2011 3rd International Conference on Signal Processing Systems (ICSPS 2011)
IPCSIT vol. 48 (2012) © (2012) IACSIT Press, Singapore
DOI: 10.7763/IPCSIT.2012.V48.26

High Range Resolution Radar Imaging from Partial and Non-uniform Frequency Measurements

Hu Lei+, Zhou Jianxiong, Shi Zhiguang, Fu Qiang

ATR Key Laboratory, School of Electronic Science and Engineering
National University of Defense Technology
Changsha, China

Abstract. To reduce the number of transmitted pulses required for obtaining the high range resolution profile (HRRP) by step frequency radar (SFR), we propose a HRRP construction approach based on the philosophy of compressed sensing (CS). The approach first reconstructs the target responses over all linearly-stepped frequencies from a random subset of frequency measurements, and then constructs the target HRRP by FFT-processing the recovered complete frequency samples. To improve the imaging quality, the approach models the sparsifying dictionary of complete frequency samples as a parameterized Fourier dictionary and refines the dictionary via optimizing its underlying parameters. Experimental results on the anechoic chamber scattering data of a missile model validate the effectiveness and robustness of the proposed approach.

Keywords: step frequency radar; range profile; compressed sensing;, non-uniform sampling; EM algorithm

1. Introduction

Step frequency radar (SFR) is a typical wideband radar scheme to obtain the high range resolution profile (HRRP) of a radar target. Conventionally, a SFR transmits pulses with linearly-stepped frequencies and then synthesizes the target HRRP by fast Fourier transform (FFT)-processing of the received frequency measurements. The range resolution of the synthesized HRRP is inversely proportional to the total covered bandwidth and the unambiguous imaging interval is inversely proportional to the frequency step length. As a result, to obtain the HRRP of large-sized targets, it may well be that the required frequency measurements are considerably huge.

In the high-frequency region, however, it has been shown that the backscattering of a target can be approximated as coming from a few dominant scattering centers. Thus, the target response in this region can be mathematically modeled as the superposition of several amplitude-modulated and time-shifted replicas of the transmitted signal. For a SFR, it can be seen that the target response can be viewed as the superposition of a few sinusoids. By the concept in the emerging theory of compressed sensing (CS) [1] [2], such a signal can be considered sparse or compressible with respect to some Fourier dictionary [3]. According to conclusions in CS, if a signal is sparse or compressible on a known dictionary, it may be accurately reconstructed from its under-sampled measurements. Applying this to SFR, we may reconstruct target responses corresponding to the overall frequencies from those measured over a subset of frequencies; that is, we may obtain the target HRRP of specified resolution with reduced frequency measurements.

Given a group of measurements over a random subset of frequencies, the most natural way to obtain the HRRP may be applying a zero-filled FFT to the measurements. However, due to the existence of the spectrum discontinuity, there will be significantly high side-lobes in the HRRP formed this way [4]. Another approach extensively employed to construct target HRRP is the eigenanalysis-based line spectral estimation method [5].

+ Corresponding author.
E-mail address: vanilla_hugh_hl@163.com

Unfortunately, this approach requires the frequency measurements should be shift-invariant and thus is only applicable to uniform sampling.

In this paper, we study a novel CS-based approach to construct target HRRP from partial and non-uniform frequency measurements. Our strategy is first reconstructing the target responses corresponding to overall frequencies and then constructing the HRRP by FFT-processing of the complete frequency samples. Therefore, the performance of our approach is dependent on the accuracy of the complete frequency samples reconstruction. According to CS, the accurate reconstruction of a signal lies in our complete knowledge of the signal's sparsifying dictionary. Since we do not know the ranges of target scattering centers *a priori*, the exact sparsifying Fourier dictionary of the complete frequency samples is unknown to us prior to HRRP construction. To handle this, we consider the sparsifying Fourier dictionary as tunable, treating the sampled frequency grid points as adjustable parameters. Hopefully, the accuracy in reconstructing the complete frequency samples will be improved by the optimization of the sparsifying dictionary. To validate the proposed approach, we apply it to the anechoic chamber scattering data of a missile model. The experimental results demonstrate the effectiveness and robustness of the proposed approach, and thus indicate the feasibility of high range resolution imaging from partial and non-uniform frequency measurements.

2. DATA Model

Conventionally, a SFR transmits *N* pulses $p_n(t)=\exp\left[2\pi\left(f_0+n\Delta f\right)t\right]$, where f_0 is the start frequency and Δf is the frequency step length. Suppose the considered stationary target consists of *K* scattering centers. The range and scattering coefficient of the *k*-th scattering center are respectively denoted by R_k and A_k. Then, the received signal can be formulated as:

$$q_n(t)=\sum_{k=1}^{K}A_k\exp\left[j2\pi\left(f_0+n\Delta f\right)\left(t-\frac{2R_k}{c}\right)\right]. \tag{1}$$

In this case, the output of the quadrature mixer can then be written as:

$$x_n=\sum_{k=1}^{K}A_k\exp\left[-j\frac{4\pi R_k}{c}\left(f_0+n\Delta f\right)\right]=\sum_{k=1}^{K}a_k e^{-j\omega_k n}, \tag{2}$$

where

$$a_k=A_k e^{-j4\pi f_0 R_k/c}, \tag{3}$$

$$\omega_k=4\pi\Delta f R_k/c. \tag{4}$$

Now, the parameters of scattering centers are modeled as parameters of a harmonic signal. By Fourier transform, the signal parameters can be retrieved and thus the target HRRP can be constructed. The unambiguous range interval R_u and the range resolution δR of the resulted HRRP are given by

$$R_u=\frac{c}{2\Delta f}, \tag{5}$$

$$\delta R=\frac{c}{2N\Delta f}. \tag{6}$$

Therefore, for scenarios where Δf is required to be small, it is necessary to transmit a large amount of pulses to achieve a high range resolution. To address this, we use a random subset of the overall pulses and manage to reconstruct the complete frequency samples from partial measurements.

Suppose the measurements are y_m, $m=1,\cdots,M$, where M is assumed to be $M<N$. The measurement model that we consider can be expressed as

$$y_m=x_{t_m}=\sum_{k=1}^{K}a_k e^{-j\omega_k t_m}+\varepsilon_m, \tag{7}$$

where t_m is an arbitrary integer in $[0,N-1]$ and ε_m is the measurement noise. Let $\mathbf{y}=\left[y_1 \;\cdots\; y_M\right]^{\mathrm{T}}$, $\mathbf{x}=\left[x_0 \;\cdots\; x_{N-1}\right]^{\mathrm{T}}$ and $\boldsymbol{\varepsilon}=\left[\varepsilon_1 \;\cdots\; \varepsilon_M\right]^{\mathrm{T}}$. Then, (7) can be formulated in vector form as

$$\mathbf{y}=\mathbf{x}_{\mathrm{T}}+\boldsymbol{\varepsilon}, \tag{8}$$

where $\mathrm{T}=\{t_1+1,\cdots,t_M+1\}\triangleq\{T_1,\cdots,T_M\}$ and $\mathbf{x}_{\mathrm{T}}$ denotes the vector with $|\mathrm{T}|$ elements chosen from those elements of $\mathbf{x}$ with indices in T.

Now, if we can reconstruct $\mathbf{x}$ from $\mathbf{y}$ with high accuracy, we can construct the target HRRP by processing the reconstructed samples with conventional approaches. Thus, how to reconstruct $\mathbf{x}$ is the key problem we are to handle.

3. Reconstruction of complete frequency samples

In essence, reconstruction of $\mathbf{x}$ from $\mathbf{y}$ is a problem of reconstructing a signal from its under-sampled measurements. Note that CS is a recently proposed tool to solve such problems. In the CS field, the sparsity property of the unknown signal is exploited to achieve accurate signal reconstruction. By (2), we see that the signal $\mathbf{x}$ is sparse with respect to some Fourier dictionary. However, due to our ignorance of ω_k, $k=1,\cdots,K$, it is difficult to preset a Fourier dictionary that can represent $\mathbf{x}$ exactly sparsely. To overcome this difficulty, we model the sparsifying Fourier dictionary as a parameterized dictionary, viewing the frequency grid points as unknown parameters to be estimated. In this way, by updating the estimate of the frequency grid, the sparsifying dictionary will be optimized during signal reconstruction.

For an arbitrary frequency grid $\mathbf{\Omega}=[\Omega_1 \quad \Omega_2 \quad \cdots \quad \Omega_P]$, there exists a corresponding Fourier dictionary $\mathbf{D}_{\mathrm{F}}(\mathbf{\Omega})$, which is constructed as follows:

$$\mathbf{D}_{\mathrm{F}}(\mathbf{\Omega})=\left[\mathbf{e}(\Omega_1) \quad \mathbf{e}(\Omega_2) \quad \cdots \quad \mathbf{e}(\Omega_P)\right], \tag{9}$$

with $\mathbf{e}(\Omega_i)=\left[e^{-j\Omega_i\cdot 0} \quad \cdots \quad e^{-j\Omega_i\cdot(N-1)}\right]^{\mathrm{T}}$. Suppose the $N\times P$ matrix $\mathbf{D}_{\mathrm{F}}(\mathbf{\Omega}_{\mathrm{S}})$ is a true sparsifying dictionary of $\mathbf{x}$ and $\mathbf{w}$ is a $P\times 1$ vector of the sparsest representation coefficients of $\mathbf{x}$ with respect to $\mathbf{D}_{\mathrm{F}}(\mathbf{\Omega}_{\mathrm{S}})$. Then, (8) can be reformulated as:

$$\mathbf{y}=\mathbf{D}_{\mathrm{FT}}(\mathbf{\Omega}_{\mathrm{S}})\mathbf{w}+\boldsymbol{\varepsilon}, \tag{10}$$

where $\mathbf{D}_{\mathrm{FT}}(\mathbf{\Omega}_{\mathrm{S}})$ denotes the matrix with $|\mathrm{T}|$ rows chosen from those rows of $\mathbf{D}_{\mathrm{F}}(\mathbf{\Omega}_{\mathrm{S}})$ with indices in T. Now, the task of reconstructing $\mathbf{x}$ can be regarded as recovering $\mathbf{\Omega}_{\mathrm{S}}$ and $\mathbf{w}$ from $\mathbf{y}$ based on (10). Following this idea, we develop an iterative algorithm for jointly recovering $\mathbf{\Omega}_{\mathrm{S}}$ and $\mathbf{w}$ based on the philosophy of expectation-maximization (EM) algorithm. In (10), if we treat $\mathbf{w}$ and $\mathbf{\Omega}_{\mathrm{S}}$ as stochastic and deterministic variables, respectively, then by utilizing EM algorithm, we can update $\mathbf{w}$ as a hidden variable in the E-step and estimate $\mathbf{\Omega}_{\mathrm{S}}$ as a deterministic parameter in the M-step. To do that, we need to solve two problems: (a) how to update $\mathbf{w}$ when the current estimate of $\mathbf{\Omega}_{\mathrm{S}}$ is given; (b) how to update $\mathbf{\Omega}_{\mathrm{S}}$ when the current estimate of $\mathbf{w}$ is known. Next, we present approaches to these problems. First of all, we give a brief introduction to the EM algorithm, concentrating on the interpretation of the algorithm from the perspective of variational Bayesian inference.

3.1 The Variational EM Algorithm

The EM algorithm is an effective method to find maximum likelihood (ML) estimates. For complicated problems, however, due to the difficulty in finding posterior densities of hidden variables, the algorithm is not applicable any more. To address this, D.Tzikas et al. [6] applied the new methodology termed "variational Bayesian inference" to find approximate posterior densities of hidden variables and established a new type of EM algorithm, which is referred to as the variational EM algorithm.

Let us denote the likelihood function by $p(\mathbf{y};\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is a vector of unknown parameters. Suppose $\mathbf{z}$ is a hidden variable. Then for an arbitrary probability density function $q(\mathbf{z})$, the log-likelihood function can be written as

$$\ln p(\mathbf{y};\boldsymbol{\theta})=F(q,\boldsymbol{\theta})+KL(q\,\|\,p), \tag{11}$$

where

$$F(q,\boldsymbol{\theta})=\int q(\mathbf{z})\ln\left(\frac{p(\mathbf{y},\mathbf{z};\boldsymbol{\theta})}{q(\mathbf{z})}\right)\mathrm{d}\mathbf{z}, \tag{12}$$

$$KL(q\,\|\,p)=-\int q(\mathbf{z})\ln\left(\frac{p(\mathbf{z}\,|\,\mathbf{y};\boldsymbol{\theta})}{q(\mathbf{z})}\right)\mathrm{d}\mathbf{z}, \tag{13}$$

and $KL(q\,\|\,p)$ denotes the Kullback-Leibler divergence between $p(\mathbf{z}\,|\,\mathbf{y};\boldsymbol{\theta})$ and $q(\mathbf{z})$. From (11), the parameter estimation procedure can be interpreted as an iterative process to optimize $q(\mathbf{z})$ and $\boldsymbol{\theta}$ alternately.

In the variational EM algorithm, $q(\mathbf{z})$ is assumed to be of factorized form, i.e., $q = \prod_i q_i$. Then, it can be shown that the optimal distribution for q_j is given by [6]

$$\ln q_j^* = \left\langle \ln p(\mathbf{y}, \mathbf{z}; \boldsymbol{\theta}) \right\rangle_{i \neq j} + \text{const}, \tag{14}$$

where $\langle \cdot \rangle$ denotes the expectation and const is the normalizing constant used to make q_j^* a true probability density function. To sum up, the main steps of the variational EM algorithm are described as follows [6]:

Variational E-step: Calculate the updated posterior density $q^{\text{NEW}}(\mathbf{z})$ by using (14);

Variational M-step: Find the updated estimate $\boldsymbol{\theta}^{\text{NEW}}$ by solving $\boldsymbol{\theta}^{\text{NEW}} = \arg\max_{\boldsymbol{\theta}} F\left(q^{\text{NEW}}, \boldsymbol{\theta}\right)$.

3.2 Update of Representation Coefficients

When the current estimate $\boldsymbol{\Omega}_{\text{S}}^{\text{OLD}}$ of $\boldsymbol{\Omega}_{\text{S}}$ is given, the update of $\mathbf{w}$ can be achieved by conducting the variational E-step for the model (10). Let $\mathbf{D}_{\text{F}}$ denote the current sparsifying dictionary $\mathbf{D}_{\text{F}}\left(\boldsymbol{\Omega}_{\text{S}}^{\text{OLD}}\right)$. Suppose $\mathbf{D}_{\text{FT}}$ is the matrix comprised of those rows of $\mathbf{D}_{\text{F}}$ that are indexed by elements in T. Then, (10) can be rewritten as

$$\mathbf{y} = \mathbf{D}_{\text{FT}} \mathbf{w} + \boldsymbol{\varepsilon}. \tag{15}$$

To exploit the sparsity property of $\mathbf{w}$, we adopt a hierarchical complex Gaussian prior [7] for it. Suppose α_i^{-1} is the prior variance of the *i*-th element of $\mathbf{w}$. Then, the prior distribution of $\mathbf{w}$ is assumed to be

$$p(\mathbf{w} \mid \boldsymbol{\alpha}) = \frac{1}{\pi^M |\mathbf{A}|^{-1}} \exp\left(-\mathbf{w}^{\text{H}} \mathbf{A} \mathbf{w}\right), \tag{16}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2 ..., \alpha_P]^{\text{T}}$ and $\mathbf{A} = \text{diag}(\alpha_1, \alpha_2 ..., \alpha_P)$. Assuming that α_i, $i = 1, \cdots, P$, are i.i.d. Gamma variables, the prior distribution of $\boldsymbol{\alpha}$ is formulated as

$$p(\boldsymbol{\alpha}; a, b) = \prod_{i=1}^{P} \text{Gamma}(\alpha_i \mid a, b), \tag{17}$$

where $\text{Gamma}(\alpha_i \mid a, b)$ is the Gamma distribution [7]. The measurement noise is assumed to be independent and complex Gaussian with zero-mean and variance equal to β^{-1}, that is,

$$p(\boldsymbol{\varepsilon} \mid \beta) = \left(\pi\beta^{-1}\right)^{-M} \exp\left(-\beta \|\boldsymbol{\varepsilon}\|_2^2\right). \tag{18}$$

A Gamma distribution is assumed as the prior for β, i.e.,

$$p(\beta; c, d) = \text{Gamma}(\beta \mid c, d). \tag{19}$$

Consequently, the likelihood function can be expressed as follows:

$$p(\mathbf{y} \mid \mathbf{w}, \beta) = \left(\pi\beta^{-1}\right)^{-M} \exp\left(-\beta \|\mathbf{y} - \mathbf{D}_{\text{FT}} \mathbf{w}\|_2^2\right). \tag{20}$$

Utilizing (14) and (16)–(20), we can obtain the approximate posterior distributions of $\mathbf{w}$, $\boldsymbol{\alpha}$ and β. Specifically, the posterior density of $\mathbf{w}$ is given by

$$q(\mathbf{w}) = CN\left(\mathbf{w} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}\right), \tag{21}$$

i.e., $q(\mathbf{w})$ is complex Gaussian with the mean vector $\boldsymbol{\mu} = \langle \beta \rangle \boldsymbol{\Sigma} \mathbf{D}_{\text{FT}}^{\text{H}} \mathbf{y}$ and the covariance matrix $\boldsymbol{\Sigma} = \left(\langle \beta \rangle \mathbf{D}_{\text{FT}}^{\text{H}} \mathbf{D}_{\text{FT}} + \langle \mathbf{A} \rangle\right)^{-1}$. The posterior density of $\boldsymbol{\alpha}$ is

$$q(\boldsymbol{\alpha}) = \prod_{i=1}^{P} \text{Gamma}\left(\alpha_i \mid \tilde{a}, \tilde{b}_i\right), \tag{22}$$

with $\tilde{a} = a + 1$ and $\tilde{b}_i = \left\langle |w_i|^2 \right\rangle + b$. The posterior density of β is

$$q(\beta) = \text{Gamma}\left(\beta \mid \tilde{c}, \tilde{d}\right), \tag{23}$$

with $\tilde{c} = c + M$ and $\tilde{d} = \|\mathbf{y} - \mathbf{D}_{\text{FT}} \boldsymbol{\mu}\|_2^2 + \text{tr}\left(\mathbf{D}_{\text{FT}}^{\text{H}} \mathbf{D}_{\text{FT}} \boldsymbol{\Sigma}\right) + d$.

To update the estimate of $\mathbf{w}$, we first update the posterior densities of $\boldsymbol{\alpha}$ and β using the current posterior distribution of $\mathbf{w}$, and then obtain the new posterior density of $\mathbf{w}$ based on the updated statistics of $\boldsymbol{\alpha}$ and β. In this procedure, a, b, c and d are typically set to very small values [7].

3.2.1 Update of Frequency Grid Points

Let $q\left(\mathbf{w},\boldsymbol{\alpha},\beta;\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{OLD}}\right)$ denote the current posterior density of $\mathbf{w}$, $\boldsymbol{\alpha}$ and β obtained in the E-step. According to the variational M-step, the estimate of $\boldsymbol{\Omega}_{\mathrm{S}}$ is updated via the following formulation:

$$\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{NEW}} = \arg\max_{\boldsymbol{\Omega}_{\mathrm{S}}} \left\langle \ln p\left(\mathbf{y},\mathbf{w},\boldsymbol{\alpha},\beta;\boldsymbol{\Omega}_{\mathrm{S}}\right)\right\rangle_{q\left(\mathbf{w},\boldsymbol{\alpha},\beta;\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{OLD}}\right)} \tag{24}$$

where $\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{NEW}}$ denotes the updated estimate of $\boldsymbol{\Omega}_{\mathrm{S}}$. After simplification (24) can be expressed as

$$\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{NEW}} = \arg\max_{\boldsymbol{\Omega}_{\mathrm{S}}} \left\langle \ln p\left(\mathbf{y}\mid\mathbf{w},\beta;\boldsymbol{\Omega}_{\mathrm{S}}\right)\right\rangle_{q\left(\mathbf{w},\boldsymbol{\alpha},\beta;\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{OLD}}\right)}. \tag{25}$$

By (20), (25) can be formulated as

$$\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{NEW}} = \arg\min_{\boldsymbol{\Omega}_{\mathrm{S}}} \left\langle \left\|\mathbf{y}-\mathbf{D}_{\mathrm{FT}}\left(\boldsymbol{\Omega}_{\mathrm{S}}\right)\mathbf{w}\right\|_2^2\right\rangle_{q\left(\mathbf{w},\boldsymbol{\alpha},\beta;\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{OLD}}\right)}. \tag{26}$$

Utilizing (21)–(23), we obtain

$$\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{NEW}} = \arg\min_{\boldsymbol{\Omega}_{\mathrm{S}}} H\left(\boldsymbol{\Omega}_{\mathrm{S}}\right), \tag{27}$$

where

$$H\left(\boldsymbol{\Omega}_{\mathrm{S}}\right) = \left\|\mathbf{y}-\mathbf{D}_{\mathrm{FT}}\left(\boldsymbol{\Omega}_{\mathrm{S}}\right)\boldsymbol{\mu}\right\|_2^2 + \mathrm{tr}\left(\mathbf{D}_{\mathrm{FT}}^{\mathrm{H}}\left(\boldsymbol{\Omega}_{\mathrm{S}}\right)\mathbf{D}_{\mathrm{FT}}\left(\boldsymbol{\Omega}_{\mathrm{S}}\right)\boldsymbol{\Sigma}\right) \tag{28}$$

with $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ denote respectively the mean vector and covariance matrix of $q\left(\mathbf{w};\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{OLD}}\right)$. Clearly (28) is a penalized nonlinear least-squares problem. We use the pure Newton method [8] to solve this problem.

Denoting by $\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}$ the parameter estimation at iteration *k*, the updated estimate $\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k+1}$ is then computed by

$$\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k+1} = \hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k} - \nabla^2 H\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right)^{-1}\nabla H\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right), \tag{29}$$

where $\nabla H\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right)$ and $\nabla^2 H\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right)$ respectively represent the gradient and Hessian evaluated at $\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}$. By derivation and simplification, it can be shown that

$$\nabla H\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right) = -2\,\mathrm{Im}\left(\hat{\mathbf{Q}}_k^{\mathrm{T}}\mathbf{C}_{\mathrm{T}}\bar{\mathbf{u}}\right) - 2\,\mathrm{Im}\left(\mathrm{diag}\left(\hat{\mathbf{R}}_k\boldsymbol{\Sigma}\right)\right), \tag{30}$$

$$\begin{aligned}\nabla^2 H\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right) = {} & 2\,\mathrm{Re}\left(\hat{\mathbf{Q}}_k^{\mathrm{H}}\mathbf{C}_{\mathrm{T}^2}\hat{\mathbf{Q}}_k + \hat{\mathbf{S}}_k \odot \bar{\boldsymbol{\Sigma}}\right) + \\ & 2\mathrm{diag}\left(\mathrm{Re}\left(\hat{\mathbf{Q}}_k^{\mathrm{H}}\mathbf{C}_{\mathrm{T}^2}\mathbf{u} - \mathrm{diag}\left(\hat{\mathbf{S}}_k\boldsymbol{\Sigma}\right)\right)\right),\end{aligned} \tag{31}$$

where

$$\mathbf{u} = \mathbf{y} - \mathbf{D}_{\mathrm{FT}}\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right)\boldsymbol{\mu}, \tag{32}$$

$$\mathbf{C}_{\mathrm{T}} = \mathrm{diag}\left(\mathbf{T}_{-1}\right), \tag{33}$$

$$\mathbf{T}_{-1} = \left[T_1-1 \quad T_2-1 \quad \cdots \quad T_M-1\right]^{\mathrm{T}}, \tag{34}$$

$$\mathbf{C}_{\mathrm{T}^2} = \mathrm{diag}\left(\mathbf{T}_{-2}\right), \tag{35}$$

$$\mathbf{T}_{-2} = \left[\left(T_1-1\right)^2 \quad \left(T_2-1\right)^2 \quad \cdots \quad \left(T_M-1\right)^2\right]^{\mathrm{T}}, \tag{36}$$

$$\hat{\mathbf{Q}}_k = \mathbf{D}_{\mathrm{FT}}\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right)\mathrm{diag}\left(\boldsymbol{\mu}\right), \tag{37}$$

$$\hat{\mathbf{R}}_k = \mathbf{D}_{\mathrm{FT}}^{\mathrm{H}}\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right)\mathbf{C}_{\mathrm{T}}\mathbf{D}_{\mathrm{FT}}\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right), \tag{38}$$

$$\hat{\mathbf{S}}_k = \mathbf{D}_{\mathrm{FT}}^{\mathrm{H}}\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right)\mathbf{C}_{\mathrm{T}^2}\mathbf{D}_{\mathrm{FT}}\left(\hat{\boldsymbol{\Omega}}_{\mathrm{S}}^{k}\right). \tag{39}$$

In addition, the symbol $\overline{(\cdot)}$ denotes the complex conjugate and $\odot$ denotes the Hadamard (elementwise) matrix product.

Now, the updated estimate of $\boldsymbol{\Omega}_{\mathrm{S}}$ can be obtained by performing (29) iteratively until convergence.

3.2.2 Procedure of Complete Data Reconstruction

By performing iteratively the above procedures, we can obtain the recovery of $\boldsymbol{\Omega}_{\mathrm{S}}$, $\mathbf{w}$ and then the reconstruction of $\mathbf{x}$. However, when the dimension of $\boldsymbol{\Omega}_{\mathrm{S}}$ is high, the computational burden of the above procedures may be very large. To address this, we prune the current estimate of $\boldsymbol{\Omega}_{\mathrm{S}}$ in the E-step, so that the computational complexity will decrease with the gradual shrinkage of the frequency grid considered. Specifically, pruning of $\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{OLD}}$ can be accomplished via thresholding of elements in $\langle\boldsymbol{\alpha}\rangle$. From the expression of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, it can be seen that when $\langle\alpha_i\rangle$ is large enough, μ_i and Σ_{ii} will be quite small, which means that the contribution of the *i*-th frequency grid point for synthesizing $\mathbf{x}$ is negligible, and thus this point can be removed from the current grid. Denoting by α_{th} the threshold used to measure the size of elements in $\langle \boldsymbol{\alpha} \rangle$, the process for pruning $\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{OLD}}$ can be realized through

$$\tilde{\boldsymbol{\Omega}}_{\mathrm{S}}^{\mathrm{OLD}} = \boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{OLD}} \left[\left\{ i \middle| \langle \alpha_i \rangle \le \alpha_{th} \right\} \right], \tag{40}$$

where $\tilde{\boldsymbol{\Omega}}_{\mathrm{S}}^{\mathrm{OLD}}$ represents the pruned frequency grid. Note that along with the pruning of $\boldsymbol{\Omega}_{\mathrm{S}}^{\mathrm{OLD}}$, the posterior mean vector and covariance matrix of $\mathbf{w}$ should be pruned accordingly.

Now, the algorithm for reconstructing $\mathbf{x}$ from $\mathbf{y}$ can be summarized as follows:

1. Initialization: set the initial frequency grid to be $\boldsymbol{\Omega}_0$ and form the initial dictionary $\mathbf{D}_{\mathrm{F}}(\boldsymbol{\Omega}_0)$ using $\boldsymbol{\Omega}_0$. Meanwhile, initialize the posterior mean vector and covariance matrix of $\mathbf{w}$.

2. Update respectively the posterior densities of $\boldsymbol{\alpha}$ and β. Then, update the posterior density of $\mathbf{w}$ by using the current posterior densities of $\boldsymbol{\alpha}$ and β.

3. Prune the current frequency grid. Then, prune the posterior mean vector and covariance matrix of $\mathbf{w}$ accordingly.

4. Update the current frequency grid. Then, form the updated Fourier dictionary by using the new frequency grid.

5. If the halting criterion is satisfied, then go to step 6; otherwise, perform steps 2–4.

6. Output the current frequency grid and representation coefficients as the estimates of the component frequencies and complex amplitudes of $\mathbf{x}$. Using these estimates, obtain the reconstruction $\hat{\mathbf{x}}$.

Specifically, the halting criterion can be set to be such that there is no significant variation in the measurement residual.

4. experimental results

To validate the proposed approach, we apply it to the anechoic chamber scattering data of a missile model. The overall frequency samples of this model are obtained according to parameters as follows: the frequency steps from 9 to 11GHz with the step length of 20MHz; the elevation is fixed at 0° and the azimuth varies between 0° and 180° with the angle interval of 0.2°.

Fig. 1(a) shows the original range profiles obtained by FFT-processing the overall frequency samples. The pixel color is proportional to the logarithmic magnitude of the range cell. The widths of the prominent lines indicate the range resolving ability of the image. We now from the partial and non-uniform frequency measurements by randomly selecting 30 out of the total 101 samples at each target attitude. Given a set of measurements acquired that way, we recover the target profiles by utilizing respectively our proposed method and the conventional zero-filled FFT approach. The reconstructed profiles corresponding to these

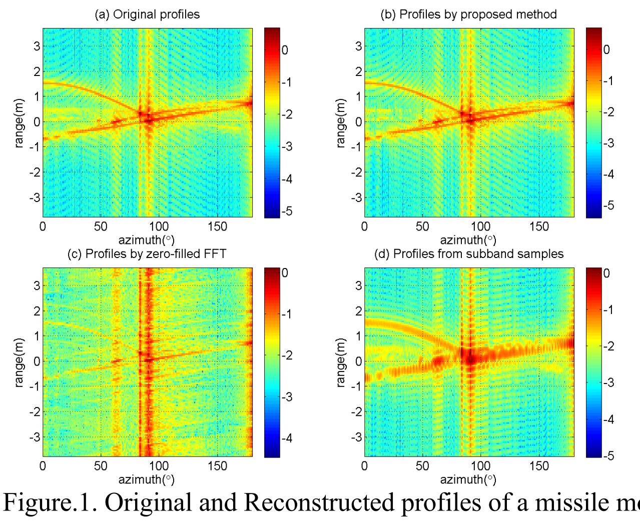

Figure.1. Original and Reconstructed profiles of a missile model

methods are shown in Fig. 1(b) and Fig. 1(c). As a comparison, Fig. 1(d) also shows the profiles obtained by FFT-processing the first 30 frequency samples.

It can be seen that the profiles reconstructed by the proposed method are almost visually indistinguishable from the original ones. On the contrary, the profiles recovered by the other two methods either suffer form high sidelobes or from degraded range resolving ability. These results indicate the improved performance and effectiveness of the proposed method in recovering radar target range profiles from incomplete measurements.

Now, we test the robustness of the proposed method to different realizations of the selected frequency measurements. To do so, we recover the target profile at one specified azimuth repeatedly by using different sets of measurements. The azimuth considered is 90°. The total number of measurement sets is 500, and each set contains 30 randomly selected samples. Fig. 2(a) demonstrates the histogram of relative errors of all trials in reconstructing the complete frequency samples. It can be seen that the overwhelming majority of the measurement sets lead to relative errors below 0.1. For the purpose of illustration, we show the profile corresponding to the reconstructed samples of a single run in Fig. 2(b). The relative error in complete data reconstruction of this trial is about 0.09. It is clear that the reconstructed and the original profiles are close enough that they almost overlap completely. Based on these results, we conclude that the proposed method is robust to the choice of the partial frequency measurements in acquiring high-quality target range profiles.

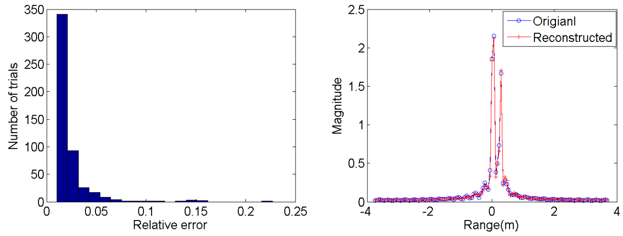

Figure.2. Reconstruction results of complete frequency samples at azimuth 90° using different sets of measurements. (a) Histogram of relative reconstruction errors; (b) Target profile corresponding to reconstructed samples with relative error close to 0.1.

5. Conclusions

To reduce the overhead in acquiring HRRP of radar targets, we proposed a CS-based approach to constructing HRRP from partial and non-uniform frequency measurements. The approach first recovers the complete frequency samples and then constructs the HRRP via FFT-processing. To improve the reconstruction accuracy of the complete samples, we modeled the sparsifying Fourier dictionary as a parameterized dictionary, and then developed an iterative algorithm for joint representation coefficients estimation and dictionary parameters optimization. For the purpose of validation, we demonstrated experimental results using the anechoic chamber scattering data of a missile model.

6. Acknowledgment

This research was supported by National Natural Science Foundation of China under Grant No. 60972113.

7. References

[1] E. J. Candès, J. K. Romberg, and T. Tao, “Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information,” IEEE. Trans. Inform. Theory, vol. 52, no. 2, pp. 489–509, 2006.

[2] D. L. Donoho, “Compressed sensing,” IEEE Trans. Inf. Theory, vol. 52, no. 4, pp. 1289–1306, 2006.

[3] E. J. Candès, Y. C. Eldar, D. Needell, and P. A. Randall, “Compressed sensing with coherent and redundant dictionaries,” 2010, in press.

[4] P. Babu, P. Stoica, “Spectral analysis of nonuniformly sampled data – a review,” Digital Signal Processing, vol. 20, pp. 359–378, 2010.

[5] P. Stoica, R. L. Moses, “Introduction to spectral analysis,” Upper Saddle River, NJ: Prentice Hall, 1997.

[6] D. G. Tzikas, A. C. Likas, and N. P. Galatsanos, “The variational approximation for Bayesian inference,” IEEE Signal Process. Mag., vol. 25, no. 6, pp. 131–146, 2008.

[7] M. E. Tipping, “Sparse Bayesian learning and the relevance vector machine,” J. Mach. Learn. Res, vol. 1, pp. 211–244, 2001.

[8] D. G. Luenberger and Y. Ye, Linear and Nonlinear Programming, 3rd ed., Springer, 2004, pp. 246-247.