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New Exact Traveling Wave Solutions for Two Nonlinear Evolution Equations

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Abstract. In this paper, a generalized sub-ODE method is proposed to construct exact solutions of two nonlinear equations. As a result, some new exact traveling wave solutions for them are found.

Keywords:sub-ODE method, traveling wave solution, exact solution, nonlinear equation,

1. Introduction

In scientific research, seeking the exact solutions of nonlinear equations is a hot topic. Many approaches have been presented so far. Some of these approaches are the homogeneous balance method [1,2], the hyperbolic tangent expansion method [3,4], the trial function method [5], the tanh-method [6-8], the nonlinear transform method [9], the inverse scattering transform [10], the Backlund transform [11,12], the Hirotas bilinear method [13,14], the generalized Riccati equation [15,16], the theta function method [17-19], the sine-Ccosine method [20], the Jacobi elliptic function expa-nsion [21,22], the complex hyperbolic function method [23-25], and so on..

In this paper, we proposed a sub-ODE method to construct exact traveling wave solutions for NLEES. The rest of the paper is organized as follows. In Section 2, we describe the sub-ODE method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the method to find exact traveling wave solutions of the Boussinesq equation and (2+1) dimensional Boussinesq equation. In the last Section, some conclusions are presented.

2. Description of the Bernoulli Sub-ODE method

In this section we present the solutions of the following ODE:

$$G' + \lambda G = \mu G^2, \tag{2.1}$$

where $\lambda \neq 0, G = G(\xi)$

When $\mu \neq 0$, Eq. (2.1) is the type of Bernoulli equation, and we can obtain the solution as

$$G = \frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}, \tag{2.2}$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t , is given by

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$$P(u,u_{t,}u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xx},u_{yy}......)=0 \tag{2.3}$$

where u = u(x, y, t) is an unknown function, P is a polynomial in u = u(x, y, t) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. By using the solutions of Eq. (2.1), we can construct a serials of exact solutions of nonlinear equations:.

Step 1. We suppose that

$$u(x,y,t)=u(\xi),\xi=\xi(x,y,t) \tag{2.4}$$

the traveling wave variable (2.4) permits us reducing Eq. (2.3) to an ODE for $u=u(\xi)$

$$P(u,u',u'',......)=0 \tag{2.5}$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in G as follows:

$$u(\xi)=\alpha_m G^m+\alpha_{m-1}G^{m-1}+...... \tag{2.6}$$

where $G=G(\xi)$ satisfies Eq. (2.1), and $\alpha_m,\alpha_{m-1}...$ are constants to be determined later, $\alpha_m\neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the same order of G together, the left-hand side of Eq. (2.5) is converted into another polynomial in G. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m,\alpha_{m-1},...\lambda,\mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (2.1), we can construct the traveling wave solutions of the nonlinear evolution equation (2.5).

In the subsequent sections we will illustrate the proposed method in detail by applying it to Boussinesq equation and (2+1) dimensional Boussinesq equation.

3. Application for Boussinesq Equation

In this section, we will consider the following Boussinesq equation:

$$u_{tt}+\alpha u_{xx}+\beta(u^2)_{xx}+\gamma u_x{}^{(4)}=0,\alpha<0 \tag{3.1}$$

Suppose that

$$u(\xi),\xi=k(x-ct) \tag{3.2}$$

where the constants c,k can be determined later.

By using (3.2), (3.1) is converted into an ODE

$$(\alpha+c^2)u''+\beta(u^2)''+\gamma k^3u^{(5)}=0 \tag{3.3}$$

Integrating (3.3) twice, and take the integration constant for zero, then we have

$$(\alpha+c^2)u+\beta u^2+\gamma k^3u'''=0 \tag{3.4}$$

Suppose that the solution of (3.4) can be expressed by a polynomial in G as follows:

$$u(\xi)=\sum_{i=0}^{m}a_iG^i \tag{3.5}$$

where a_i are constants, and $G=G(\xi)$ satisfies Eq. (2.1). Balancing the order of u^2 and u''' in Eq. (3.5), we obtain that $2m=m+3\Rightarrow m=3$ .So Eq. (3.5) can be rewritten as

$$u(\xi)=a_3G^3+a_2G^2+a_1G+a_0,a_3\neq 0$$

where a_3,a_2,a_1,a_0 are constants to be determined later.

Substituting (3.5) into (3.3) and collecting all the terms with the same power of G together, the left-hand side of Eq. (3.3) is converted into another polynomial in G. Equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0:\beta a_0{}^2+\alpha a_0+c^2a_0=0$$

$$G^1:2\beta a_0a_1+\alpha a_1+c^2a_1+\gamma k^3a_1\mu^3=0$$

$$G^2:-7\gamma k^3\lambda a_1\mu^2+2\beta a_0a_2+(c^2+\alpha)a_2+8\gamma k^3a_2\mu^3+\beta a_1^2=0$$

$$G^3:-38\gamma k^3a_2\mu^2\lambda+(c^2+\alpha)a_3+2\beta a_0a_3+2\beta a_1a_2+12\gamma k^3\lambda^2a_1\mu+27\gamma a_3k^3\mu^3=0$$

$$G^4:54\gamma a_2\mu k^3\lambda^2+\beta a_2^2+2\beta a_1a_3-111\gamma a_3\lambda k^3\mu^2-6\gamma a_1k^3\lambda^3=0$$

$$G^5:144\gamma a_3\mu k^3\lambda^2+2\beta a_2a_3-24\gamma a_2k^3\lambda^3=0$$

$$G^6:-60\gamma a_3k^3\lambda^3+\beta a_3^2=0$$

Solving the algebraic equations above, yields:

Case 1:

$$a_3=\frac{60\gamma k^3\lambda^3}{\beta},a_2=a_1=a_0=0,k=k,\ c=\sqrt{-\alpha},\lambda=\lambda,\mu=0 \tag{3.6}$$

Substituting (3.6) into (3.5), we have

$$u_1(\xi)=\frac{60\gamma k^3\lambda^3}{\beta}G^3\ ,\ \xi=k(x-\sqrt{-\alpha}t) \tag{3.7}$$

Combining with Eq. (2.2) and we can obtain the traveling wave solutions of (3.1) as follows:

$$u_1(x,t)=\frac{60\gamma k^3\lambda^3}{\beta}[de^{-\lambda k(x-\sqrt{-\alpha}t)}]^3 \tag{3.8}$$

where d is an arbitrary constant.

Case 2:

$$a_3=\frac{60\gamma k^3\lambda^3}{\beta},a_2=a_1=a_0=0,k=k,\ c=-\sqrt{-\alpha},\lambda=\lambda,\mu=0 \quad (3.9)$$

Substituting (3.6) into (3.5), we have

$$u_1(\xi)=\frac{60\gamma k^3\lambda^3}{\beta}G^3,\ \xi=k(x+\sqrt{-\alpha}t) \quad (3.10)$$

Combining with Eq. (2.2) and we can obtain the traveling wave solutions of (3.1) as follows:

$$u_1(x,t)=\frac{60\gamma k^3\lambda^3}{\beta}[de^{-\lambda k(x+\sqrt{-\alpha}t)}]^3 \quad (3.11)$$

where d is an arbitrary constant

4. Application for (2+1) dimensional Boussinesq Equation

In this section, we will consider the following (2+1) dimensional Boussinesq equation:

$$u_{tt}-u_{xx}-u_{yy}-(u^2)_{xx}-u_{xxxx}=0 \quad (4.1)$$

Suppose that

$$u(x,y,t)=u(\xi),\xi=kx+ly+mt+d \quad (4.2)$$

where l,k,m,d are constants that to be determined later.

By (4.2), (4.1) is converted into an ODE

$$(m^2-k^2-l^2)u''-2k^2(u'^2+uu'')-k^4u''''=0 \quad (4.3)$$

Integrating (4.3) once we obtain

$$(m^2-k^2-l^2)u'-2k^2uu'-k^4u'''=g \quad (4.4)$$

where g is the integration constant.

Suppose that the solution of (4.4) can be expressed by a polynomial in G as follows:

$$u(\xi)=\sum_{i=0}^{m}a_iG^i \quad (4.5)$$

where a_i are constants, and $G=G(\xi)$ satisfies Eq.(2.1).

Balancing the order of uu' and u''' in Eq.(4.4), we have $2m+1=m+3\Rightarrow m=2$ .So Eq. (4.5) can be rewritten as

$$u(\xi)=a_2G^2+a_1G+a_0,a_2\neq 0 \quad (4.6)$$

where a_2,a_1,a_0 are constants to be determined later.

Substituting (4.6) into (4.4) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0: -g = 0$$

$$G^1: (k^2 + l^2 - m^2)a_1\lambda + k^4 a_1\lambda^3 + 2k^2 a_0 a_1 = 0$$

$$G^2: -7k^4\mu a_1\lambda^2 + 8k^4 a_2\lambda^3 + 2(k^2 + l^2 - m^2)a_2\lambda + (m^2 - k^2 - l^2)\mu a_1 - a_1^2 + 2k^2 a_1^2\lambda + 4k^2 a_0 a_2\lambda - 2k^2 a_0 a_1\mu = 0$$

$$G^3: 2(m^2 - k^2 - l^2)a_2\mu - 4k^2 a_0 a_2\mu + 6k^2 a_1 a_2\lambda - 38a_2\mu k^4\lambda^2 - 2k^2 a_1^2\mu + 12a_1\lambda k^4\mu^2 = 0$$

$$G^4: -6k^2 a_1 a_2\mu + 54a_2\lambda k^4\mu^2 - 6a_1 k^4\mu^3 + 4k^2 a_2^2\lambda = 0$$

$$G^5: -24a_2 k^4\mu^3 - 4k^2 a_2^2\mu = 0$$

Solving the algebraic equations above, yields:

$$a_2 = -6k^2\mu^2, a_1 = -6k^2\mu\lambda, a_0 = -\frac{1}{2}\frac{l^2 + k^2 - m^2 + \lambda^2 k^4}{k^2} \quad k = k, l = l, m = m, d = d \tag{4.7}$$

where k, l, m, d are arbitrary constants.

Substituting (4.7) into (4.6), we get that

$$u(\xi) = -6k^2\mu^2 G^2 - 6k^2\mu\lambda G - \frac{1}{2}\frac{l^2 + k^2 - m^2 + \lambda^2 k^4}{k^2} \quad \xi = kx + ly + mt + d \tag{4.8}$$

Combining with Eq. (2.2), we can obtain the traveling wave solutions of (4.1) as follows:

$$u(\xi) = -6k^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 - 6k^2\mu\lambda\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) - \frac{1}{2}\frac{l^2 + k^2 - m^2 + \lambda^2 k^4}{k^2} \tag{4.9}$$

Remark : Our result (4.9) is new exact traveling wave solutions for Eq. (4.1).

5. Conclusions

In the present work, we propose a new sub-ODE method, and then test its power by finding some new traveling wave solutions of Boussinesq equation and (2+1) dimensional Boussinesq equation. This method is one of the most effective approaches handling nonlinear evolution equations. One can see the method is concise and effective. Also this method can be used to many other nonlinear problems.

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