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New Analytical Solutions for Two Equations by a Proposed Sub-ODE Method

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Abstract. In this paper, we derive exact traveling wave solutions of mBBM equation and 2D-Burgers equation by a proposed Bernoulli sub-ODE method. The method appears to be efficient in seeking exact solutions of nonlinear equations.

Keywords: Bernoulli sub-ODE method, traveling wave solutions, exact solution, evolution equation, mBBM equation, 2D-Burgers equation

1. Introduction

Recently searching for exact traveling wave solutions of nonlinear equations has gained more and more popularity, and many effective methods have been presented so far. Some of these approaches are the homogeneous balance method [1,2], the hyperbolic tangent expansion method [3,4], the trial function method [5], the tanh-method [6-8], the nonlinear transform method [9], the inverse scattering transform [10], the Backlund transform [11,12], the Hirotas bilinear method [13,14], the generalized Riccati equation [15,16], the theta function method [17-19], the sine-Ccosine method [20], the Jacobi elliptic function expansion [21,22], the complex hyperbolic function method [23-25], and so on.

In this paper, we propose a Bernoulli sub-ODE method to construct exact traveling wave solutionns for NLEES, and try to apply the method to obtain the traveling wave solutions for the mBBM equation and the 2D-Burgers equation.

2. Description of the Bernoulli Sub-ODE method

In this section we present the solutions of the following ODE:

$$G' + \lambda G = \mu G^2, \tag{2.1}$$

where $\lambda \neq 0, G = G(\xi)$

When $\mu \neq 0$, Eq. (2.1) is the type of Bernoulli equation, and we can obtain the solution as

$$G = \frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}, \tag{2.2}$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t , is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy} ......) = 0 \tag{2.3}$$

where u = u(x, y, t) is an unknown function, P is a polynomial in u = u(x, y, t) and its various partial derivatives,

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in which the highest order derivatives and nonlinear terms are involved. By using the solutions of Eq. (2.1), we can construct a serials of exact solutions of nonlinear equations:.

*Step 1.*We suppose that

$$u(x, y, t) = u(\xi), \xi = \xi(x, y, t) \tag{2.4}$$

the traveling wave variable (2.4) permits us reducing Eq. (2.3) to an ODE for $u = u(\xi)$

$$P(u, u', u'', ...) = 0 \tag{2.5}$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in G as follows:

$$u(\xi) = \alpha_m G^m + \alpha_{m-1} G^{m-1} + ... \tag{2.6}$$

where $G = G(\xi)$ satisfies Eq. (2.1), and $\alpha_m, \alpha_{m-1}...$ are constants to be determined later, $\alpha_m \neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the same order of G together, the left-hand side of Eq. (2.5) is converted into another polynomial in G. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \alpha_{m-1}, ...\lambda, \mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (2.1), we can construct the traveling wave solutions of the nonlinear evolution equation (2.5).

In the subsequent sections we will illustrate the proposed method in detail by applying it to mBBM equation and the 2D-Burgers equation.

3. Application Of Bernoulli Sub-ODE Method For mBBM Equation

In this section, we will consider the following mBBM equation:

$$u_t + u_x + u^2 u_x + u_{xxt} = 0 \tag{3.1}$$

In order to obtain the traveling wave solutions of Eq.(3.1), we suppose that

$$u(x,t) = u(\xi), \xi = x - ct \tag{3.2}$$

where c is a constant that to be determined later.

By using (3.2), (3.1) is converted into an ODE

$$(1-c)u' + u^2 u' - cu''' = 0 \tag{3.3}$$

Integrating (3.3) once it follows:

$$(1-c)u + \frac{u^3}{3} - cu'' = g \tag{3.4}$$

where g is the integration constant.

Suppose that the solution of (3.4) can be expressed by a polynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i \tag{3.5}$$

where a_i are constants, and G. satisfies Eq. (2.2). Balancing the order of u'' and u^3 in Eq. (3.3), we have $m + 2 = 3m \Rightarrow m = 1$. So

$$u(\xi) = a_0 + a_1 G \tag{3.6}$$

where a_1, a_0 are constants to be determined later.

Substituting (3.6) into (3.4) and collecting all the terms with the same power of G together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0 : a_0 - ca_0 - g + \frac{1}{3} a_0^{\ 3} = 0$$

$$G^1 : a_1 + a_0^{\ 2} a_1 - ca_1 \lambda^2 - ca_1 = 0$$

$$G^2 : 3ca_1\mu\lambda + a_0 a_1^{\ 2} = 0$$

$$G^3 : -2ca_1\mu^2 + \frac{1}{3}a_1^{\ 3} = 0$$

Solving the algebraic equations above, yields:

Case 1:

$$a_1 = 2\sqrt{-\frac{3}{\lambda^2-2}}\mu, a_0 = -\sqrt{-\frac{3}{\lambda^2-2}}\lambda, c = -\frac{2}{\lambda^2-2}, g = 0 \tag{3.7}$$

Substituting (3.7) into (3.6), we have

$$u_1(\xi) = -\sqrt{-\frac{3}{\lambda^2-2}}\lambda + 2\sqrt{-\frac{3}{\lambda^2-2}}\mu G, \ \xi = x + \frac{2}{\lambda^2-2}t \tag{3.8}$$

Combining with Eq. (2.2) and(3.8), we can obtain the traveling wave solutions of (3.1) as follows:

$$u_1(x,t) = -\sqrt{-\frac{3}{\lambda^2-2}}\lambda + 2\sqrt{-\frac{3}{\lambda^2-2}}\mu\frac{1}{\frac{\mu}{\lambda} + de^{\lambda(x+\frac{2}{\lambda^2-2}t)}} \tag{3.9}$$

where d is an arbitrary constant.

Case 2:

$$a_1 = -2\sqrt{-\frac{3}{\lambda^2-2}}\mu, a_0 = \sqrt{-\frac{3}{\lambda^2-2}}\lambda, c = -\frac{2}{\lambda^2-2}, g = 0 \tag{3.10}$$

Substituting (3.7) into (3.6), we have

$$u_2(\xi) = \sqrt{-\frac{3}{\lambda^2-2}}\lambda - 2\sqrt{-\frac{3}{\lambda^2-2}}\mu G, \ \xi = x + \frac{2}{\lambda^2-2}t \tag{3.11}$$

Combining with Eq. (2.2) and(3.8), we can obtain the traveling wave solutions of (3.1) as follows:

$$u_2(x,t) = \sqrt{-\frac{3}{\lambda^2-2}}\lambda - 2\sqrt{-\frac{3}{\lambda^2-2}}\mu(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda(x+\frac{2}{\lambda^2-2}t)}}) \tag{3.12}$$

where d is an arbitrary constant.

Remark : Our results (3.9) and (3.12) are new exact traveling wave solutions for Eq. (3.1).

4. Application Of Bernoulli Sub-ODE Method For 2d Burgers Equation

In this section, we will consider the following 2D Burgers equation:

$$u_t - 2uu_x - u_{xx} - u_{yy} - 2vu_y = 0 \tag{4.1}$$

$$v_t - 2uv_x - v_{xx} - v_{yy} - 2vv_y = 0 \tag{4.2}$$

In order to obtain the traveling wave solutions of Eq.(4.1), we suppose that

$$u(x,t) = u(\xi), \xi = kx + \omega y + st \tag{4.3}$$

where k, ω, s are constants that to be determined later.

By using (4.3), (4.1) and (4.2) is converted into ODEs

$$su' - 2kuu' - (k^2 + \omega^2)u'' - 2\omega vu' = 0 \tag{4.4}$$

$$sv' - 2kuv' - (k^2 + \omega^2)v'' - 2\omega vv' = 0 \tag{4.5}$$

Suppose that the solution of (4.4) and (4.5) can be expressed by a polynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i \tag{4.6}$$

$$v(\xi)=\sum_{i=0}^{n} b_i G^i \quad (4.7)$$

where a_i, b_i are constants, G. satisfies Eq. (2.2). Balancing the order of uu' and vu' in Eq. (4.6), the order of uv' and v'' in Eq. (4.7), we have $2m+1=m+n+1, m+n+1=n+2 \Rightarrow m=n=1$. So

$$u(\xi)=a_0+a_1 G \quad (4.8)$$

$$v(\xi)=b_0+b_1 G \quad (4.9)$$

where a_1, a_0, b_1, b_0 are constants to be determined later.

Substituting (4.8), (4.9) into (4.4) , (4.5), and collecting all the terms with the same power of G together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

For Eq. (4.4):

$$G^1: sa_1\mu - a_1\omega^2\mu^2 - 2ka_1a_0\mu - 2\omega a_1 b_0\mu - a_1k^2\mu^2 = 0$$

$$G^2: 2ka_1a_0\lambda - sa_1\lambda + 3a_1k^2\mu\lambda - 2ka_1^2\mu + 2\omega a_1b_0\lambda + 3a_1\omega^2\mu\lambda - 2\omega a_1b_1\mu = 0$$

$$G^3: -2a_1\omega^2\lambda^2 - 2a_1k^2\lambda^2 + 2a_1^2k\lambda + 2\omega a_1b_1\lambda = 0$$

For Eq. (4.5):

$$G^1: sb_1\mu - b_1(k^2+\omega^2)\mu^2 - 2kb_1a_0\mu - 2\omega b_1b_0\mu = 0$$

$$G^2: 2kb_1a_0\lambda - sb_1\lambda + 3b_1(k^2+\omega^2)\mu\lambda - 2kb_1a_1\mu + 2\omega b_1b_0\lambda - 2\omega b_1^2\mu = 0$$

$$G^3: -2b_1(k^2+\omega^2)\lambda^2 + 2kb_1a_1\lambda + 2\omega b_1^2\lambda = 0$$

Solving the algebraic equations above, yields:

$$a_1=\frac{\omega^2\lambda+k^2\lambda-\omega b_1}{k}, a_0=a_0, b_0=b_0, b_1=b_1,\ \omega=\omega, k=k, s=\omega^2\mu+2ka_0+2\omega b_0+k^2\mu \quad (4.10)$$

Substituting (4.10) into (4.8) and (4.9), we have

$$u(\xi)=a_0+\frac{\omega^2\lambda+k^2\lambda-\omega b_1}{k}G \quad (4.11)$$

$$v(\xi)=b_0+b_1 G \quad (4.12)$$

where k,ω,a_0,b_0,b_1 are arbitrary constants, and $k,\omega,b_1\neq 0$, $\xi=kx+\omega y+(\omega^2\mu+2ka_0+2\omega b_0+k^2\mu)t$. Combining with Eq. (2.2), we can obtain the traveling wave solutions of (4.1) and (4.2) as follows:

$$u(x,y,t)=a_0+\frac{\omega^2\lambda+k^2\lambda-\omega b_1}{k}\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right) \quad (4.13)$$

$$v(x,y,t)=b_0+b_1\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right) \quad (4.14)$$

where $\xi=kx+\omega y+(\omega^2\mu+2ka_0+2\omega b_0+k^2\mu)t$

Remark: Our result (4.13) and (4.14) are new exact traveling wave solutions for Eqs. (3.1)-(3.2).

5. Conclusions

We have seen that some new traveling wave solution of mBBM equation is successfully found by using the Bernoulli sub-ODE method. One can see the method is concise and effective. Also this method can be used to many other nonlinear problems.

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