaluate cluster validitywithBezdek as follows:

- The partition coefficient:

$$V_{pc}(U) = \frac{1}{n} (\sum_{k=1}^n \sum_{i=1}^c U_{ik}^2) \quad (5)$$

- The partition entropy:

$$V_{PE}(U) = -\frac{1}{n} (\sum_{k=1}^n \sum_{i=1}^c U_{ik} \log(U_{ik})) \quad (6)$$

When VPC is the maximum value, the clusters are most compact. and when VPE is the minimum value, the clusters are the most separation.

Since many heuristic methods are proposed to improve FCM algorithm to obtain maximum compaction and separation.

3. Proposed algorithm

We present a new fuzzy clustering algorithm based on coulomb's law. This algorithm seeks to offer the most compactness of data into clusters and to satisfy the maximum separation between clusters. The algorithm is usedto calculate the membership degrees of data clustering and updating the membership degree matrix of a heuristic function based on coulomb's law.

Proposed algorithm like FCM algorithm followed by minimizing the following function:

$$J = \sum_{i=1}^c \sum_{k=1}^n U_{ik}^m d_{ik}^2 = \sum_{i=1}^c \sum_{k=1}^n U_{ik}^m \|X_k - V_i\|^2$$

Where (n) is data vector of objects, (c) is number of clusters, V is cluster centers matrix and (U) is membership function matrix that this membership function matrix have to satisfy follows constraints:

$$I. \quad \mu_{ik} \in [0,1]; 1 \leq i \leq c, 1 \leq k \leq n \quad (7)$$

$$II. \quad \sum_{i=1}^c \mu_{ik} = 1; 1 \leq k \leq n \quad (8)$$

$$III. \quad 0 < \sum_{k=1}^n \mu_{ik} < n; 1 \leq i \leq c \quad (9)$$

The memberships function and center matrix update equations all obtained through alternating optimization. The steps of the FCM method are presenting as follows:

Step1. Select appropriate values for m, c, and a small positive number . initialize the center matrix V randomly. Set step variable t=0;

Step2. Update the membership matrix U by

$$U_{ij}^{(t+1)} = \frac{[\sqrt{q_i q_j} / D^2(q_i, q_j)]^{m-1}}{(\sum_{l=1}^c [\sqrt{q_i q_l} / D^2(q_j, q_l)]^{m-1})} \quad (10)$$

Step3. Update the center matrix V by

$$V_i = \frac{\sum_{k=1}^n U_{ik}^m q_j}{\sum_{k=1}^n U_{ik}^m}; \text{ for } i=1, \dots, c; \quad (11)$$

Step4. Repeat steps 2 and 3 until $\max\{\|U_{t+1}-U_t\|\leq\epsilon\}$.

Calculate objective function $J(p)$ as follows:

$$J = \sum_{i=1}^c \sum_{k=1}^n U_{ik}^m d_{ik}^2 = \sum_{i=1}^c \sum_{k=1}^n U_{ik}^m \|X_k - V_i\|^2$$

4. Experimental results

4.1. Evaluating measures

There are many ways to evaluate how exactly our clustering algorithm performs. We can use many validity indexes to evaluate and compare performance between our algorithm and FCM algorithm. Therefore, we use Bezdek's validity indexes for measuring how exactly of our method.

4.2. Iris Data set and Wine Data set

In this section, we examine performance of proposed algorithm and compare it with FCM algorithm on the Iris and Wine database available in the UC Irvine Machine Learning Repository. The Iris database contains 150 Number of Instances, each instance has 4 Number of Attributes: 4 numeric, predictive attributes and the class and the Wine Database contains 178 number of Wine recognition data kind of wines, each kind of wine has 13 Number of Attributes. Figure (1) shows the comparison results of our algorithm and FCM algorithm in Iris data set on (3,4,...,10) number of clusters for VPC value. In number clusters 3,4 our algorithm shows value of VPC better than FCM algorithm.

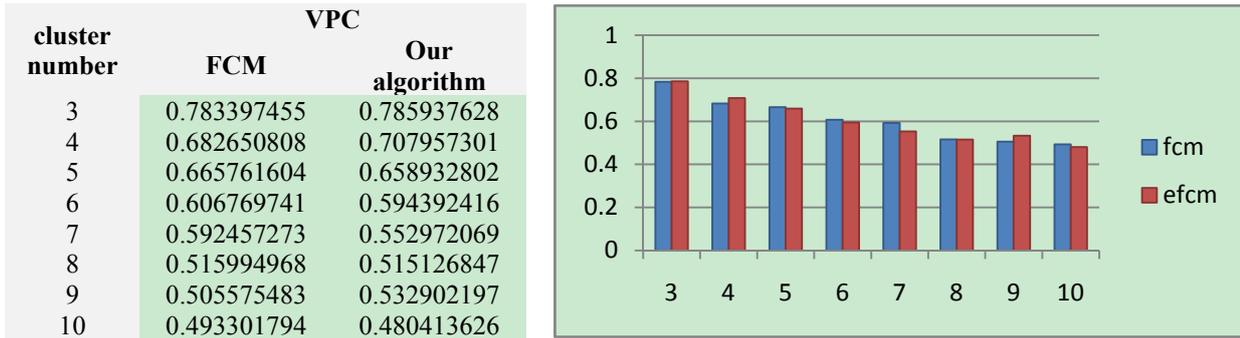


Fig 1.comparing VPC result between FCM and EFCM on Iris data set

Fig 2 shows the comparison results of our algorithm and FCM algorithm in Iris data set on (3,4,...,10) number of clusters for VPE value. In number clusters 3,4 our algorithm shows value of VPE better than FCM algorithm.

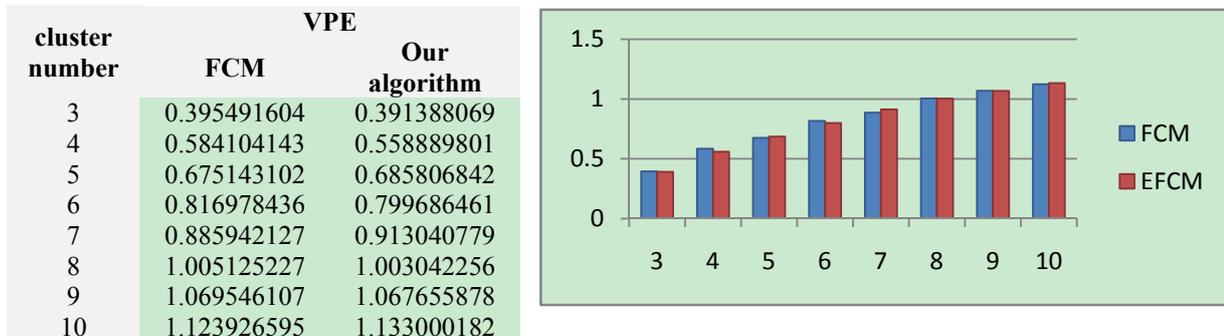


Fig 2.comparing VPE result between FCM and EFCM on Iris data set

Fig 3 shows the comparison results of our algorithm and FCM algorithm in Iris data set on (3,4,...,10)number of clusters for Objective function value.

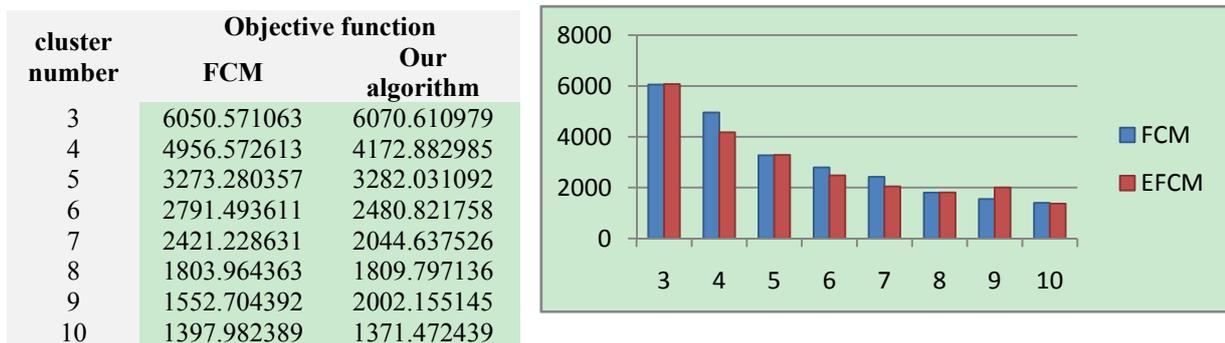


Fig 3.comparing VPE result between FCM and EFCM on Iris data set

Fig 4 shows the comparison results of our algorithm and FCM algorithm in Wine data set on (3,4,...,10) number of clusters for VPC value. In number clusters 3,4 our algorithm shows value of VPC better than FCM algorithm.

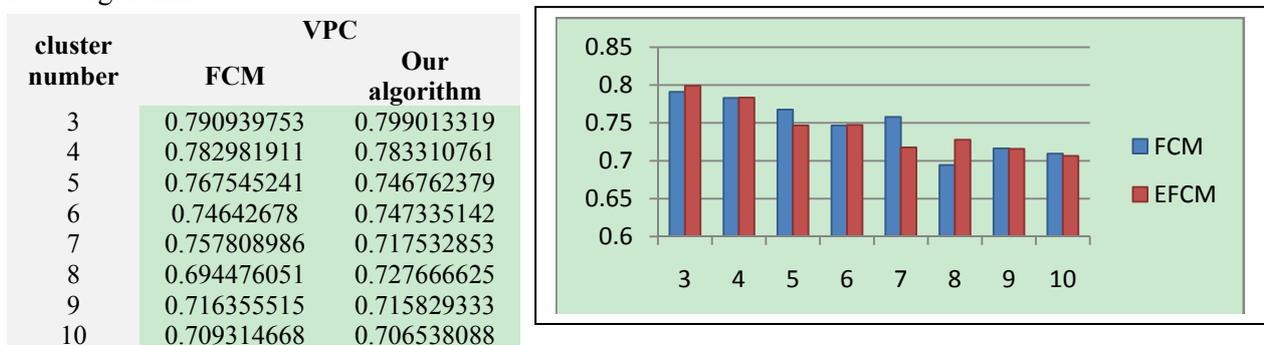


Fig 4.comparing VPC result between FCM and EFCM on Wine data set

Fig 5 shows the comparison results of our algorithm and FCM algorithm in Wine data set on (3,4,...,10) number of clusters for VPE value. In number clusters 3,4 our algorithm shows value of VPE better than FCM algorithm.

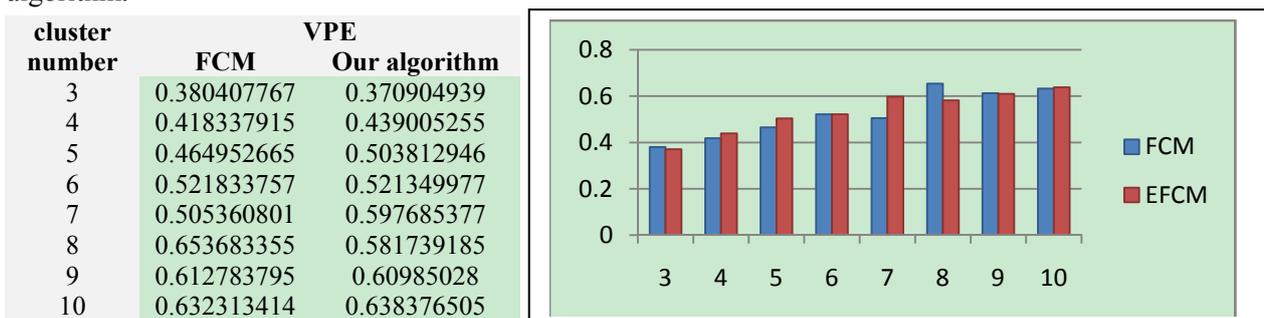


Fig 5.comparing VPE result between FCM and EFCM on Wine data set

5. Conclusion

In generally our algorithm ideas come from similarity between specifications of electrical fields and clustering algorithms and our proposed algorithm based on heuristic approach. In this paper, we present a new clustering algorithm based on coulomb's law that provide 2 validity indexes of Bezdek (VPC, VPE) compare with FCM algorithm.

We provide separation and compactness of clusters more than FCM method. Experimental results on document datasets suggest the success of our methodology.

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