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New Analytical Solutions For (2+1) Dimensional BKK Equation

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Abstract. In this paper, we derive exact traveling wave solutions of (2+1) dimensional BKK equation by a proposed Bernoulli sub-ODE method and the known (G'/G) expansion method.

Keywords: Bernoulli sub-ODE method, (G'/G) expansion method, traveling wave solutions, exact solution, evolution equation, (2+1) dimensional BKK equation

1. Introduction

During the past four decades or so searching for explicit solutions of nonlinear evolution equations by using various different methods have been the main goal for many researchers, and many powerful methods for constructing exact solutions of nonlinear evolution equations have been established and developed such as the inverse scattering transform, the Darboux transform, the tanh-function expansion and its various extension, the Jacobi elliptic function expansion, the homogeneous balance method, the sine-cosine method, the rank analysis method, the exp-function expansion method and so on [1-20]..

In this paper, we proposed a Bernoulli sub-ODE method to construct exact traveling wave solutions for NLEES.

The rest of the paper is organized as follows. In Section 2, we describe the Bernoulli sub-ODE method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the Bernoulli Sub-ODE method and the known (G'/G) expansion method to find exact traveling wave solutions of the (2+1) dimensional BKK equation. In the last Section, some conclusions are presented.

2. Description of the Bernoulli Sub-ODE method

In this section we present the solutions of the following ODE:

$$G' + \lambda G = \mu G^2, \tag{2.1}$$

where $\lambda \neq 0, G = G(\xi)$

When $\mu \neq 0$, Eq. (2.1) is the type of Bernoulli equation, and we can obtain the solution as

$$G = \frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}, \tag{2.2}$$

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where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t , is given by

$$P(u,u_{t,}u_x,u_y,u_{tt},u_{xt},u_{yt},u_{xx},u_{yy}......)=0 \quad (2.3)$$

where u = u(x, y, t) is an unknown function, P is a polynomial in u = u(x, y, t) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. By using the solutions of Eq. (2.1), we can construct a serials of exact solutions of nonlinear equations:.

*Step 1.*We suppose that

$$u(x,y,t)=u(\xi),\xi=\xi(x,y,t) \quad (2.4)$$

the traveling wave variable (2.4) permits us reducing Eq. (2.3) to an ODE for $u=u(\xi)$

$$P(u,u',u'',......)=0 \quad (2.5)$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in G as follows:

$$u(\xi)=\alpha_m G^m+\alpha_{m-1}G^{m-1}+...... \quad (2.6)$$

where $G=G(\xi)$ satisfies Eq. (2.1), and $\alpha_m,\alpha_{m-1}...$ are constants to be determined later, $\alpha_m\neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the same order of G together, the left-hand side of Eq. (2.5) is converted into another polynomial in G . Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m,\alpha_{m-1},...\lambda,\mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (2.1), we can construct the traveling wave solutions of the nonlinear evolution equation (2.5).

In the subsequent sections we will illustrate the proposed method in detail by applying it to (2+1) dimensional BKK equation.

3. Application Of the Bernoulli Sub-ODE Method For (2+1) dimensional BKK Equation

In this section, we will consider the following (2+1) dimensional BKK equation:

$$u_{ty}-u_{xxy}+2(uu_x)_y+2v_{xx}=0 \quad (3.1)$$

$$v_t+v_{xxx}+2uv_x=0 \quad (3.2)$$

Supposing that

$$\xi=kx+ly+st \quad (3.3)$$

By (3.3), (3.1) and (3.2) are converted into ODEs

$$slu''-k^2lu'''+2kl(uu')'+2k^2v''=0 \quad (3.4)$$

$$sv'+k^2v''+2k(uv)'=0 \quad (3.5)$$

Integrating (3.4) and (3.5) once, we have

$$slu' - k^2 lu'' + 2kluu' + 2k^2 v' = 0 \tag{3.6}$$

$$sv + k^2 v' + 2kuv = 0 \tag{3.7}$$

Suppose that the solution of (3.6) and (3.7) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i (\tfrac{G'}{G})^i \tag{3.8}$$

$$v(\xi) = \sum_{i=0}^{n} b_i (\tfrac{G'}{G})^i \tag{3.9}$$

where a_i, b_i are constants, $G = G(\xi)$ satisfies Eq. (2.2). Balancing the order of uu' and v' in Eq. (3.6), the order of v'' and uv in Eq.(3.7), we can obtain $2m+1 = n+1, n+1 = m+n \Rightarrow m = 1, n = 2$. So Eqs. (3.8) and (3.9) can be rewritten as

$$u(\xi) = a_1 G + a_0, a_1 \neq 0 \tag{3.10}$$

$$v(\xi) = b_2 G^2 + b_1 G + b_0, b_2 \neq 0 \tag{3.11}$$

where a_1, a_0, b_2, b_1, b_0 are constants to be determined later.

Substituting (3.10) and (3.11) into (3.6) and (3.7) and collecting all the terms with the same power of G together and equating each coefficient to zero, yields a set of simultaneous algebraic equations. Solving the algebraic equations above, yields:

$$a_1 = -k\mu, a_0 = a_0, b_2 = -k^2 l \mu^2, b_1 = k^2 l \mu \lambda, b_0 = 0,\ k = k, l = l, s = -k\lambda + 2a_0, g_1 = g_2 = 0 \tag{3.12}$$

where a_0, k, l are arbitrary constants.
Substituting (3.13) into (3.11) and (3.12), yields:

$$u(\xi) = -k\mu G + a_0 \tag{3.13}$$

$$v(\xi) = -k^2 l \mu^2 G^2 + k^2 l \mu \lambda G \tag{3.14}$$

where $\xi = kx + ly + (-k\lambda + 2a_0)t$.

Substituting the general solutions of (2.2) into (3.13) and (3.14), we have:

$$u(\xi) = -k\mu(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}) + a_0 \tag{3.15}$$

$$v(\xi) = -k^2 l \mu^2 (\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}})^2 + k^2 l \mu \lambda (\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}) \tag{3.16}$$

where $\xi = kx + ly + (-k\lambda + 2a_0)t$, and k, l, μ, λ are arbitrary constants.

4. Application Of (G’/G) expansion Method For (2+1) dimensional BKK Equation

In this section, we apply the (G'/G) expansion method to obtain the traveling wave solutions of (2+1) dimensional BKK equations (3.1)-(3.2).

Suppose that the solution of (3.6) and (3.7) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi)=\sum_{i=0}^{m}a_i(\tfrac{G'}{G})^i \tag{4.1}$$

$$v(\xi)=\sum_{i=0}^{n}b_i(\tfrac{G'}{G})^i \tag{4.2}$$

where a_i , b_i are constants, $G=G(\xi)$ satisfies the second order LODE in the form:

$$G''+\lambda G'+\mu G=0 \tag{4.3}$$

where λ and μ are constants. Balancing the order of uu' and v' in Eq.(3.6), the order of v'' and uv in Eq.(4.7), we can obtain $2m+1=n+1, n+1=m+n \Rightarrow m=1, n=2$. So Eq.(4.8) and (4.9) can be rewritten as

$$u(\xi)=a_1(\tfrac{G'}{G})^1+a_0, a_1\neq 0 \tag{4.4}$$

$$v(\xi)=b_2(\tfrac{G'}{G})^2+b_1(\tfrac{G'}{G})^1+b_0, b_2\neq 0 \tag{4.5}$$

where a_1, a_0, b_2, b_1, b_0 are constants to be determined later.

Substituting (4.4) and (4.5) into (3.6) and (3.7) and collecting all the terms with the same power of $(\frac{G'}{G})$ together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

For Eq. (4.6):

$$(\frac{G'}{G})^0: -k^2la_1\lambda\mu-lsa_1\mu-2k^2b_1\mu-g_1-2kla_1a_0\mu=0$$

$$(\frac{G'}{G})^1: -lsa_1\lambda-2k^2la_1\mu-k^2la_1\lambda^2-2lka_1a_0\lambda-4k^2b_2\mu-2k^2b_1\lambda-2kla_1^2\mu=0$$

$$(\frac{G'}{G})^2: -2kla_1^2\lambda-4k^2b_2\lambda-2kla_1a_0-lsa_1-2k^2b_1-3lk^2a_1\lambda=0$$

$$(\frac{G'}{G})^3: -2kla_1^2-4k^2b_2-2k^2la_1=0$$

For Eq. (4.7):

$$(\frac{G'}{G})^0: sb_0+2kb_0a_0-g_2-k^2b_1\mu=0$$

$$(\frac{G'}{G})^1: -k^2b_1\lambda+sb_1+2kb_1a_0-2k^2b_2\mu+2ka_1b_0=0$$

$$(\frac{G'}{G})^2: -k^2b_1-2k^2b_2\lambda+2kb_2a_0+2kb_1a_1+sb_2=0$$

$$(\frac{G'}{G})^3: -2k^2b_2+2ka_1b_2=0$$

Solving the algebraic equations above, yields:

$$a_1 = k, a_0 = a_0, b_2 = -kl, b_1 = -kl\lambda, b_0 = -kl\mu,\ k = k, l = l, s = k^2\lambda - 2ka_0, g_1 = g_2 = 0 \tag{4.6}$$

where a_0, k, l are arbitrary constants.

Substituting (4.13) into (4.11) and (4.12), yields:

$$u(\xi) = k(\tfrac{G'}{G}) + a_0 \tag{4.7}$$

$$v(\xi) = -kl(\tfrac{G'}{G})^2 - kl\lambda(\tfrac{G'}{G})^1 - kl\mu \tag{4.8}$$

where $\xi = kx + ly + (k^2\lambda - 2ka_0)t$.

Substituting the general solutions of (4.3) into (4.7) and (4.8), we have:

When $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = -\frac{k\lambda}{2} + \frac{k\sqrt{\lambda^2-4\mu}}{2} \cdot \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right) + a_0$$

$$v_1(\xi) = \frac{kl\lambda^2}{4} - \frac{kl}{4}(\lambda^2-4\mu) \cdot \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)^2 - kl\mu$$

where $\xi = kx + ly + (k^2\lambda - 2ka_0)t$, a_0, k, l, C_1, C_2 are arbitrary constants.

When $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = -\frac{k\lambda}{2} + \frac{k\sqrt{4\mu-\lambda^2}}{2} \cdot \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right) + a_0$$

$$v_2(\xi) = \frac{kl\lambda^2}{4} - \frac{kl}{4}(\lambda^2-4\mu) \cdot \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2 - kl\mu$$

where $\xi = kx + ly + (k^2\lambda - 2ka_0)t$, a_0, k, l, C_1, C_2 are arbitrary constants.

When $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{k(2C_2 - C_1\lambda - C_2\lambda\xi)}{2(C_1 + C_2\xi)} + a_0$$

$$v_3(\xi) = \frac{k\lambda^2 l}{4} - \frac{klC_2^{\,2}}{(C_1 + C_2\xi)^2} - kl\mu$$

where $\xi = kx - \frac{3b_1^{\,2}}{2k}t$, a_0, k, l, C_1, C_2 are arbitrary constants..

Remark: As one can see from Section III and Section IV, the traveling wave solutions obtained by the Bernoulli Sub-ODE method are different from those by the known (G'/G) expansion method.

5. Conclusions

We have seen that some new traveling wave solutions of (2+1) dimensional BKK equation are successfully found by using the Bernoulli sub-ODE method. The main points of the method are that assuming the solution of the ODE reduced by using the traveling wave variable as well as integrating can be expressed by an m-th degree polynomial in G, where $G = G(\xi)$ is the general solutions of a Bernoulli sub-ODE equation. The positive integer m can be determined by the general homogeneous balance method, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations. Also this method can be used to many other nonlinear problems.

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