A Survey on Compressed Sensing Based Banknote Classification

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Abstract. In this paper, we propose a Compressed Sensing based Banknote Classification Method (CSBCM), the aim of which is to study the image classification of banknote in the complex lighting environment. After briefly analyzing the traditional methods of banknote classification and introducing the relevant concept of the compressed sensing theory, we explore the robustness of CS theory in the application of pattern recognition. The proposed method involves the creation of banknote image database, the construction of banknote dictionary bases and so on. All our experiments are made on the self-built banknote database and we compare the traditional ways of banknote classification with the CSBCM to lead to the conclusion of this text: the feature extraction is not critical in CSBCM and the calculating efficiency and the classification accuracy of the CSBCM is superior to that of the traditional method.

Keywords: Feature extraction; Compressed sensing; Banknote dictionary base; Banknote classification

1. Introduction

With the development of modern technology and the prosperity of world economy, the importance of banknote classification, whose purpose is to classify automatically the currency according to the banknote images, is increasingly notice-taking. As an important branch of pattern recognition, the result of banknote classification can not only provide people with rich information of currency, but can also free people from the tedious work as notes counting. Thus, banknote classification has a broad business application prospect.

The study of the traditional methods of banknote classification made its initial progress in the 1990s. After the preprocessing to the banknote images with random mask, F. Takeda and S. Omatu classified them by using the multi-layer neural network in 1993[1]. A. Frosini and M. Gori showed a banknote identifying and confirming system based on the neural network in 1996[2]. M. Tanaka applied the probabilistic principal component analysis (PPCA) into banknote classification in 1999[3]. M. Teranishi, after using the acoustic cepstrum patterns, classified banknote by the competitive neural network in 2000[4]. A. Ahmadi, first extract feature from the banknote image by PCA, used learning vector quantization network as classifier in 2003 [5]. Gai Shan and Liu Jiafeng identified the banknote by using the Hidden Markov Model which was based on the principle of structural risk minimization in 2009 [6].

In recent years, with the rising of the CS theory [7] [8], a new technology has been introduced to the field of pattern recognition. When used in the face recognition (a branch of pattern recognition), it shows great robustness to the identification of those faces which are either occlusion or corruption [9]. This paper is trying to explore the application of the compressed sensing in the Pattern Classification and take banknote classification as an example to illustrate it. The method of CSBCM is proposed based on the complete of the preprocessing works like the image acquisition, segmentation, normalization and so on.
The structure of the paper is as follows. Section 2 analyses the traditional methods of banknote classification. Section 3 briefly introduces the relevant content of compressed sensing theory. Section 4 describes the application of CS theory in banknote classification. And then, it comes to the experiments and discussions. Finally, conclusions and future work are summarized.

2. The Traditional Methods of Banknote Classification

The fulfillment of the traditional methods of banknote classification is mainly divided into three parts: banknote image acquisition and preprocessing, feature extraction and denomination classification (as show in Fig.1).

The acquisition and preprocessing of the banknote image is the base for the final completion of denomination classification. Its aim is to find the sub-window which consist the banknote image and to segment that image through preprocessing. The preprocessing of the original image is most of the time going like this: brightness compensating, color space transformation, edge detection, skew correction, image segmentation and normalization and some other steps when necessary.

![Fig.1. The flow chart of the system](image)

The part of feature extraction is to show the denomination information of banknote with a feature vector of high efficiency in describing the information through extracting certain features. And this part can provide strong support to the denomination classification. The general ways of feature extraction are: geometry-based feature extraction, feature extraction based on K-L transform, feature extraction based on Fisher criterion, feature extraction based on random mask, grid feature extraction and Harr-like rectangle feature extraction.

The part of denomination classification is to get the feature vector of banknote through the feature extraction, design the classifier by training or learning the input banknote feature vector and finally make the judgment of the denomination of the banknote. The common classifiers are minimum distance classifier, artificial neural networks, support vector machines and Hidden Markov Model-based classifier.

3. Brief Introduction of Compressed Sensing

Compressed sensing is a new theory about signal measurement and reconstruction proposed by Candès and Donoho in 2006[8]. It subverts the traditional Nyquist-Shannon Theorem and can perform the reconstruction of certain signal well with samples or measurements which are far less than the Nyquist frequency. The main contents of the CS theory are sparse representation of signal, the measurement of
encoding and reconstruction algorithm. Note the only priori condition of CS is that the signal must be sparse or compressible. The mathematical descriptions of the CS theory are summarized as follows:

Given A, the compressible signal \( x \) can be represented over the base \( V = [V_1, V_2, \ldots, V_n] \in \mathbb{R}^{m \times n} \) with \( n < k \). Signal \( x \) can be described as \( x = V\alpha, \alpha \in \mathbb{R}^k \), \( \|\alpha\|_0 < A \ll n \) (\( \|\alpha\|_0 \) is the \( l_0 \)-norm of \( \alpha \)). In CS theory, signal \( x \) is compressed by a projection matrix \( U \in \mathbb{R}^{R \times n} \) and then we can get the sensing vector \( y = UX \in \mathbb{R}^R \). Given dictionary \( V \), projection matrix \( U \) and \( y = UX = UV\alpha \), we can reconstruct the original signal \( x \). Let \( W = UV \in \mathbb{R}^{m \times k} \), then, \( x \) can be described as column vector \( \alpha \in \mathbb{R}^k \) which satisfying formula (1):

\[
y = W\alpha
\]

Because \( p \ll k \), formula (1) is underdetermined, so it has no unique solution.

\[
\hat{\alpha}_0 = \arg \min_{\alpha} \|\alpha\|_0 \quad s.t. \quad y = W\alpha
\]

The problem of finding the sparsest solution of an underdetermined system, as formula (2), is NP-hard. With the development of CS theory, Donoho proves that if the solution \( \alpha_0 \) is sparse enough, the solution of the \( l_0 \)-norm minimization problem (2) is equal to the following \( l_1 \)-norm minimization problem (3):

\[
\hat{\alpha}_1 = \arg \min_{\alpha} \|\alpha\|_1 \quad s.t. \quad y = W\alpha
\]

By finding the sparsest solution among all the possible \( \alpha \) which satisfying \( y = W\alpha \) can we reconstruct \( y \) from \( x \), however we should know that if we want reconstruct \( K \)-sparse signal, the measurement times \( M \) must satisfy formula \( M = O(K \log(n)) \) and measurement matrix \( W \) must satisfy restricted isometry property (RIP). After getting \( \hat{\alpha} \), we can get the reconstructed signal through \( \hat{x} = V\hat{\alpha} \). Several methods have been developed to solve the problem of finding sparse representation \( \alpha \) to reconstruct the original signal \( x \). There are three main reconstruction algorithms used: matching pursuit (MP), basis pursuit (BP) and orthogonal matching pursuit (OMP).

The above descriptions are under the assumption of formula (1), however, the real data are usually having noise. To the signal which has noise, formula (1) can be modified as:

\[
y = W\alpha + z
\]

Here \( z \in \mathbb{R}^R \) is a noise with bounded energy \( \|z\|_2 < \varepsilon \). So, we can also get the sparse solution \( \alpha \) by solving the \( l_1 \)-norm minimization problem like this:

\[
\hat{\alpha}_1 = \arg \min_{\alpha} \|\alpha\|_1 \quad s.t. \quad \|W\alpha - y\|_2 \leq \varepsilon
\]

4. Compressed Sensing Based Banknote Classification

The original intention to study the compressed sensing is for signal compression and representation, in which each basis function has no practical significance. Since the signal selects the base which can mostly represent it, the sparsest representation of signals has a good sense of identification. Yi Ma, together with some other people, used the sparse representation's sense of identification to realize face recognition effectively by using the training samples as the super-complete bases [9]. If each type of samples in the training samples is sufficient, the testing sample can be expressed as a linear combination of the training samples in the same type, whose representation is naturally sparse comparing with the whole entire sample set and thus the problem of classification can be solved through CS. This section will first articulate the details of CSBCM algorithm, and then the geometrical description of CSBCM will be given.

4.1. Details of CSBCM algorithm

Presume that there is no noise, \( B \) is the training sample set, \( y \) is the testing sample, and the same training samples are in the same sub-space, then the testing sample can be regarded as the linear combination of the same type of training sample. After the banknote images are transformed into \( m \) dimensional column vector \( V \), for each class \( i \), all the samples are the columns of the matrix \( V_i = [V_{i,1}, V_{i,2}, \ldots, V_{i,n}] \in \mathbb{R}^{m \times N} \). So if given a new testing sample \( y \) from class \( i \), it can be writing like this:

\[
y = Vx \in \mathbb{R}^m
\]

Here \( x = [0, \ldots, 0, a_{i,1}, a_{i,2}, \ldots, a_{i,10}, 0, \ldots, 0]^T \), \( T \) represents transpose, and the coefficients which have relation to \( i \) are not zero while the rest are all zero.
We can get the sparse representation \( \hat{x}_i \) through formula (3), and in the ideal condition, the non-zero coefficients in \( \hat{x}_i \) are the items which have relation to \( i \). For each class \( i \), let \( \eta_i \) be the function to show the relation to class \( i \), \( \eta \in \mathbb{R} \). And \( \eta(x) \in \mathbb{R} \) is a new vector which only selects the item that has relation to class \( i \). We can approximately estimate the testing sample \( y \) as \( \hat{y}_i = B \eta_i(\hat{x}_i) \), and then finish classification task by classify it to the class which can minimum the estimated deviation \( \rho(y) \) between \( y \) and \( \hat{y}_i \).

\[
\rho_i(y) = \left\| y - B \eta_i(\hat{x}_i) \right\| 
\]

(7)

The CSBCM algorithm is summarized as follows (Fig.2 will help you understand):

1. Construct the dictionary base \( B \) for the notes images, and choose a testing sample \( y \).
2. Form a random projection matrix \( P \), the dimension of which is \( r \times m \), \( r \) is the dimension of the reduced feature space.
3. Project the training samples and the testing sample into the feature space to get respectively \( \tilde{B} \) and \( \tilde{y} \).
4. Normalize the column vector \( \tilde{B} \) and \( \tilde{y} \).
5. Solve the \( l_1 \)-norm minimization problem.
6. Calculate the estimated deviation.
7. Minimize the estimated deviation to get the result of banknote classification.

4.2. Geometric interpretation of CSBCM

The CSBCM algorithm treats every banknote image as a column vector, and assumes that the dimension of high-dimensional space corresponds to the size of the image. In this case each picture in the database can be regarded as one point in the high-dimensional space and the testing picture is in the same space with the pictures of the database. The random projecting matrix projects the information of the banknote from a high dimensional space into a low dimensional space to form a feature space. Banknote classification can be described as the testing sample be expressed as minimum linear combination of all the training samples to form a sparse vector, that is mean the testing picture could be represented sparse by the subset of the database.
images. In another word, the corresponding data point of the testing picture is classified into the corresponding data points’ collection of the database images which can represent testing picture sparse.

5. **Experiments and Discussions**

Feature extraction methods in the traditional banknote classification are usually time and calculation consuming, so we tried to find other ways to replace that. The emergence of compressed sensing theory is really a great surprise to us (the selection of the best feature is no longer the key point, and the computation has also been greatly reduced). The task of classification can be finished quite simply, because we only need to extract the random feature and calculate the sparse representation on the dictionary bases of the banknote image. In this section, we will first introduce the image database of banknote created by us and then give the experimental details, finally, compare the proposed CSBCM algorithm with traditional classifier (SVMs).

5.1. **Image database of banknote**

To validate the proposed method, we have established the image database of RMB banknote (2005 edition). There are all together 1440 pieces in the database. We have processed with the image acquisition and preprocessing of the notes with denominations of 100, 50, 20, 10, 5, and 1. As each denomination of the RMB has four existing states, the banknote can then be classified into 24 categories. Through controlling the lighting conditions of the image acquisition, each banknote get 60 different images, and through preprocessing, we get those normalized banknotes pictures of 480 × 240 pixel.

5.2. **Experimental details**

After image acquisition and preprocessing, we finish the classification by using the nature that the testing banknote image can be represented by the subset of the training banknote image.

In the experiment, we divide the self-built banknote image database into training and testing note database, choose randomly 45 pictures from each category of note and put them into the training note database, the rest 15, to the testing database. The images in the training database are used to construct the dictionary bases of the banknote image, and the random projection matrix is constricted with the Gaussian random matrix. The solving of the minimization problem of $l_1$-norm is completed by using the function in the typical gradient projection algorithm of package GPSR_6.0 [10]. We calculate the classification accuracy of when the dimension of feature space is 28, 112 and 200. To avoid the influence of image selecting when constructing the training note database and the testing database, we repeat the above-mentioned experiment three times and the results is shown in Table 1.

<table>
<thead>
<tr>
<th>Feature space dimension</th>
<th>Down sample rate</th>
<th>Min-accuracy(%)</th>
<th>Max-accuracy(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>1/64</td>
<td>76.7</td>
<td>86.5</td>
</tr>
<tr>
<td>112</td>
<td>1/32</td>
<td>79.8</td>
<td>90.0</td>
</tr>
<tr>
<td>200</td>
<td>1/24</td>
<td>83.1</td>
<td>91.6</td>
</tr>
</tbody>
</table>

5.3. **Comparison CSBCM with SVMs**

In CSBCM, the selection of the best feature is no longer the key point, and the computation has also been greatly reduced. For traditional methods (see Figure 1 for details), in our experiment, we extract the grid features of banknote as the currency information, each note image is divided into 24 × 8 grids, the average gray scale of pixels in each grid is computed as the feature value, at last, the value of all the grid features are combined as a feature vector and be sent to the classifier to be classified. For traditional classifier, we choose
SVMs (OSU_SVM3.00) [11] and use radial-basis function as the kernel function. Fig.3 shows the classification accuracy by using different classifiers.

![Classification Accuracy Graph](image)

**Fig.3. Comparison of accuracy**

### 6. Conclusions and Future Work

By exploring the application of the CS theory in the pattern classification, this paper proposed a method of banknote image classification based on CS with the establishment of banknote image database, constructing of the banknote dictionary bases and the experiments of banknote image data in the complex lighting surroundings. After analyzing the traditional methods of banknote classification, we chose the grid feature as the input feature information, and adopting the SVMs as classifier to finish the classification task. The comparison of experimental results shows that the CSBCM algorithm proposed in the paper is superior to the traditional methods in computing efficiency and recognition rate. The future researching direction of us is to explore the recognition of banknote, esp. the defaced banknote in the more complex real world environment.

### 7. References


