

A Multiple Signal Classification Algorithm Based on Niche Differential Evolution

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Abstract. In order to overcome the disadvantage of slow speed of the multiple signal classification (MUSIC) caused by huge computational load, MUSIC algorithm based on niche differential evolution (DE) is proposed in this paper. Firstly, DE is improved by dividing the mutation operation into within-niche mutation and global mutation, besides the parameters are self-adapted to improve exploration ability, as well as to speed up the local convergence rate. And then the improved DE is combined with niche technique to solve multi-modal problems. The simulation results illustrate that the new algorithm can effectively estimate the directions of arrival (DOA) from multiple one-dimensional or two-dimensional angles with better accuracy and rate.

Keywords: niche; multiple signal classification; differential evolution; multi-modal problem.

1. Introduction

DOA estimates is a primary research direction of array signal processing, which is widely applied to many fields such as radar, sonar and communication. Over the past two decades, one kind of DOA estimation approaches based on sub-eigenspace analysis represented by MUSIC is highly valued. MUSIC can be applied to arbitrary array geometry with its error variance approaching the Cramer-Rao accuracy bound. The estimates of DOA by MUSIC can be divided into two stages: the first stage is to estimate the number of signals and spatial spectrum according to the response of antenna array; the second is to search the extreme points corresponding to DOAs, shortened as spectral peak searching. Spectral peak searching is ergodic within detectable directions of antenna array which results in huge computation load in the case of high resolution, as step size should be very small. Spectral peak searching belongs to multi-modal problems. Therefore, MUSIC based on niche differential evolution [1] is proposed to solve spectral peak searching to reduce the computational load and increase the searching speed.

2. Multiple Signal Classification

MUSIC [2, 3] is to decompose the eigenvectors of covariance matrix of signals, and get the signal subspace U_s and noise subspace U_n . Then find the vector that is orthogonal to U_n by angle searching. The DOAs of incidences are thus obtained. Spectral peak searching of DOAs by MUSIC can be written as:

$$\hat{P}_{MUSIC} = \frac{1}{b^H(\theta)U_nU_n^Hb(\theta)}. \quad (1)$$

Where \hat{P}_{MUSIC} is the function value of space spectrum of any angles by MUSIC, $b(\theta)$ continuum is a function of θ .

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As a result, Eq.1 is the fitness function of niche DE, the goal of this paper is to search all its extreme points.

3. Niche Technique

A self-adaptive niche recognition technique is proposed in [4], which is the most effective method at present. It determines whether the individual is on the slope or on the bottom of the niche, according to the tendency of fitness function. Niche technique can be described as follows:

Step 1: Calculate the distances between every two individuals of the population. Set all the individuals unlabeled.

Step 2: Choose the individual with the highest fitness value among all the unlabeled individuals, denoted by x_c . Then label x_c .

Step 3: Sort all the unlabeled individuals in order of the distance from x_c from small to big, which form unlabeled sequence.

Step 4: Check the fitness values of every individual of the unlabeled sequence .If the latter one is larger than the former one, namely $fitness(x_{c,i}) < fitness(x_{c,i+1})$ go to Step 4.1, and else go to Step 4.2.

Step 4.1: Calculate the fitness value of point $e = x_{c,i} + \delta(x_c - x_{c,i})$, if it is lower than that of $x_{c,i}$, go to Step 4.1.1, else go to Step 4.1.2, where e is a point in the vicinity of $x_{c,i}$ and δ is the test step length, a very small positive number.

Step 4.1.1: If $i \geq 2$, x_c is the peak point of a niche, and all the $i-1$ individuals in the front of the sequence belong to the niche, and then label them. Otherwise, x_c doesn't belong to any niche. Go to step 5.

Step 4.1.2: Calculate the fitness value of point $e = x_{c,i} + \delta(x_{c,i+1} - x_{c,i})$, if it is lower than that of $x_{c,i}$, x_c is the peak point of a niche, and all the i individuals in the front of the sequence belong to the niche, then label them and go to step 5. Otherwise, continue to check if the latter one is larger than the former one.

Step 5: If the number of unlabeled individuals is no more than two, the unlabeled individual doesn't belong to any niche; the estimation of niches is over. Otherwise go to Step 2.

4. Niche Differential Algorithm

The searching performance of DE depends on the balance of exploration and exploitation, which is largely determined by parameter settings [5]. The scaling degree of differential vector is controlled by scaling factor F . The larger the F is the larger contribution differential vector of parent generation makes, which improves the diversity of population and global searching ability. Adversely, the local searching ability and the convergence rate are enhanced. In like manner, the larger crossover probability CR is, the larger contribution mutation individual makes to the test individual, which enhances the local searching ability and convergence rate. Adversely, the population is more diverse and the global searching ability is improved. With regard to the multi-modal problem, F should be larger and CR smaller in the early period, which can increase the diversity of the population. With the increase of iterations, CR should be larger and F smaller, which can focus on the local searching in order to ensure convergence of the algorithm. Based on the analysis above, a linear incremental CR and a linear descending F are adopted as follows:

$$F = F_{\max} - \frac{(iter - 1)(F_{\max} - F_{\min})}{total - 1}. \quad (2)$$

$$CR = CR_{\min} + \frac{(iter - 1)(CR_{\max} - CR_{\min})}{total - 1}. \quad (3)$$

Where CR_{\min} and CR_{\max} are the minimum and maximum value of the crossover probability, F_{\max} and F_{\min} are the minimum and maximum value of scaling factor respectively. $total$ is the maximum number of iterations and $iter$ is the current number of iteration.

As the niche technology adopted in this paper can roughly estimate every peak point, the searching process should be mainly within the niche, namely every target individual selects the individuals in the same niche of its own to mutate to speed up the convergence rate. However, the peak point of a niche may be

mistaken for the individual point of another niche or a niche isn't covered with individuals due to the randomly initial distribution. As a result, global searching should also be carried on with certain probability, namely every individual selects among all the other individuals randomly during the mutation operation, in order to avoid omission of peak points. Besides, the peak point of every niche should be saved to speed up the convergence rate. To keep the population number invariant, when a peak point is saved, the point whose fitness value is smaller than that of the peak point and distance is the shortest from the peak point, is abandoned. Usually, the fitness value of the point is relatively high in its niche as it's near the central point. To some extent, it will restrain the phenomena that points congregate around its peak points on account of greedy selection operation of DE, which may cause peak values lost. Besides, better solutions will not be lost. The niche DE can be described as follows:

- 1) Set parameters and randomly initialize the population.
- 2) Recognize niches. Determine the niche every individual belongs to, and save the peak point of each niche.
- 3) Generate a random number between 0 and 1. If it is smaller than a fixed probability P , the target individual select the individuals within the range of its own niche to mutate. Otherwise, select among all the other individuals randomly to mutate. Here P should be close to 1.
- 4) Operate crossover and selection.
- 5) Abandon M points, where M is the number of niches. The remaining points and the peak points make up of the population of next generation.
- 6) Determine whether or not it reaches the maximum value of iterations. If so, output the best solution. If not, go back to step 2).

5. Experiment result and analysis

As spectral peak searching belongs to multi-modal problems, niche DE is first tested on 5 standard benchmark functions to approve its validity in solving multi-modal problems. After many experiments, the parameters are set as follows: $F_{\min} = 1.2$, $F_{\max} = 2.4$, $CR_{\min} = 0.1$, $CR_{\max} = 0.9$, $P = 0.9$, $total = 20$.

And the benchmark functions are listed as follows:

- 1) $f_1(x) = \sin^6(5\pi x), x \in [0, 1]$
- 2) $f_2(x) = \sin^4(5\pi x) / \exp(1.25(x - 0.1))^2, x \in [0, 1]$
- 3) $f_3(x) = \sin^6[5\pi(x^{0.75} - 0.05)], x \in [0, 1]$
- 4) $f_4(x) = x(x + 1) \sin((2x - 0.5)^2 \pi - 1), x \in [-1.5, 2.5]$
- 5) $f_5(x, y) = \cos(2\pi x) \cos(2\pi y) \exp((-x^2 - y^2)/10), x \in [-1, 1], y \in [-1, 1]$

The peak points of $f_1(x)$, $f_2(x)$, $f_3(x)$, $f_4(x)$, $f_5(x, y)$ found by niche DE are shown in Fig.1~Fig.5. The performances of test functions are shown in Table 1.

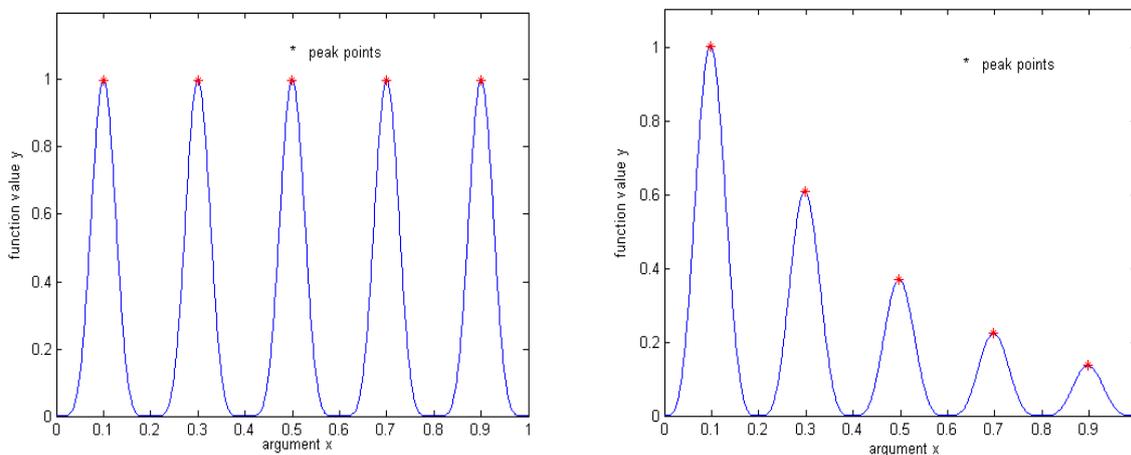


Figure 1 the simulation of $f_1(x)$

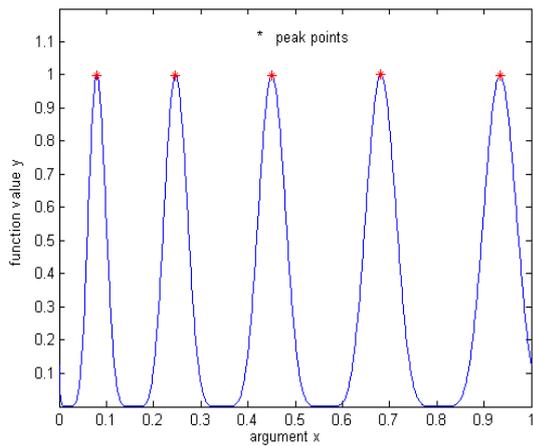


Figure 3 the simulation of $f_3(x)$

Figure 2 the simulation of $f_2(x)$

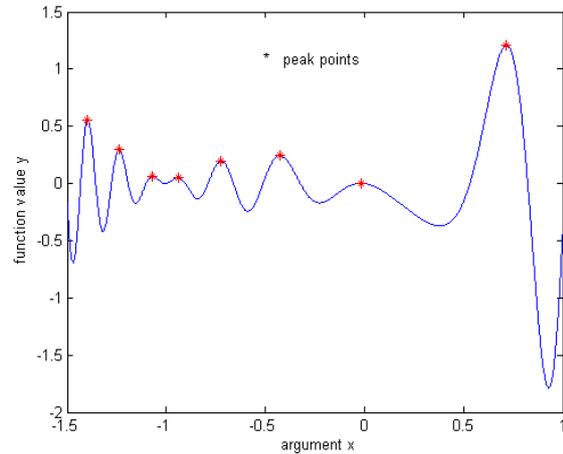


Figure 4 the simulation of $f_4(x)$

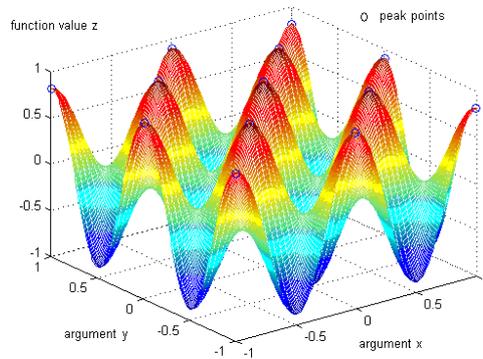


Figure 5 the simulation of $f_5(x, y)$

TABLE 1 PERFORMANCE OF TEST FUNCTIONS

| <i>Function</i> | <i>Population size</i> | <i>Running time</i> | <i>Average peak number</i> |
|-----------------|------------------------|---------------------|----------------------------|
| $f_1(x)$ | 200 | 0.1723[s] | 5 |
| $f_2(x)$ | 200 | 0.1657[s] | 5 |
| $f_3(x)$ | 200 | 0.1836[s] | 5 |
| $f_4(x)$ | 200 | 0.22822[s] | 8 |
| $f_5(x, y)$ | 350 | 1.6808[s] | 12.8 |

From the table and figure above, it's clear that in univariate functions, niche DE finds all the peak points with high accuracy and rate. Even in the bivariate function, which is difficult to search, the performance is also great.

In order to verify the effectiveness of the algorithm proposed in this paper, the algorithm is in contrast with original MUSIC.

The evaluation criterion of searching precision is defined as error ε :

$$\varepsilon = \sqrt{\sum_{i=1}^n (\tilde{x} - x)^2} . \quad (4)$$

Where \tilde{x} is the location of the peak estimated, x is the actual location of the peak, and n the number of spectral peaks.

Select incidences of various number and different distribution to verify. The step length of original MUSIC is 0.01 while the number of snapshots is 1000. The size of population of the algorithm proposed is 250. We assume a uniform linear array of six sensors with half wavelength spacing, and assume that noise is uncorrelated from sensor to sensor and also uncorrelated with signal. In each experiment, the number of snapshots taken is 1000, signal/noise ratio is 30dB in the environment of Gaussian white noise, and the step length of original MUSIC is 0.01. A total of 100 independent simulation runs are done to compute the errors of original MUSIC and the algorithm proposed.

Case 1: Three one-dimensional incidences arrive from 60° , 20° , and -15° . The number of iterations was 30. The comparing performance is shown in Fig.6.

Case 2: Four one-dimensional incidences arrive from 80° , 50° , 10° and -20° . The number of iterations was 35. The comparing performance is shown in Fig.7.

In the cases of two-dimensional incidences, we assume a uniform circular array of five sensors. The step length of original MUSIC is 0.1 while the number of snapshots is 100. The size of population of the algorithm proposed is 350. And all the angles below are generated randomly, other hypothesis are the same with one-dimension.

Case 3: The pitching angles of two incidences are 45.3456° and 50.1529° , and their corresponding azimuth angles are 147.496° and 95.7401° . The number of iterations is 80. The comparing performance is shown in Fig.8.

Case 4: The pitching angles of three incidences are 73.3251° , 81.5213° and 11.4288° their corresponding azimuth angles are 164.4077° , 113.8247° and 17.5573° . The number of iterations was 150. The comparing performance is shown in Fig.9.

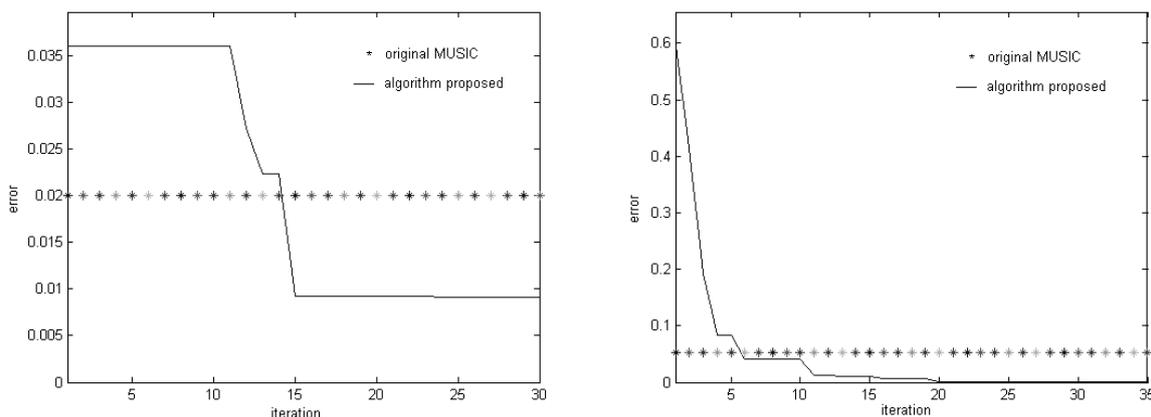


Figure 6 comparing performance of 3 one-dimensional incidences; Figure 7 comparing performance of 4 one-dimensional incidences

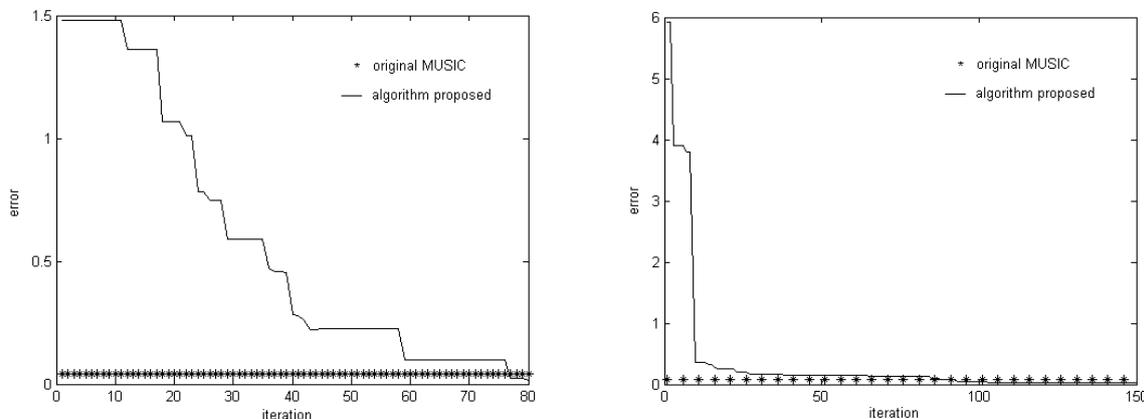


Figure 8 comparing performance of 2 two-dimensional incidences; Figure 9 comparing performance of 3 two-dimensional incidences

As shown in Fig.6, with the increase of iterations, the error of the proposed algorithm decreases rapidly, lower than that of original MUSIC in the iteration of 14. At the 30th iteration, the error of the proposed algorithm is 0.1. Other cases are similar with case 1.

The algorithm proposed can guarantee that the extreme points corresponding to the DOAs can be searched accurately. And the performance is irrelevant to the number of DOAs and their distribution in the feasible region.

To compare the rate of the two algorithms, the running time of the two algorithms are listed in Table 2.

TABLE 2 RUNNING TIME OF TWO ALGORITHMS

| | <i>Algorithm proposed</i> | <i>Original MUSIC</i> |
|------------------------------|---------------------------|-----------------------|
| 3 one-dimensional incidences | 1.464[s] | 6.854[s] |
| 4 one-dimensional incidences | 1.705[s] | 8.074[s] |
| 2 two-dimensional incidences | 8.923[s] | 268.352[s] |
| 3 two-dimensional incidences | 12.589[s] | 270.336[s] |

6. Conclusion and discussion

The multiple signal classification based on niche differential evolution (DE) was proposed in this paper. DE was first improved by dividing the mutation operation into within-niche mutation and global mutation, and the parameters are self-adapted. Then the improved DE was combined with niche technique to construct niche DE technique. The results of experiments showed the effectiveness of niche DE when dealing with multi-modal problems. At last, niche DE is applied to the spectral peak searching. The experimental results illustrated that the errors of both one-dimensional and two-dimensional DOAs' estimation are smaller than that of the original MUSIC, and the results are independent of the number of spectral peaks and the distribution of angles in feasible region. Besides, the running time of the proposed algorithm is far less than that of original MUSIC.

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8. References

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