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Traveling Wave Solutions for a Class of Nonlinear Equation

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Abstract. In this paper, we derive exact traveling wave solutions of (3+1) dimensional Burgers equation by a proposed Bernoulli sub-ODE method. The method appears to be efficient in seeking exact solutions of nonlinear equations. We also make a comparison between the present method and the known (G'/G) expansion method.

Keywords: Bernoulli sub-ODE method, traveling wave solutions, exact solution, evolution equation, (3+1) dimensional Burgers equation

1. Introduction

Recently searching for exact traveling wave solutions of nonlinear equations has gained more and more popularity, and many effective methods have been presented so far. Some of these approaches are the homogeneous balance method [1,2], the hyperbolic tangent expansion method [3,4], the trial function method [5], the tanh-method [6-8], the nonlinear transform method [9], the inverse scattering transform [10] and so on.

In this paper, we proposed a Bernoulli sub-ODE method to construct exact traveling wave solutions for NLEES.

The rest of the paper is organized as follows. In Section 2, we describe the Bernoulli sub-ODE method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the Bernoulli Sub-ODE method and the known (G'/G) expansion method to find exact traveling wave solutions of the (3+1) dimensional Burgers equation. In the last Section, some conclusions are presented.

2. Description of the Bernoulli Sub-ODE method

In this section we present the solutions of the following ODE:

$$G' + \lambda G = \mu G^2, \tag{2.1}$$

where $\lambda \neq 0, G = G(\xi)$

When $\mu \neq 0$, Eq. (2.1) is the type of Bernoulli equation, and we can obtain the solution as

$$G = \frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}, \tag{2.2}$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t , is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy} \ldots\ldots) = 0 \tag{2.3}$$

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where u = u(x, y, t) is an unknown function, P is a polynomial in u = u(x, y, t) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. By using the solutions of Eq. (2.1), we can construct a serials of exact solutions of nonlinear equations:.

Step 1. We suppose that

$$u(x,y,t)=u(\xi),\xi=\xi(x,y,t) \tag{2.4}$$

the traveling wave variable (2.4) permits us reducing Eq. (2.3) to an ODE for $u=u(\xi)$

$$P(u,u',u'',......)=0 \tag{2.5}$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in G as follows:

$$u(\xi)=\alpha_m G^m+\alpha_{m-1}G^{m-1}+...... \tag{2.6}$$

where $G=G(\xi)$ satisfies Eq. (2.1), and $\alpha_m,\alpha_{m-1}...$ are constants to be determined later, $\alpha_m\neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the same order of G together, the left-hand side of Eq. (2.5) is converted into another polynomial in G. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m,\alpha_{m-1},...\lambda,\mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (2.1), we can construct the traveling wave solutions of the nonlinear evolution equation (2.5).

In the subsequent sections we will illustrate the proposed method in detail by applying it to (3+1) dimensional Burgers equation.

3. Application Of Bernoulli Sub-ODE Method For (3+1) dimensional Burgers Equation

In this section, we will consider the following (3+1) dimensional Burgers equation (3.1)-(3.3):

$$u_t=\alpha uu_y+\beta vu_x+\gamma wu_x+u_{xx}+u_{yy}+u_{zz} \tag{3.1}$$

$$u_x=v_y \tag{3.2}$$

$$u_z=w_y \tag{3.3}$$

Suppose that

$$u(x,y,t)=u(\xi),\xi=k(x+ly+mz-ct) \tag{3.4}$$

where c,k,m,l are constants that to be determined later.

By (3.4), (3.1)-(3.3) are converted into ODEs

$$-cu'=\alpha luu'+\beta vu'+\gamma wu'+ku''+kl^2u''+km^2u'' \tag{3.5}$$

$$u'=v' \tag{3.6}$$

$$u'=w' \tag{3.7}$$

Suppose that the solution of (3.5)-(3.7) can be expressed by a polynomial in G as follows:

$$u(\xi)=\sum_{i=0}^{p}a_iG^i \tag{3.8}$$

$$v(\xi)=\sum_{i=0}^{n}b_iG^i \tag{3.9}$$

$$w(\xi)=\sum_{i=0}^{q}c_iG^i \tag{3.10}$$

where a_i, b_i, c_i are constants, and $G = G(\xi)$ satisfies Eq. (2.1). Balancing the order of u'' and vu' in Eq. (3.5), the order of u' and v' in Eq.(3.6) , and the order of u' and w' in Eq. (3.7) we have $p+2=n+p+1, p+1=n+1,\ p+1=q+1 \Rightarrow p=n=q=1$. So Eq.(3.8)-(3.10) can be rewritten as

$$u(\xi) = a_1 G + a_0, a_1 \neq 0 \tag{3.11}$$

$$v(\xi) = b_1 G + b_0, b_1 \neq 0 \tag{3.12}$$

$$w(\xi) = c_1 G + c_0, c_1 \neq 0 \tag{3.13}$$

where $a_1, a_0, b_1, b_0, c_1, c_0$ are constants to be determined later.

Substituting (3.11)-(3.13) into (3.5)-(3.7) and collecti-ng all the terms with the same power of G together, equati-ng each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

For Eq. (3.5)

$$G^1 : -a_1k^2\lambda^2 + ca_1\lambda - a_1l^2\lambda^2 + \alpha la_1a_0\lambda - a_1m^2\lambda^2 + \beta ka_1b_0\lambda = 0$$

$$G^2 : -c\mu a_1 - \alpha la_1a_0\mu + 3m^2a_1\mu\lambda + \alpha la_1{}^2\lambda - \gamma ka_1c_0\mu + \gamma ka_1c_1\lambda\ \ +3a_1l^2\mu\lambda + 3a_1k^2\mu\lambda + \beta ka_1b_1\lambda - \beta ka_1b_0\mu = 0$$

$$G^3 : -\alpha la_1{}^2\mu - \gamma ka_1c_1\mu - 2a_1l^2\mu^2 - 2a_1m^2\mu^2 - \beta ka_1b_1\mu - 2a_1k^2\mu^2 = 0$$

For Eq. (3.6)

$$G^1 : -a_1k\lambda + lb_1\lambda = 0$$

$$G^2 : k\mu a_1 - lb_1\mu = 0$$

For Eq. (3.7)

$$G^1 : -a_1k\lambda + lb_1\lambda = 0$$

$$G^2 : k\mu a_1 - lb_1\mu = 0$$

Solving the algebraic equations above, yields:

$$a_1 = a_1, a_0 = a_0, k = k, l = l, m = m, b_1 = \frac{ka_1}{l}, b_0 = b_0,\quad c_0 = c_0, c_1 = -\frac{a_1\alpha l^2 + a_1\beta k^2 + 2\mu lk^2 + 2\mu l^3 + 2\mu lm^2}{\gamma kl}$$

$$c = k^2\lambda + l^2\lambda - \alpha la_0 + m^2\lambda - \beta kb_0 - \gamma kc_0 \tag{3.14}$$

where $a_1, a_0, k, l, m, b_0, c_0$ are arbitrary constants.

Substituting (3.4) into (3.11)-(3.13), we obtain the traveling wave solutions of (3+1) dimensional Burgers equation as follows

$$u(\xi) = a_1 G + a_0, a_1 \neq 0 \tag{3.15}$$

$$v(\xi) = \frac{ka_1}{l} G + b_0, k \neq 0, a_1 \neq 0 \tag{3.16}$$

$$w(\xi) = -\frac{a_1\alpha l^2 + a_1\beta k^2 + 2\mu lk^2 + 2\mu l^3 + 2\mu lm^2}{\gamma kl} G + c_0 \tag{3.17}$$

where $\xi = k[x + ly + mz - (k^2\lambda + l^2\lambda - \alpha la_0 + m^2\lambda - \beta kb_0 - \gamma kc_0)t]$. Combining with Eq. (2.2), we can obtain the traveling wave solutions of (3.1) as follows:

$$u(\xi) = a_1(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}) + a_0, a_1 \neq 0 \tag{3.18}$$

$$v(\xi) = \frac{ka_1}{l}(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}) + b_0, k \neq 0, a_1 \neq 0 \tag{3.19}$$

$$w(\xi) = -\frac{a_1\alpha l^2 + a_1\beta k^2 + 2\mu lk^2 + 2\mu l^3 + 2\mu lm^2}{\gamma kl} \times (\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}) + c_0 \tag{3.20}$$

Since $\xi = k[x + ly + mz - (k^2\lambda + l^2\lambda - \alpha la_0 + m^2\lambda - \beta kb_0 - \gamma kc_0)t]$ then furthermore from (3.18)-(3.20) we have

$$u(\xi) = \frac{a_1}{\frac{\mu}{\lambda} + de^{\lambda k[x+ly+mz-(k^2\lambda+l^2\lambda-\alpha la_0+m^2\lambda-\beta kb_0-\gamma kc_0)t]}} + a_0$$

$$v(\xi) = \frac{ka_1}{l}\frac{1}{\frac{\mu}{\lambda} + de^{\lambda k[x+ly+mz-(k^2\lambda+l^2\lambda-\alpha la_0+m^2\lambda-\beta kb_0-\gamma kc_0)t]}} + b_0$$

$$w(\xi) = -\frac{a_1\alpha l^2 + a_1\beta k^2 + 2\mu lk^2 + 2\mu l^3 + 2\mu lm^2}{\gamma kl} \times (\frac{1}{\frac{\mu}{\lambda} + de^{\lambda k[x+ly+mz-(k^2\lambda+l^2\lambda-\alpha la_0+m^2\lambda-\beta kb_0-\gamma kc_0)t]}}) + c_0$$

4. Application Of (G'/G) expansion Method For (3+1)-d Burgers Equation

In this section, we apply the (G'/G) expansion method to obtain the traveling wave solutions of (3+1) dimensional Burgers equation (3.1)-(3.3).

Suppose that the solution of (3.5)-(3.7) can be expressed by a polynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{p} a_i(\frac{G'}{G})^i \tag{4.1}$$

$$v(\xi) = \sum_{i=0}^{n} b_i(\frac{G'}{G})^i \tag{4.2}$$

$$w(\xi) = \sum_{i=0}^{q} c_i(\frac{G'}{G})^i \tag{4.3}$$

where a_i are constants, and $G = G(\xi)$ satisfies

$$G'' + \lambda G' + \mu G = 0 \tag{4.4}$$

Balancing the order of u'' and vu' in Eq.(3.5), the order of u' and v' in Eq.(3.6) , and the order of u' and w' in Eq.(3.7) we have $p+2 = n+p+1, p+1 = n+1, \ p+1 = q+1 \Rightarrow p = n = q = 1$. So Eqs. (3.8)-(3.10) can be rewritten as

$$u(\xi) = a_1(\frac{G'}{G}) + a_0, a_1 \neq 0 \tag{4.5}$$

$$v(\xi) = b_1(\frac{G'}{G}) + b_0, b_1 \neq 0 \tag{4.6}$$

$$w(\xi) = c_1(\frac{G'}{G}) + c_0, c_1 \neq 0 \tag{4.7}$$

Substituting (4.5)-(4.7) into (3.5)-(3.7) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations. Solving the algebraic equations above, yields:

$$a_1 = a_1, a_0 = a_0, k = k, l = l, m = m, b_1 = \frac{ka_1\mu}{l}, b_0 = b_0,\ c_0 = c_0, c_1 = 2k\mu\quad c = k\lambda^2 - \beta k b_0 \tag{4.8}$$

where $a_1, a_0, k, l, m, b_0, c_0$ are arbitrary constants .

Substituting (4.8) into (4.5)-(4.7), we obtain

$$u(\xi) = a_1 G + a_0, a_1 \neq 0 \tag{4.9}$$

$$v(\xi) = \frac{ka_1\mu}{l} G + b_0, k \neq 0, a_1 \neq 0 \tag{4.10}$$

$$w(\xi) = 2k\mu G + c_0 \tag{4.11}$$

Remark : As one can see from Section III and Section IV, the traveling wave solutions obtained by the Bernoulli Sub-ODE method are very similar to those by the known (G'/G) expansion method.

5. Conclusions

We have seen that some new traveling wave solutions of (3+1) dimensional Burgers equation are successfully found by using the Bernoulli sub-ODE method. The main points of the method are that assuming the solution of the ODE reduced by using the traveling wave variable as well as integrating can be expressed by an m -th degree polynomial in G , where $G = G(\xi)$ is the general solutions of a Bernoulli sub-ODE equation. The positive integer m can be determined by the general homogeneous balance method, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations. The Bernoulli Sub-ODE method can be applied to many other nonlinear problems.

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