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Side-lobe Suppression Methods for Polyphase Codes

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Abstract. Polyphase codes derived from linear-frequency-modulation (LFM), such as P3 code and P4 code, are a useful class of pulse compression waveforms. This paper discusses and compares the pulse compression performance including peak side-lobe level (PSL) and signal-to-noise ratio (SNR) loss of P4 code under different side-lobe suppression methods, besides, the Doppler tolerance and range resolution are also discussed and compared. Two new methods including mismatch filter design based on second-order cone programming and optimal design method combined arbitrary phase codes with mismatch filter optimization are introduced.

Keywords: P3, P4 codes; Side-lobe suppression; Second-order cone programming; Combined optimal design method

1. Introduction

As we know, we cannot take account of distance and dis-tance resolution at the same time in the common pulse radar system. The pulse compression concept in radar systems appears as a solution to the dichotomous problem of simul-taneously obtaining high transmitted pulse energy, in order to achieve long range, along with high local energy concern-tration after processing in the radar receiver, to yield high range resolution. Of course, using matched filter to compress signal with large time and frequency band, the output pulse can be narrow. Although pulse compression waveforms bear low side-lobes in their autocorrelations, the side-lobe levels can not satisfy the needs in many practical applications. So, further side-lobe reduction is desired. According to different signals we use variant side-lobes reduction techniques, including two kinds of configuration: the first kind is to add side-lobes reduction filter after matched filter, the other kind is to design mismatched filter directly, realizing pulse com-pression and side-lobes suppression at the same time. There are many pulse compression signals including linear chirp signal, nonlinear chirp signal, biphase codes and polyphase codes etc. Different classes of pulse compression waveforms have different pulse compression properties, and polyphase-pulse-compression codes have many useful features. The Frank, P1, P2 codes which have been derived from step-approximately-to-linear-frequency-modulation pulse comp-ression waveforms and P3, P4 codes which have been de-rived from linear-frequency-modulation pulse compression waveforms are typical polyphase codes, and the P4 code which possess these features such as low range-time-side-lobe, ease of implementation, low cross-correlation between codes, large Doppler tolerance and compatibility with band-pass limited receivers at the same time is the best of them[1-2]. In respect to side-lobe reduction for P4 code, several techniques available can be adopted including the classical window function amplitude weighting, the post-compression sliding window 2-sample processing and second-order cone programming amplitude and phase weighting[7].Of course, using these techniques can substantially reduce the side-lobe levels of the compressed pulses of the codes. However, higher distance resolution and lower side-lobe levels in many special applications are needed, for the polyphase codes with arbitrary phase, optimal design method combined with pulse compression filters was proposed and obtained good results[3-5].

This paper concerned the results of various weighting techniques to further reduce the side-lobe levels of the P4 code and the attendant tradeoffs. Therefore, we could choose the best technique according to practical applications.

2. P3 code and P4 code

P3 and P4 codes are derived from linear-frequency-modulation pulse compression waveforms. They take pos-session of the features such as large Doppler tolerance and the phase of the side-lobes near the main-lobe of the auto-correlation function appears to change by from one bin to another[6] of the linear-frequency-modulation pulse compress-ion waveforms other than the features of the phase coded pulse compression waveforms. The following paper gives the phase formulation of P3 and P4 codes through simple derivation.

The Transient frequency of LFM pulse compression waveforms is

$$f = f_0 + kt \qquad (1)$$

Where $k = B/T$, B and T represent the bandwidth and the pulse length of LFM pulse compression waveforms respectively. The bandwidth will support a compressed pulse length of approximately

$$\tau = 1/B \qquad (2)$$

And the waveform will provide a pulse compression ratio of

$$T/\tau = \mathrm{TB} = \mathrm{N} \qquad (3)$$

2.1. P3 Code

Assuming that the first sample is taken at the leading edge of wave form.the phases of successive samples taken apart is τ

$$\begin{aligned}\phi_i^{P3} &= 2\pi \int_0^{(i-1)\tau} \left[(f_0 + kt) - f_0 \right] dt \\ &= \pi k (i-1)^2 \tau^2 \qquad (4)\end{aligned}$$

Where $i = 1, 2, \cdots, N$ $k = B/T$ and from (2) $\tau = 1/B$, therefore (4) can be written as

$$\phi_i^{P3} = \pi (i-1)^2 / N \qquad (5)$$

2.2. P4 Code

The P4 code is conceptually derived from the same waveform as the P3 code. However, in this case, the local oscillator frequency is set equal to $f_0 + kT./2$

With this frequency, the phases of successive samples taken apart is

$$\begin{aligned}\phi_i^{P4} &= 2\pi \int_0^{(i-1)\tau} \left[(f_0 + kt) - (f_0 + kT/2) \right] dt \\ &= \pi k (i-1)^2 \tau^2 - \pi kT (i-1)\tau \qquad (6)\end{aligned}$$

Or

$$\phi_i^{P4} = \pi (i-1)^2 / N - \pi (i-1) \qquad (7)$$

It should be noted that the largest phase increments from code element to code element are in the middle of the P3 code but are on the two ends of the P4 code. Thus the P3 code is less precompression bandwidth limitation tolerant than P4 code. This follows since precompression bandwidth limitations average the code phase increments and would attenuate the P3 code in the middle which decreases the PSL of the compressed pulse and attenuate the P4 code on the ends which increases the PSL of the compressed pulse.

In fact, P3 and P4 code are similar to LFM pulse compression waveforms. The following discussion will take P4 code as a example.

3. Side-lobe Suppression Methods for Polyphase Codes

P4 code is derived from LFM pulse compression waveforms as discussed in previous. So some side-lobe suppression methods for LFM pulse compression waveforms such as classical window function amplitude weighting and post-compression sliding window 2-sample processing can be used to P4 code, otherwise, designing mismatched filter directly also can be used to P4 code. The following paper will introduce these techniques simply.

3.1. Classical window function amplitude weighting

Generally, this technique is carried out by weighting the match filter of the transmitted waveform with window function in the time domain. As illustrated in Figure 1.

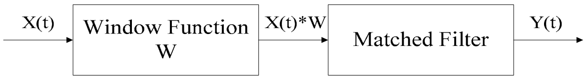

Fig.1. Window function weighting in time domain

Bartlett, Hamming, Hann, Blackman, Kaiser and Chebwin etc., are commonly used. For a given polyphase code with definite length, the reduced side-lobe level is dependent upon the window function selected. Take P4 code with the definite length of 100 as a example, the compressed pulse for 100-elment P4 code with matched filter is shown in Figure 2, we can see that the PSL is -26.3232dB.

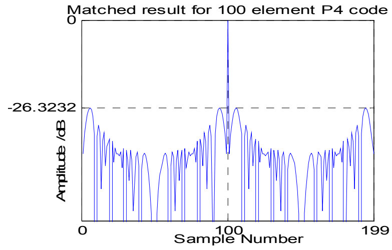

Fig.2. –ACF of 100-element P4 Code

After classical window function amplitude weighting, the side-lobes of compressed pulse can be largely reduced and the PSL is approximately -40dB as illustrated in table 1. Figure 3 presents the performance of P4 code under hamming and chebwin window function weighting when Doppler shift is zero.

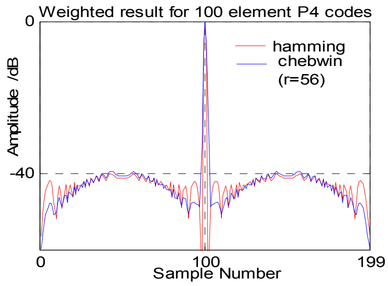

Fig.3. Compressed pulses of 100-element P4 Code with hamming and chebwin

The doppler properties under hamming window function is shown in Figure 4 , the relationship between PSL and Doppler frequency shift of different classical window functions is shown in Figure 5.

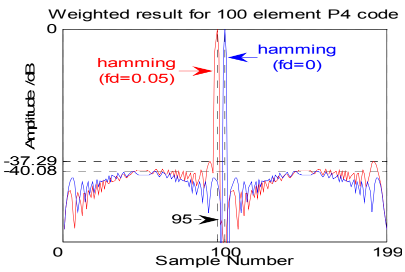

Fig.4. Compressed pulses of 100-element P4 Code with hamming weighting for normalized Doppler shifts being zero and 0.05, respectively

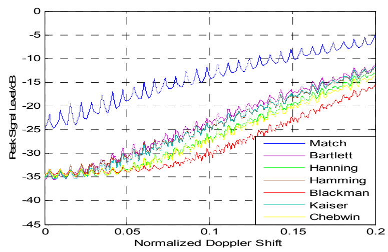

Fig.5. PSL's versus Doppler shift for P4 Code under different window function weighting

From Figure 4 and Figure 5, we could know that good Doppler tolerance have been obtained when P4 code is weighted by classical window functions, however, some negative effects are introduced, such as the spreading of the main-lobe of the compressed pulse and SNR loss, while the window function reduces the side-lobes. The better the PSL gets, the wider the compressed pulse is, and similarly, the greater the SNR loss is.

3.2. Post-Compression, 2-sample Averager

As illustrated in previous, the phase of the side-lobes near the main-lobe of the LFM pulse compression waveform autocorrelation function appears to change by π from one bin to another. According to the relationship between the P3, P4 code and LFM pulse compression waveform, Lewis,B.L proposed this technique, and its implementing diagram is illustrated in Figure 6.

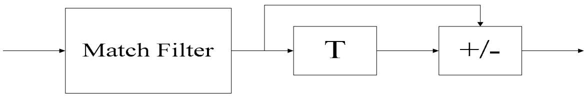

Fig. 6. Pulse compressor followed by silding window 2-sample processing

In Figure 6, T denotes one sample delay, the signs + and – denote operations of addition and subtraction respectively. The selection of addition operation or subtraction operation depends upon which kind of polyphase is used, P3 and P4 codes correspond to subtraction and addition respectively. After the processing shown in Figure 6, the PSL of compressed pulse attainable is -20log(N/2), where N is pulse compression ratio. Figure 7 illustrated the performance of P4 code with 100 length after this processing, and the PSL is 33.9dB.

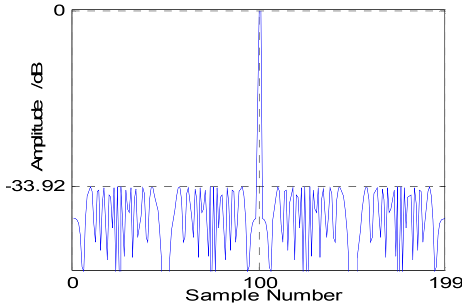

Fig. 7. P4 code compressed pulse with 2-sample Sliding window adder

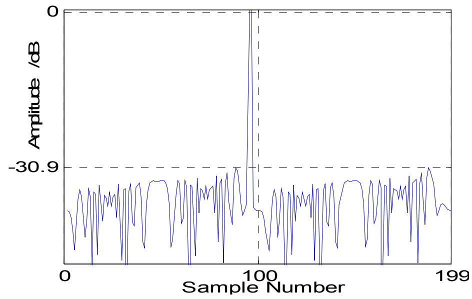

Fig.8. Result of sliding window 2-sample adder on output of 100 to 1 P4 code compressor with 0.05 normalized Doppler shift on signal

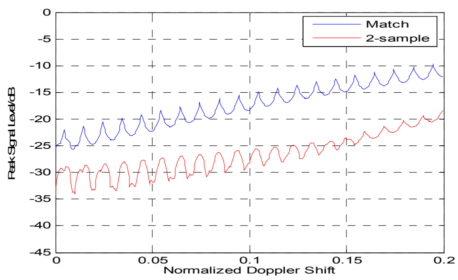

Fig.9. PSL's versus Doppler shift for P4 Code under different weighting

From Figure 8 and Figure 9, we can see that good Doppler tolerance also have been obtained by this technique. Of course, it also brings in some negative effects such as a spread by a factor of 2 in main-lobe and a loss in SNR.

3.3. Side-lobe suppresion filter design based on second-order cone programming

Classical window function amplitude weighting in time domain and post compression, 2-sample Averager processing are two kind s of classical side-lobe suppression methods specific to polyphase codes, the two methods are belong to the structure of using side-lobe suppression filter after mat-ching pulse compression. In addition, we can suppress the side-lobe through designing mismatched filter directly for p4 code, there are lots of ways to design mismatched filter, such as including the least square method, linear programming and neural network, etc. They are able to suppress range sidelobes well but have some certain shortcomings, among them the least square method is able to get the optimal filter of minimum integral sidelobes, but it need to iterate for a lot of times and it's hard to control iteration times and convergence; Generally the linear programming method isn't suitable for polyphase codes; Convergence speed of neural network method is slow, so it affects its practical application.

However, using above methods to design mismatched filter is only considering to minimize the side-lobes of compressed pulse, but not considering SNR loss. Document[7] proposed a method to design side-lobe suppression filter based on Second-order cone programming, this method translates the design of side-lobe suppression filter under the condition of biggest SNR loss into the second-order cone issue , and using interior point method to solve it efficiently. The specific details will be discussed as follows.

We can get the coefficient f of side-lobe suppression filter from the following formula

$$\min_f \quad \left\| R^H f \right\|_\infty, \quad s.t. \quad s^H f = 1, \quad \left\| f \right\| \le 10^{(\varepsilon/20)} \tag{8}$$

The formula above can be explained like this, under the constraints of the maximum SNR loss , solve the minimum peak side-lobe filter when the main-lobe value of comp-ressed pulse is set to 1. where s denotes the signal sequence which filled with zero to both ends ,and the middle of the sequence is the code sequence, and the length of s is equal to f. R is a matrix consists of signal sequence s and zeros, shown as follows.

$$R = \begin{bmatrix} 0 & \cdots & 0 & s_1 & \cdots & s_{M-1} \\ 0 & \cdots & s_0 & s_2 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ s_0 & \cdots & s_{M-2} & 0 & \cdots & 0 \end{bmatrix}_{M\times(2M-2)} \tag{9}$$

ε is the biggest SNR loss, the unit is dB.

The mismatch filter designed for P4 code with length of 100 based on this method at a certain case of SNR loss can suppress the side-lobe efficiently.

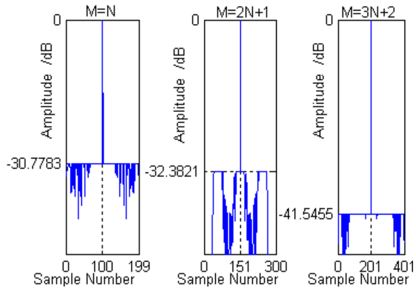

Fig.10. PSL's versus filter length for 100 element P4 Code(SNR Loss=0.5dB)

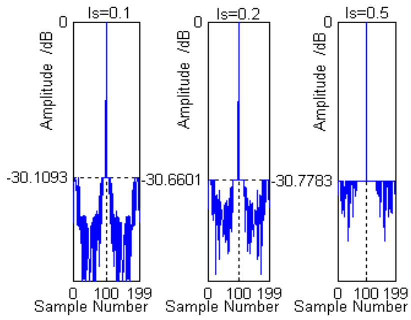

Fig.11. PSL's versus SNR Loss for 100 element P4 Code (Filter Length=100)

From Figure 10, we could know that the higher filter order is, the better pulse compression results we can get. If the order of side-lobe suppression filter is definite, the greater SNR loss is, the better pulse compression performance we can get, as shown in Figure 11. In Figure 10 and Figure 11, N denotes code length, M represents mismatched filter order, ls refers to the biggest SNR loss. In short, We can know that the P4 code under this technique can get a good pulse compression performance while the SNR loss is very small.

3.4. Optimal design method combined arbitrary phase codes with side-lobe suppression filters optimization

This method finds the required codes and mismatch filters through applying the optimal algorithm to joint optimization of the arbitrary phase codes and the mismatch filter. Genetic algorithms(GA), simulated annealing algorithms(SA), gradient search procedure and nonlinear constrained optimization method are common optimization algorithms. Of course, these methods above reflect a certain advantages and disadvantages when used in the combined optimization. Inspired by the optimization waveform design method of the mutual information, document[5] put forward a kind of optimal design method combined arbitrary phase codes with side-lobe suppression filters optimization, it could be seen as an improvement of the method based on second-order cone programming. This method can obtain the final mismatch filter and the corresponding optimization code through lots of iterations. the core ideas of this method is, do some certain operations to the filter coefficients and the initial input signals at this time's iteration, then form a new phase encoding signal as the input signal for the next iteration, it can further reduce the range side-lobes of pulse compression results through iterating for a lot of times.

Using this method, under the condition of the maximum iteration times is 200 and maximum SNR loss is 0.75 dB, to design the optimized codes and the corresponding filter for the p4 codes of different length, we can get the results as illustrated in table 2. Among them, the practical optimized side-lobe refers to the highest side-lobe obtained from the compressed pulse by using joint optimization, the theoretic optimized side-lobe refers to the highest side-lobe obtained according to the matching theory. From the data of table 2, we can know that this method can get the best side-lobe close to matching filter theory only at the cost of small SNR loss, and filter order equals to the input code, so it will not increase the implement complexity of the practical system.

Take P4 code with the length of 100 as a example, firstly, we get the optimized filter coefficient and the corres-ponding code, and then , using the optimized filter coeffi-cient with the corresponding code for filtering processing, finally, we get the corresponding ambiguity function dia-gram as illustrated in Figure 12.

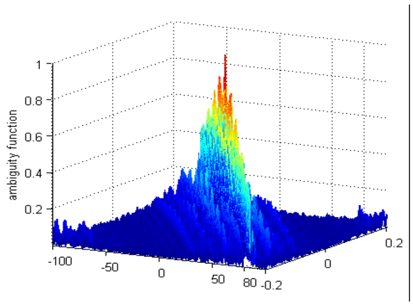

Fig.12. Partial ambiguity function for 100-element optimized code and filter

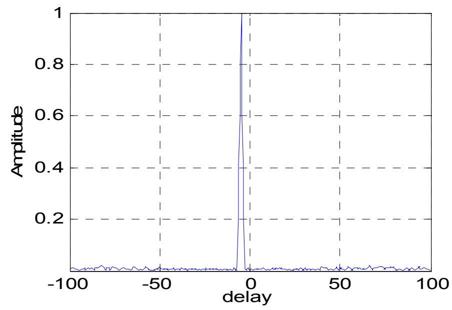

Fig.13. Ambiguity function versus dealy

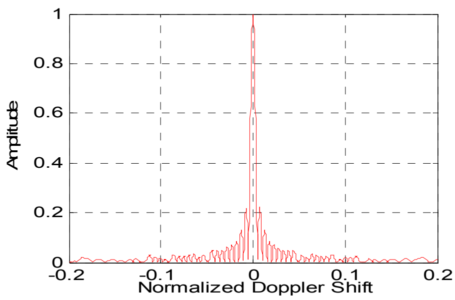

Fig.14. Ambiguity function versus frequency

Meanwhile, Figure 13 and Figure 14 give the corresponding distance ambiguity function diagram and speed ambiguity function diagram. We can see that this method could get very low and flat distance side-lobe from Figure 12 to 14.

4. Conclusions

This paper discusses and shows the results of the pulse compression performance of P4 code under different side-lobe reduction methods. The classical window function amplitude weightings can largely reduce the side-lobes but result in a SNR loss and degraded resolution similar to that of LFM waveform. For post compression 2-sample averager processing, the reduced side-lobes and the range resolution are higher than classical window function weighting, and good Doppler tolerance is maintained which is similar to classical window function weighting. It was shown that, the mismatch filter based on second order cone programming result in low PSL and very low SNR loss, the higher the mismatch filter order is, the lower the PSL we can get. The design method combined arbitrary phase codes with mismatch filters optimization result in the lowest PSL, the lowest SNR loss and the highest range resolution in these methods. However, good Doppler tolerance is not main-tained in second-order cone programming and combined optimal design. It is to deserved to be noted that, for classical window function weighting method, the side-lobes decrease as the number of P4 code elements increase which is not similar to LFM waveforms.

In fact, we could not find the best method to reduce side-lobes for pulse compression because all methods exist certain defect. According to the practical applications, we should choose the suitable method to play up strengths and avoid weakness, of course, do some corresponding improvements when needed, finally, we could realize the design better.

5. Acknowledgment

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Table 1. Performance of classical amplitude weightings for 100 element P4 code

Window Functions (Parameters)	**Target**				
	SNR Loss (dB)	***Broadening (-3dB)***	***Peak Side Level(dB)***	***Integrated Sidelobes(dB)***	***PSL (fd=0.05)***
Bartlett	1.2935	1.381	39.8245	19.743	36.8
Hann	1.8046	1.51	40.08	19.99	39.69
Hamming	1.3758	1.41	39.9105	19.731	37.29
Blackman	2.416	1.6797	38.3947	19.825	38.24
Kaiser(Beta=2)	0.20864	1.1876	30.2217	15.445	26.48
Kaiser(Beta=3)	0.57735	1.2610	34.85	17.738	30.18
Kaiser(Beta=4)	0.98957	1.3359	39.3431	19.19	34.31
Kaiser(Beta=5)	1.3702	1.4105	40.5533	19.822	38.37
Chebwin(r=30)	0.62551	1.2590	31.2004	12.224	27.98
Chebwin(r=40)	1.0318	1.3474	37.4455	18.126	33.93
Chebwin(r=50)	1.4725	1.4337	39.2296	19.813	37.88
Chebwin(r=60)	1.8472	1.519	39.4603	20.026	39.00

Table 2. The result of compressed pulse under optimized code and filter

Code Length	**Target**		
	Practical Optimized side-lobe(dB)	***Theoretic Optimized side-lobe(dB)***	***SNR Loss(dB)***
50	-34.5014	-33.9794	≤0.75
100	-39.9957	-40.0000	≤0.75
150	-41.6021	-43.5218	≤0.75
200	-42.5585	-46.0206	≤0.75
250	-44.6261	-47.9588	≤0.75
300	-45.6505	-49.5424	≤0.75