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Adaptive Array Filter Using a Combined QLMS-CLMS Algorithm

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Abstract. A new adaptive filter, called quaternion least mean square plus complex least mean square (QLMS-CLMS) filter, is proposed for an electromagnetic vector sensor array. The QLMS-CLMS filter involves the use of two LMS-based sections. The first one is the quaternion LMS algorithm, the other is the complex LMS algorithm. Computer simulation results show that QLMS-CLMS filter is superior in the output signal-to-noise ratio over earlier CLMS filter. In addition, the proposed QLMS-CLMS algorithm is robust when the interference signal is coherent with the desired signal.

Keywords: Adaptive filter; Convergence; QLMS-CLMS; CLMS; SINR; Stepsize;

1. Introduction

In recent years, adaptive filter has become a key component for various applications, such as radar, sonar, communications, seismic, and cellular mobile communications [1]. And in the field of adaptive filter, there are two research directions. One is to achieve fast convergence with an adaptive algorithm, the other is to achieve higher output SINR in complex electromagnetic environment.

Because of its simplicity and robustness, the least mean square (LMS) algorithm has become one of the most popular adaptive signal processing techniques adopted in many applications including array filter [2], [3].

In many other fields, the simultaneous processing of the two dimensions of a signal can lead to a more efficient signal processing algorithm than processing each dimension separately [4]. As the quaternion domain represents an extend of the complex field, it has stronger constraint than complex.

This paper presents a very different approach to achieve higher value of output SINR with a LMS-based filter. As shown in fig. 1, the proposed quaternion least mean square plus complex least mean square (QLMS-CLMS) filter involves the use of two LMS algorithm sections, QLMS and CLMS. The first one is the quaternion LMS algorithm which the desired signal is separated from the sample signal, and the other is the complex LMS algorithm which the remain interference and noise are cancelled.

2. Derivation of the Qlms Algorithm

If quaternion-valued $x(n)$ and quaternion-valued $d(n)$ denote respectively the output vector and the desired signal, we assume $x(n)$ is a random vector and employ a linear model to estimate the desired signal. It is given by $\hat{d}(n)=w^{\triangleleft}(n)x(n)$, where $w(n)$ is the quaternion-valued weight vector and the symbol $(\cdot)^{\triangleleft}$ denotes the quaternion transposition-conjugation operator. The estimated error is defined as

$$e(n) = d(n) - \hat{d}(n) = d(n) - w^{\triangleleft}(n)x(n) \tag{1}$$

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Defining the real-valued quadratic, time-varying cost function based on MSE as

$$J(n)=|e(n)|^2=e(n)e^{\mathrm{C}}(n) \tag{2}$$

The symbol $(\cdot)^{\mathrm{C}}$ denotes the quaternion conjugation operator.

Based on the cost function, we adopt the steepest descent optimization to derive the optimal weight vector. Since the C-expansion of quaternion $w(n)$, $x(n)$, $d(n)$ and $e(n)$ are respectively $w(n)=w_1(n)+jw_2(n)$, $x(n)=x_1(n)+jx_2(n)$, $d(n)=d_1(n)+jd_2(n)$ and $e(n)=e_1(n)+je_2(n)$, the estimated error can be rewritten as

$$e(n)=[d_1(n)-\overline{w}^{\mathrm{H}}(n)\overline{x}_1(n)]+j[d_2(n)-(\overline{w}^{\mathrm{H}}(n)\overline{x}_2(n))^*] = e_1(n)+je_2(n) \tag{3}$$

where $\overline{w}(n)=\begin{bmatrix} w_1(n) \\ w_2(n) \end{bmatrix}$, $\overline{x}_1(n)=\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix}$, $\overline{x}_2(n)=\begin{bmatrix} x_2^*(n) \\ -x_1^*(n) \end{bmatrix}$. The symbol $(\cdot)^{\mathrm{H}}$ denotes the complex transposition-conjugation operator. Substituting (3) in (2), the cost function can be written as

$$\begin{aligned} J(n) &= e(n)e^{\mathrm{C}}(n) = (e_1(n)+je_2(n))(e_1(n)+je_2(n))^{\mathrm{C}} \\ &= d_1(n)d_1^*(n)-d_1(n)(\overline{w}^{\mathrm{H}}(n)\overline{x}_1(n))^* \\ &\quad -(\overline{w}^{\mathrm{H}}(n)\overline{x}_1(n))d_1^*(n)+\overline{w}^{\mathrm{H}}(n)(\overline{x}_1(n)\overline{x}_1^{\mathrm{H}}(n))\overline{w}(n) \\ &\quad +d_2^*(n)d_2(n)-(\overline{w}^{\mathrm{H}}(n)\overline{x}_2(n))d_2(n) \\ &\quad -d_2^*(n)(\overline{w}^{\mathrm{H}}(n)\overline{x}_2(n))^*+\overline{w}^{\mathrm{H}}(n)(\overline{x}_2(n)\overline{x}_2^{\mathrm{H}}(n))\overline{w}(n) \end{aligned} \tag{4}$$

Based on the definition of complex gradient in [5], the following gradients need to be calculated

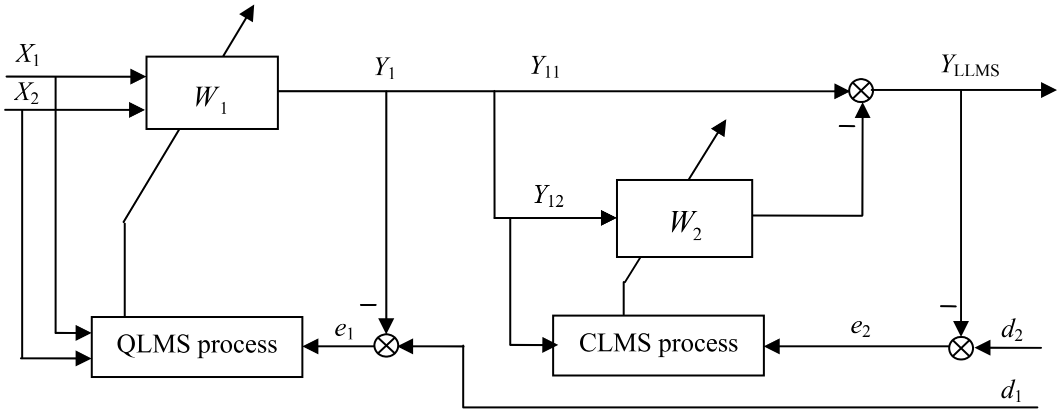

Figure 1. QLMS-CLMS filter

$$\frac{\partial J(n)}{\partial \overline{w}^*(n)} = -\overline{x}_1(n)e_1^*(n)-\overline{x}_2(n)e_2(n) \tag{5}$$

Subsequently, the update of the adaptive weight vector can be expressed as

$$\overline{w}(n+1)=\overline{w}(n)-\mu\frac{\partial J(n)}{\partial \overline{w}^*(n)}=\overline{w}(n)+\mu(\overline{x}_1(n)e_1^*(n)+\overline{x}_2(n)e_2(n)) \tag{6}$$

where μ denotes the iterative stepsize. Multiplying the two left side of equation (6) by $[I, jI]$, we have

$$w(n+1)=w(n)+\mu x(n)e^{\mathrm{C}}(n) \tag{7}$$

3. Convergence of the Qlms Algorithm

The error signal for updating the QLMS algorithm at the n^{th} iteration is given by

$$e(n)=d(n)-y(n)=w_{opt}^{\triangleleft}(n)x(n)-w^{\triangleleft}(n)x(n)=-V^{\triangleleft}(n)x(n) \tag{8}$$

where w_{opt} is the optical weight vector and $V(n)=w(n)-w_{opt}(n)$.

Now, the convergence performance of the QLMS algorithm can be analyzed in terms of the expected value of $|\mathrm{e(n)}|^2$ such that

$$\xi(n) = E[|e(n)|^2] = E[|-V^{\triangleleft}(n)x(n)|^2]$$
$$= V(n)E[x(n)x^{\triangleleft}(n)]V^{\triangleleft}(n) = V^{\triangleleft}(n)Q(n)V(n) \tag{9}$$

where $E[\cdot]$ denotes expectation and Q is the correlation of the input signals, given by $Q(n) = E[x(n)x^{\triangleleft}(n)]$.

Subtracting w_{opt} the both sides of equation (7), we have

$$w(n+1) - w_{opt}(n+1) = w(n) - w_{opt}(n) + \mu x(n)e^{\mathrm{C}}(n) \tag{10}$$

By substituting (8) into (10), the weight vector error is given by

$$V(n+1) = V(n) - \mu x(n)x^{\triangleleft}(n)V(n)$$
$$= (I - \mu x(n)x^{\triangleleft}(n))V(n) = (I - \mu Q(n))V(n) \tag{11}$$

Using eigenvalue decomposition (EVD) of Q

$$Q = q\Lambda q^{-1} = q\Lambda q^{\triangleleft} \tag{12}$$

where $\Lambda = diag[\lambda_1, \lambda_2, ..., \lambda_N]$ is the diagonal matrix of eigenvalues of Q for a N element array; $diag[...]$ is the diagonal of Q.

Alternatively, (11) can be written as

$$V(n) = q(I - \mu\Lambda)q^{\triangleleft}V(n-1) = q(I - \mu\Lambda)^n q^{\triangleleft}V(0) \tag{13}$$

By substituting (13) into (9), the MSE at the n^{th} iteration is given by

$$\xi(n) = V^{\triangleleft}(0)q\left|(I - \mu\Lambda)^n\right|^2 q^{\triangleleft}V(0) \tag{14}$$

From (14), with the term $I - \mu\Lambda$ converging, the mean square error will finally approach its minimum value, such that

$$\lim_{n\to\infty}\xi(n) = 0 \tag{15}$$

4. Range of Stepsize Values for the Qlms Algorithm

In the adaptive algorithm, the range of the stepsize is directly related to the convergence or divergence. Here, we will discuss the range of stepsize for QLMS algorithm.

Defining the priori error $e(n)$ and posteriori error $e_p(n)$ of QLMS algorithm as

$$e(n) = d(n) - w^{\triangleleft}(n)x(n)$$
$$= [d_1(n) - \overline{w}^{\mathrm{H}}(n)\overline{x}_1(n)] + j[d_2(n) - (\overline{w}^{\mathrm{H}}(n)\overline{x}_2(n))^*]$$
$$= e_1(n) + je_2(n)$$
$$e_p(n) = d(n) - w^{\triangleleft}(n+1)x(n)$$
$$= [d_1(n) - \overline{w}^{\mathrm{H}}(n+1)\overline{x}_1(n)] + j[d_2(n) - (\overline{w}^{\mathrm{H}}(n+1)\overline{x}_2(n))^*] \qquad = e_{p1}(n) + je_{p2}(n)$$

According to [6], $\|e_p(n)\|$ can be written as

$$\|e_p(n)\|^2 = \|e(n)\|^2 + \Delta\overline{w}^{\mathrm{H}}(n)\frac{\partial\|e(n)\|^2}{\partial\overline{w}^*(n)} \tag{16}$$

For steepest descent, the weight vector for the gradient can be expressed as

$$\Delta\overline{w}(n) = \overline{w}(n+1) - \overline{w}(n) = -\mu\frac{\partial J(n)}{\partial\overline{w}^*(n)} \tag{17}$$

By substituting (17) into (16), the posteriori error $e_p(n)$ is given by

$$\|e_p(n)\|^2 = \|e(n)\|^2 - \mu\left|\frac{\partial\|e(n)\|^2}{\partial\overline{w}^*(n)}\right|^2 \tag{18}$$

We may rewrite (18) according to (5)

$$\left\|e_p(n)\right\|^2 = \left\|e(n)\right\|^2 - \mu\left|-\bar{x}_1(n)e_1^*(n) - \bar{x}_2(n)e_2(n)\right|^2$$
$$= \left\|e(n)\right\|^2(1-\mu\left\|x(n)\right\|^2) \qquad (19)$$

Thus, for QLMS algorithm, convergence can be satisfied if $\| e_p(n) \| < \| e(n) \|$. This gives

$$0 < \mu < \frac{1}{\left\|x(n)\right\|^2} \qquad (20)$$

5. The Qlms-clms Filter

Assume that two narrowband, completely polarized plane-waves impinge on the 2-components vector-sensor [7]. One is the desired signal characterized by its arrival angles (θ_d, ϕ_d) and polarization parameters (γ_d, η_d), whereas the other is the interference signal characterized by its arrival angles (θ_i, ϕ_i) and polarization parameters (γ_i, η_i). Thus, the quaternion-valued output of the 2-components vector-sensor is given by

$$X(t) = P(\theta_d, \phi_d, \gamma_d, \eta_d)\bar{s}_s(t) + P(\theta_i, \phi_i, \gamma_i, \eta_i)\bar{s}_i(t) + e(t) \qquad (21)$$

The series $X(n)$ obtained by sampling $X(t)$ is multiplied by a quaternion-valued weight $W_1(n)$ to produce the first output of filter, it is given by

$$Y_1(n) = W_1^{\lhd}(n)X(n) = W_1^{\lhd}(n)[P_s\bar{s}_s(n) + P_i\bar{s}_i(n) + e(n)] \qquad (22)$$

where $P(\theta_d, \phi_d, \gamma_d, \eta_d)$ is denoted by P_s and $P(\theta_i, \phi_i, \gamma_i, \eta_i)$ is denoted by P_i.

Here we assume $W_1(n) = W_{opt1}(n) = Q_1^{-1}(n)Z_1(n)$ and $d_1(n) = \bar{s}_s(n) + j*0$.

where $Q_1(n) = E[X_1(n)X_1^{\lhd}(n)]$ and $Z_1(n) = E[X_1(n)d_1^{\lhd}(n)]$.

According to (21), $Q_1(n)$ and $Z_1(n)$ can be written as

$$Q_1(n) = E[X_1(n)X_1^{\lhd}(n)] = P_sP_s^{\lhd} + P_iP_i^{\lhd} + 1 \qquad (23)$$
$$Z_1(n) = E[X_1(n)d_1^{\lhd}(n)] = [P_s]_1 + j[P_s]_2 = P_s \qquad (24)$$

Now the optical weight vector can be written as

$$W_1(n) = W_{opt1}(n) = Q_1^{-1}(n)Z_1(n) = (P_sP_s^{\lhd} + P_iP_i^{\lhd} + 1)^{-1}P_s \qquad (25)$$

By substituting (25) into (22), the first output of filter is given by

$$Y_1(n) = (P_sP_s^{\lhd} + P_iP_i^{\lhd} + 1)^{-1}P_s^{\lhd}X(n)$$
$$= (P_sP_s^{\lhd} + P_iP_i^{\lhd} + 1)^{-1}P_s^{\lhd}[P_s\bar{s}_s(n) + P_i\bar{s}_i(n) + e(n)] \qquad (26)$$

As shown in fig. 1, the second output of filter at the n^{th} iteration is given by

$$Y_{LLMS}(n) = [Y(n)]_1 - W_2^H(n)[Y(n)]_2 \qquad (27)$$

From (26) and (27), it is noted that $[Y_1(n)]_1$ include the desired signal mainly, the interference signal and noise at the same time, and $[Y_1(n)]_2$ doesn't include the desired signal but include the interference signal and noise. The second part of QLMS-CLMS filter is to cancel the remain interference signal and noise of $[Y_1(n)]_1$.

6. Simulation Results

In this section we give numerical examples to show the performance of the proposed algorithm. We will consider an array that two 2-components vector-sensors are placed uniformly with an interelement spacing of half wavelength. Since all formulated problems in the proposed algorithm are quaternion, we adopt the quaternion toolbox, qtfm [8] to solve the formulated problems. For rigor, the performance of QLMS-CLMS filter is compared with CLMS filter.

Assume that two narrowband, completely polarized plane-waves impinge on this array. One is the desired signal characterized by its arrival angles $(40^\circ, 60^\circ)$ and polarization parameters $(30^\circ, 120^\circ)$, whereas the other is the interference signal characterized by its arrival angles $(20^\circ, -45^\circ)$ and polarization parameters $(60^\circ, -60^\circ)$. These results have been obtained from 100 individual simulation runs.

Fig. 2 shows the convergence behaviors of the two adaptive schemes for SNR value of 10 dB and input INR value of 10 dB. It is observed that under the given conditions, the QLMS-CLMS filter exhibits similar performance of convergence as the CLMS filter.

Fig. 3 shows the output SINR with input SNR value of 10 dB and input INR value of 10 dB. As discussed in section Ⅴ, we know that the proposed algorithm can cancel the interference signal and noise from the sample signal. As expected, the proposed filter achieves higher value of output SINR than CLMS filter..

As shown in fig. 4, it illustrates that the robustness of QLMS-CLMS filter is superior over CLMS fiter when the interference signal is coherent with the desired signal, with input SNR value of 10 dB and input INR value of 10 dB and the snapshot is defined as 40.

7. Conclusion

In this paper, a new adaptive filter，called quaternion least mean square-complex least mean square (QLMS-CLMS) filter, has been applied to the EM vector-sensor processing field. A rigorous analysis has shown that QLMS-CLMS filter can achieve higher value of output SINR to make advantage of quaternion. In addition, the proposed algorithm is robust when the interference signal is coherent with the desired signal.

8. References

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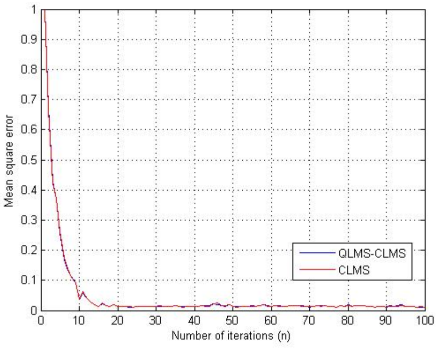

Figure 2. The convergence of QLMS-CLMS and CLMS filters with input SNR=INR=10dB

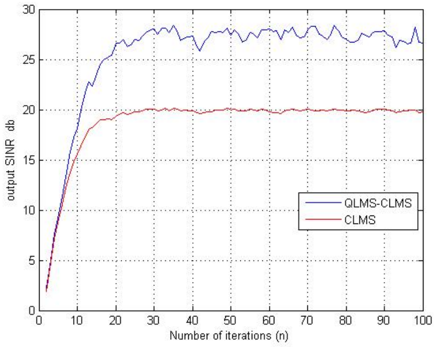

Figure 3. The output SINR of QLMS-CLMS and CLMS filters for iterations with input SNR=INR=10dB

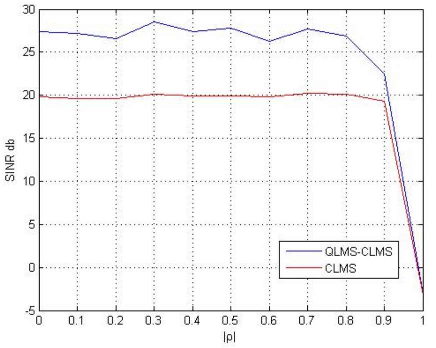

Figure 4. The influence of correlation coefient ρ on the output SINR.