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Enhanced DOA Tracking Based on the PASTd Algorithm with Outlier Rejection

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Abstract. Sometimes it is necessary to estimate the real-time direction of arrival (DOA); however, in real systems, there exist some negative factors, such as interference and non-Gaussian noise, resulting in outliers in the received data. It is not practical to estimate the real-time DOA by using subspace methods that are based on eigen decomposition of the covariance matrix, because they are of high complexity and sensitive to outliers. A novel approach is proposed to reject outliers and track DOA. Combined with the characteristics of array signals, the research theoretically analyzes the effects on DOA tracking caused by outliers, redefines the correlation between two adjacent snapshots and puts forward a method to detect and reject outliers automatically based on this correlation, and then adopts the projection approximation subspace tracking algorithm with deflation(PASTd) to track signal subspace. Finally, the method estimates the real-time DOA based on the estimated and updated signal subspace. Computer simulation verifies the effectiveness of the method.

Keywords: DOA; PASTd algorithm; correlation; subspace tracking; outlier rejection

1. Introduction

The super-resolution methods, represented by multiple signal classification (MUSIC) [1,2], are used a lot in DOA estimation. The prominent characteristic of these methods is they decompose the received data into two orthogonal subspaces—signal subspace, which is in accordance with the array steering vector, and noise subspace.

Most sensors work under terrible circumstances and sometimes the signal source to be tracked may be moving, so its DOA changes with time and it is necessary to estimate it in real-time. Traditional MUSIC-like subspace-based methods fail to work under these situations, as they need to calculate the real time covariance matrix of the received data and decompose the matrix to get the signal subspace, which make it hard to put them into practice. To estimate the real time subspace, there come many fast algorithms, among which one way is to turn the problem of seeking the signal subspace into an optimization problem[3]. Some methods are constrained optimization problem methods [3], represented by Conjugate Gradient; the others are unconstrained optimization methods, including the PAST and PASTd algorithm [4-6] proposed by Bin Yang, and these methods do not need to calculate the covariance matrix of the received data, but track the signal subspace iteratively and adaptively directly based on the received data. The PASTd algorithm is widely used for its quick convergence and excellent tracking performance. The research also adopts the algorithm.

Meanwhile, there also exist unstable factors in practice, such as unstable received channels. The received data of the array will also be affected by non-Gaussian noise, which will degrade the reliability and availability of the received data, resulting in wrong measurements, namely outliers. Outliers can be significantly large in amplitude or abnormal in phase, so they are not consistent with most measurements in

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the time series and will seriously affect the subspace-based algorithms, finally leading to wrong DOA estimations and bad DOA tracking. As a result, it is necessary to preprocess the data, detect and reject outliers online before subspace tracking.

The paper investigates how to weaken the influence of outliers and estimate the real time DOA to improve the performance of DOA tracking. The organization of the paper is as follows. Section II is a brief introduction to array signal processing. Section III describes the PASTd algorithm. Section IV analyzes the influence of outliers and how to reject outliers, in which a novel method is proposed to reject outliers and then the method is applied in DOA tracking. Section V compares the results of DOA tracking between before and after rejecting outliers with numerical simulation. Finally, some conclusions are given in Section VI.

2. Signal Model and Assumptions[2,3,7]

A general model for array signal processing is a uniform linear array (ULA) with M elements spaced by d=λ/2, λ denotes the wavelength of the center frequency w_0. Let the first element be the reference point. The channel number is the same as that of elements and there are P narrowband signal sources in the far field, and their complex envelopes are $s_1(t), s_2(t), ..., s_P(t)$. Angles between the signal incidence and the array axial direction are $\theta_1, \theta_2, \ldots, \theta_p$, which are within [0°,180°] and can change with time. Meanwhile there exist noises in the system. Both the array and sources are confined in a plane. Outputs of the array are $x_1(t), x_2(t), \ldots,$ $x_M(t)$. Let $n_m(t)$ be the additive noise for the mth channel at time t and $g_m e^{j\varphi_m}$ the complex gain of the mth channel. Both the noise and gain are unrelated to the mth signal. Sampling the received data by f_s, we can get the nth snapshot in vector form as follows.

$$\overrightarrow{X}(n) = GA(q)\vec{S}(n) + \overrightarrow{N}(n) \tag{1}$$

$$\begin{cases} \overrightarrow{X}(n) = \left[x_1(n), x_2(n), ..., x_M(n)\right]^T \\ G = diag\{[g_1 e^{j\phi_1}, g_2 e^{j\phi_2}, \cdots, g_M e^{j\phi_M}]\} \\ \vec{S}(n) = \left[s_1(n), s_2(n), ..., s_P(n)\right]^T \\ \overrightarrow{N}(n) = \left[n_1(n), n_2(n), ..., n_M(n)\right]^T \end{cases} \tag{2}$$

Where $\varphi_i = 2\pi d / \lambda \cos(\theta_i)$ denotes the phase delay of the received signals from the ith signal source between two adjacent elements, (·)T denotes the operation of transpose, let

$$\vec{a}(\theta_i) = \left[1, e^{-j\varphi_i}, \cdots, e^{-j(M-1)\varphi_i}\right]^T. \tag{3}$$

then,

$$\begin{aligned} A(\theta) &= \left[\vec{a}(\theta_1), \vec{a}(\theta_2), ..., \vec{a}(\theta_P)\right] \\ &= \begin{bmatrix} 1, & 1, & \cdots, & 1 \\ e^{-j\varphi_1}, & e^{-j\varphi_2}, & \cdots, & e^{-j\varphi_P} \\ \vdots & & \vdots & \\ e^{-j(M-1)\varphi_1}, & e^{-j(M-1)\varphi_2}, & \cdots, & e^{-j(M-1)\varphi_P} \end{bmatrix}. \end{aligned} \tag{4}$$

Some common assumptions in array signal processing are as follows.

1.Signal sources are narrowband signals in the far field and they are wide-sense stationary.

2.Additive noises are zero mean complex Gaussian processes and are independent of each other.

3. Signals and noises are independent of each other.

4. P is less than M.

5. Sensors in the array are isotropic and not cross coupled. Slow inconsistency of each channel is already corrected.

3. PASTd Algorithm

The paper adopts the PASTd algorithm to track the real time signal subspace based on the above model. Refer to [3-6] for more information about this algorithm. The following is the procedure of the PASTd algorithm.

$\overrightarrow{X}(n)$ denotes the received data at time *n*. The key problem is how to quickly update the signal subspace based on the newly received snapshot and the old signal subspace that is already computed.

1. initiate a proper $\vec{d}(0)$ and $W(0)$, let *k*=0, where $W(0)$ includes both the signal subspace and the noise subspace;
2. let *k*= *k*+1 and $\overrightarrow{X}_\Sigma = \overrightarrow{X}(k)$;
3. let *i*=1, and repeat the following procedure until *i*=*M*;
   3.1. calculate $y = W_i^H(k-1)\overrightarrow{X}_\Sigma$,where $W_i^H(k-1)$ is the *i*th column of the matrix $W(k\text{-}1)$;
   3.2. calculate $d_i(k) = \beta d_i(k-1) + |y|^2$,where $d_i(k)$ is the *i*th eigenvalue of $W(k)$, β is a constant forget factor [8], $|\cdot|$ denotes the absolute value sign;
   3.3. calculate the error $\vec{e} = \overrightarrow{X}_\Sigma - W_i(k-1)y$;
   3.4. calculate $W_i(k) = W_i(k-1) + [y^H / d_i(k)]\vec{e}$, $(\cdot)^H$ denotes the operation of conjugate transpose;
   3.5. $\overrightarrow{X}_\Sigma = \overrightarrow{X}_\Sigma - W_i(k)y$;
4. extract the signal subspace and the noise subspace from $\{W_i(n), i = 1, 2, \cdots, M\}$, and the signal subspace is the former *P* eigen vectors and the noise subspace the rest.
5. stop when *k*=*n*, or else turn to step 2.

Thus, the updated signal subspace can be obtained through the above procedure. Meanwhile, it shows that an inaccurate subspace estimation at time *n* will badly affect the next few subspace estimations.

4. Analysis of Effect of Outliers and Outlier Rejection Method [7]

4.1. Analysis of the Effects of Outliers

Although the occurrence of outlier is a small probability event, its effect on data quality cannot be neglected. In nature, the received data directly affect the estimation precision of the signal subspace. In this section, we analyze the effects of outliers based on covariance matrix [7]. In general, outliers will largen the variance of the amplitude or phase of the channel gain. The study attributes the occurrence of outliers to the instability of the channel gain, which can be viewed as a complex random variable and unusually remains stable, while sometimes the gain may suddenly become unstable. This is a fast inconsistency of the channel, which happens between snapshots and cannot be corrected.

For simplicity, the research only discusses the situation when there is only one signal source. According to (1), the covariance matrix of array output data can be written as

$$R = \sigma_s^2 E\{Ga(\theta)a^H(\theta)G^H\} + \sigma_n^2 I \,, \tag{5}$$

where ${\sigma_s}^2$ is the variance of the signal, σ_n^2 the variance of the noise, E the operation of expectation and I a unit matrix.

Under ULA, the element in the mth row and nth column of the covariance matrix is

$$r_{mn} = {\sigma_s}^2 E\{g_m g_n e^{j(\varphi_m - \varphi_n)}\} Z^{m-n} + \sigma_n^2 \delta_{mn}\,, \tag{6}$$

where $Z = e^{j\varphi}$, $\varphi = (2\pi d / \lambda)\cos(\theta)$, $g_m e^{j\varphi_m}$ is the complex gain of the mth channel and $\delta_{mn} = \begin{cases} 1 & m = n \\ 0 & else \end{cases}$.

Suppose the channels are independent, the amplitude and phase of every channel are also uncorrelated, and they follow Gaussian distributions, $g_m \sim N(1, \sigma_g^2)$, $\varphi_m \sim N(0, \sigma_\varphi^2)$, then:

$$r_{mn} = {\sigma_s}^2 E\{g_m g_n\} E\{e^{j(\varphi_m - \varphi_n)}\} Z^{m-n} + \sigma_n^2 \delta_{mn}\ . \tag{7}$$

Let $a = (1 + \sigma_g^2 + \sigma_n^2 / \sigma_s^2) e^{\sigma_\varphi^2/2}$ and $b = e^{-\sigma_\varphi^2/2}$, so the specific form of the covariance is as follows.

$$R = {\sigma_s}^2 b \begin{bmatrix} a & Z^{-1} & \cdots & Z^{-(M-1)} \\ Z & a & \cdots & bZ^{-(M-2)} \\ \vdots & & \vdots & \vdots \\ Z^{M-1} & bZ^{M-2} & \cdots & a \end{bmatrix} \tag{8}$$

The biggest eigenvalue and the corresponding eigenvector are as follows.

$$\lambda_1 = {\sigma_s}^2 b[a + (M-2)b\sqrt{(bM)^2 + 4(b^2-1)(M-1)} / 2] \tag{9}$$

$$\vec{e}_s = [1, CZ, \cdots, CZ^{M-1}]^T / \sqrt{1 + (M-1)C^2} \tag{10}$$

where $C = (b(a - \lambda_1) - 1) / (a - b - \lambda_1)$. According to (10), if the phase is stable, namely $\sigma_\varphi^2 = 0$, b=1and c=1, then the eigenvector $\vec{e}_s \in \vec{a}(\theta)$ and the correct estimation of signal subspace is achieved. However, an unstable phase leads to a larger σ_φ^2, then $\vec{e}_s \notin \vec{a}(\theta)$; When both the phase and amplitude are unstable, their influences on the whole are a larger σ_φ^2 and σ_g^2 and a worse signal subspace estimate.

In conclusion, the signal subspace estimation can be badly influenced by outliers, with the same effects as a more unstable channel phase. A bad signal subspace estimation will lead to a wrong DOA estimation. In practice, there exist approximations in the PASTd algorithm and the computation of the covariance matrix. As a result, it is necessary to evaluate the real time received data and reject possible outliers to improve the performance of DOA tracking. For further details about the influence of instability of the channel gain, please refer to [7].

4.2. A Novel Method to reject outliers

In other fields, such as failure analysis and remote sensing data processing, there are many methods about how to detect and eliminate outliers [9-13], for example, extrapolation fitting and difference detection. Some of them are introduced briefly next.

1) *kalman filter[10].*

This method needs to model the motion state of the target and then it rejects outliers based on prediction error filtering. However, it is impossible to model the motion state of the signal source in passive DOA tracking using kalman filter.

2) 3σ *criterion[13].*

It requires the received data follow Gaussian distribution. If the difference value between the data and the expectation of the distribution is three times more than the standard deviation, the data will be regarded as an outlier.

Some of the aforementioned methods are of high computation, some cannot deal with complex data, and some ask for a special data model. Overall, most of them cannot be applied in this research. The study proposes a novel method to reject possible outliers automatically.

The study considers the array data in one snapshot as M samples of a variable, denoted by $X_{\#}(k)$. Among the M samples there may exist an outlier or more. Assuming that outliers are isolated in the time series , this agrees with general situation. The analysis finds that the correlation (redefined in this paper) between two snapshots has excellent consistency, which means that the correlation between $\vec{X}(k)$ and $\vec{X}(k+1)$ is close to the one between $\vec{X}(k+1)$ and $\vec{X}(k+2)$; however, outlier will destroy the consistency and weaken the correlation between snapshots. According to this, the novel method dynamically sets the threshold, calculates the coefficient of correlation between the current snapshot and the last snapshot without outliers, and compares it to the threshold, then decides whether an outlier exists or not in the current snapshot and finally decides whether to reject the current snapshot or use it to update the subspace. The derivation is as follows.

First redefine the covariance as follows.

$$\begin{aligned} &Cov(X_{\#}(k), X_{\#}(k+1)) = \\ &E\{[X_{\#}(k) - E(X_{\#}(k))][X_{\#}(k+1) - E(X_{\#}(k+1))]^{*}\} \end{aligned} \tag{11}$$

where $(\cdot)^*$ denotes the operation of conjugate. Then redefine the coefficient of correlation as follows.

$$r_{X_{\#}(k),X_{\#}(k+1)} = \frac{Cov(X_{\#}(k), X_{\#}(k+1))}{\sqrt{Cov(X_{\#}(k))Cov(X_{\#}(k+1))}} \tag{12}$$

When $| r_{X_{\#}(k),X_{\#}(k+1)} |=1$, $\vec{X}(k)$ and $\vec{X}(k+1)$ are linearly correlated; the smaller the $| r_{X_{\#}(k),X_{\#}(k+1)} |$, the weaker the correlation. In this study, the weaker the correlation, the more influential the environment on the received data. When the coefficient of correlation is smaller than the threshold, the study considers there exist outliers in the snapshot; now the snapshot must be abandoned and the algorithm turns to the next snapshot without outliers to update the subspace.

Likewise, the section uses a narrowband signal source $\exp(j\omega_0 t + j\phi(t))$ as an example to illustrate the method, ω_0 is the carrier frequency, $\phi(t)$ is the signal-related phase which is definite at a certain moment. Sample the signal by f_s and denote the kth and (k+1)th snapshot as $\vec{X}(k)$ and $\vec{X}(k+1)$, respectively. When M is even, it is easy to get

$$E(X_{\#}(k)) \approx (\sum_{i=1}^{M} X_i(k)) / M = 0, \tag{13}$$

where

$$\begin{aligned} \vec{X}(k) = [&\exp(j\omega_0 k / f_s + j\phi(k / f_s))a_1(\theta) + n_1(k / f_s) \\ &\exp(j\omega_0 k / f_s + j\phi(k / f_s))a_2(\theta) + n_2(k / f_s) \\ &\cdots \\ &\exp(j\omega_0 k / f_s + j\phi(k / f_s))a_M(\theta) + n_M(k / f_s)] \end{aligned} . \tag{14}$$

Next calculate the covariance digitally and approximately.

$$Cov(X_{\#}(k), X_{\#}(k+1)) = \sum_{i=1}^{M} (X_i(k) \times X_i^*(k+1)) / M \quad (15)$$

Considering the random nature of noise, Equation (15) can further be approximated as follows.

$$Cov(X_{\#}(k), X_{\#}(k+1)) \approx \exp(-j(\omega_0 / f_s + \Delta\phi)) \quad (16)$$

As $Cov(X_{\#}(k), X_{\#}(k)) = 1$, the coefficient of correlation is

$$r_{X_{\#}(k), X_{\#}(k+1)} \approx \exp(-j(\omega_0 / f_s + \Delta\phi)) . \quad (17)$$

Equation (17) shows that the coefficient of correlation between two adjacent snapshots is only relevant to the sampling rate, the carrier wave and the signal, when the signal is stationary and of narrowband.

4.3. Procedure of DOA Tracking and Outlier Rejection

Assuming the exact eigenvalues and eigenvector matrix at time *N*-1 are obtained, they are $\vec{d}(N-1)$ and $W(N-1)$. Summarize the new procedure of DOA tracking based on the PASTd algorithm with outlier rejection as follows.

1. newly received data $\vec{X}(N)$
2. set the threshold γ. First, calculate at least three coefficients of correlation and their absolute values, between $\vec{X}(N-1)$ and $\vec{X}(N-2)$, $\vec{X}(N-2)$ and $\vec{X}(N-3)$, $\vec{X}(N-3)$ and $\vec{X}(N-4)$, respectively; Second, work out the medium value (not average) of the coefficients; then set the threshold γ to be 0.97 (changeable in practice) of the medium value.
3. determine whether $\vec{X}(N)$ includes outliers or not. Calculate the coefficient of correlation between $\vec{X}(N)$ and $\vec{X}(N-1)$. If the coefficient is smaller than γ, reject $\vec{X}(N)$ and turn to next newly received snapshot; or else, continue.
4. let *k*=*N*, $\vec{X}_{\Sigma} = \vec{X}(k)$.
5. let *i*=1, and repeat the following procedure until *i*=*M*

   5.1. calculate $y = W_i^H(k-1)\vec{X}_{\Sigma}$, where $W_i^H(k-1)$ is the *i*th column of matrix $W(k-1)$;

   5.2. calculate $d_i(k) = \beta d_i(k-1) + |y|^2$, where $d_i(k)$ is *i*th the eigenvalue of $W(k)$;

   5.3. calculate the error $\vec{e} = \vec{X}_{\Sigma} - W_i(k-1)y$;

   5.4. calculate $W_i(k) = W_i(k-1) + [y^H / d_i(k)]\vec{e}$;

   5.5. $\vec{X}_{\Sigma} = \vec{X}_{\Sigma} - W_i(k)y$
6. extract the signal subspace and the noise subspace from $\{W_i(n), i = 1, 2, \ldots, M\}$, and the signal subspace is the former *P* eigen vectors and the noise subspace the rest;
7. estimate the DOAs of the *P* sources via the estimated signal or noise subspace;
8. turn to step 1 and continue.

5. Simulation and Discussion

In this part, we carry some simulations to demonstrate the effectiveness of our algorithm in DOA tracking. For comparison, the simulations include results of both before and after rejecting the outliers.

The simulation parameters are as follows. We consider two moving targets emitting uncorrelated narrowband signals that impinges on a ULA of ten sensors separated by half a wavelength. The initial DOAs of the targets are 80° and 130°,while the final DOAs are 110°and 80°, The simulations have SNRs of 15 dB, 10 dB, 5 dB, The forget factor in the PASTd algorithm is chosen to be 0.95 (empirical value), The number of snapshots is 20000.

The setup of outliers is as follows. The received signals are disturbed by abnormal values randomly with a probability of 0.3 percent and the specific operation is multiplying $X(m,n)$ with a complex number whose amplitude is fifteen times the average amplitude of the signals and phase is approximately π.

To reduce the workload, the method calculates DOA every ten snapshots, but rejects outliers and calculates the signal subspace every snapshot, so the figures just contain 2,000 snapshots. The results are displayed in Figure 1-3.

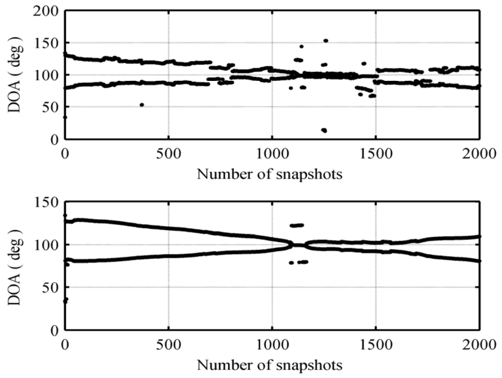

Figure 1. Comparison of DOA tracking results between before and after rejecting outliers, SNR=15dB

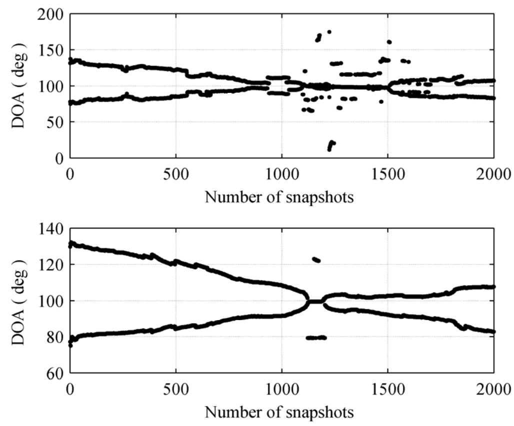

Figure 2. Comparison of DOA tracking results between before and after rejecting outliers, SNR=10dB

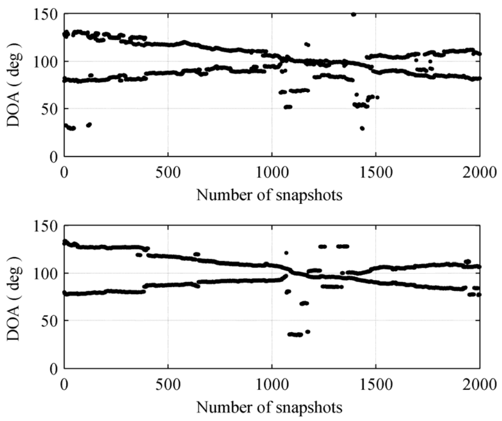

Figure 3. Comparison of DOA tracking results between before and after rejecting outliers, SNR=5dB

It can be seen the performance of DOA tracking improves considerably after outlier rejection, especially in the continuity of DOA tracking. Meanwhile, the performance of the algorithm increases with SNR. Overall, the novel algorithm with outlier rejection greatly enhances DOA tracking when outliers exist.

Also, we can see the result of DOA tracking after rejecting outliers still is not ideal (two crosswise straight lines), as there are some approximations in the algorithm and some residual outliers exist. Meanwhile, the PASTd algorithm cannot distinguish two different DOAs when they are closely spaced, because the number of the sources is set to be two through the whole tracking course and there will be a false estimate when the angles are almost the same.

Still there are some imperfections in the novel approach, for example, the threshold is determined by experience and both the number of snapshots and the sampling rate affect the performance of the PASTd algorithm. The higher the sampling rate, the better the performance.

6. Conclusion

To estimate the real time DOA and mitigate the influence of outliers, the research proposes a novel method to track DOA based the PASTd algorithm and simultaneously reject outliers automatically. The PASTd algorithm is used to update the signal subspace. This method considers that there is certain correlation between two adjacent received snapshots. Then the method rejects outliers based on this correlation, because outliers will destroy the correlation. The new method can work without much prior knowledge. The numerical results verify the effectiveness of the method.

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