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Exact Solutions for Nonlinear D-S Equation by Two Known Sub-ODE Methods

Qinghua Feng[+]

School of Science, Shandong University of Technology, Zhangzhou Road 12, Zibo, Shandong, China, 255049

Abstract. In this paper, we derive exact traveling wave solutions of nonlinear D-S equation by a proposed Bernoulli sub-ODE method and the known (G'/G) expansion method.

Keywords: Bernoulli sub-ODE method, (G'/G) expansion method, traveling wave solutions, exact solution, evolution equation, nonlinear D-S equation

1. Introduction

In scientific research, seeking the exact solutions of nonlinear equations is a hot topic. Many approaches have been presented so far [1-17].

In this paper, we proposed a Bernoulli sub-ODE method to construct exact traveling wave solutions for NLEES.

The rest of the paper is organized as follows. In Section 2, we describe the Bernoulli sub-ODE method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the Bernoulli Sub-ODE method and the known (G'/G) expansion method to find exact traveling wave solutions of the nonlinear D-S equation. In the last Section, some conclusions are presented.

2. Description of the Bernoulli Sub-ODE method

In this section we present the solutions of the following ODE:

$$G' + \lambda G = \mu G^2, \tag{2.1}$$

where $\lambda \neq 0, G = G(\xi)$

When $\mu \neq 0$, Eq. (2.1) is the type of Bernoulli equation, and we can obtain the solution as

$$G = \frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}, \tag{2.2}$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t , is given by

[+] Corresponding author. Tel.: +86-13561602410
E-mail address: fqhua@sina.com

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy} ......) = 0 \tag{2.3}$$

where u = u(x, y, t) is an unknown function, P is a polynomial in u = u(x, y, t) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. By using the solutions of Eq. (2.1), we can construct a serials of exact solutions of nonlinear equations:.

Step 1. We suppose that

$$u(x, y, t) = u(\xi), \xi = \xi(x, y, t) \tag{2.4}$$

the traveling wave variable (2.4) permits us reducing Eq. (2.3) to an ODE for $u = u(\xi)$

$$P(u, u', u'', ...) = 0 \tag{2.5}$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in G as follows:

$$u(\xi) = \alpha_m G^m + \alpha_{m-1} G^{m-1} + ... \tag{2.6}$$

where $G = G(\xi)$ satisfies Eq. (2.1), and $\alpha_m, \alpha_{m-1}...$ are constants to be determined later, $\alpha_m \neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the same order of G together, the left-hand side of Eq. (2.5) is converted into another polynomial in G. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \alpha_{m-1}, ...\lambda, \mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (2.1), we can construct the traveling wave solutions of the nonlinear evolution equation (2.5).

In the subsequent sections we will illustrate the proposed method in detail by applying it to nonlinear D-S equation.

3. Application Of the Bernoulli Sub-ODE Method For nonlinear D-S Equation

In this section, we will consider the following nonlinear D-S equations:

$$u_t + (v^2)_x = 0 \tag{3.1}$$

$$v_t - v_{xxx} + 3vu_x + 3uv_x = 0 \tag{3.2}$$

Supposing that

$$\xi = kx - \omega t \tag{3.3}$$

By (3.3), (3.1) and (3.2) are converted into ODEs

$$-\omega u' + k(v^2)' = 0 \tag{3.4}$$

$$-\omega v' - k^3 v''' + 3kvu' + 3kuv' = 0 \tag{3.5}$$

Integrating (3.4) and (3.5) once, we have

$$-\omega u + kv^2 = g_1 \tag{3.6}$$

$$-\omega v - k^3 v'' + 3kuv = g_2 \tag{3.7}$$

Suppose that the solution of (3.6) and (3.7) can be exp-ressed by a polynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i \tag{3.8}$$

$$v(\xi) = \sum_{i=0}^{n} b_i G^i \tag{3.9}$$

where a_i , b_i are constants, $G = G(\xi)$ satisfies Eq. (2.2). Balancing the order of u and v^2 in Eq. (3.6), the order of v'' and uv in Eq. (3.7), we can obtain $m = 2n, m+2 = m+n \Rightarrow m = 2, n = 1$. So Eq.(3.8) and (3.9) can be rewritten as

$$u(\xi) = a_2 G^2 + a_1 G + a_0, a_2 \neq 0 \tag{3.10}$$

$$v(\xi) = b_1 G + b_0, b_1 \neq 0 \tag{3.11}$$

where a_2, a_1, a_0, b_1, b_0 are constants to be determined later.

Substituting (3.10) and (3.11) into (3.6) and (3.7) and collecting all the terms with the same power of G together and equating each coefficient to zero, yields a set of simultaneous algebraic equations. Solving the algebraic equations above, yields:

$$a_0 = \frac{b_1^2}{2k^2\mu^2}, a_1 = -\frac{2}{3}k^2\mu\lambda, a_2 = \frac{2}{3}k^2\mu^2, \; b_0 = -\frac{1}{2}\frac{b_1\lambda}{\mu}, b_1 = b_1, k = k, g_2 = 0, \omega = \frac{3}{2}\frac{b_1^2}{k\mu^2}, \; g_1 = \frac{b_1^2(-3b_1^2 + k^4\lambda^2\mu^2)}{4k^3\mu^4} \tag{3.12}$$

where b_1, k are arbitrary constants.

Substituting (3.12) into (3.10) and (3.11), yields:

$$u(\xi) = \frac{2}{3}k^2\mu^2 G^2 - \frac{2}{3}k^2\mu\lambda G + \frac{b_1^2}{2k^2\mu^2} \tag{3.13}$$

$$v(\xi) = b_1 G - \frac{1}{2}\frac{b_1\lambda}{\mu} \tag{3.14}$$

where $\xi = kx - \frac{3}{2}\frac{b_1^2}{k\mu^2}t$., and b_1, k are arbitrary constants.

Substituting the general solutions of (2.2) into (3.13) and (3.14), we obtain the traveling wave solutions of nonlinear D-S equations as follows:

$$u(\xi) = \frac{2}{3}k^2\mu^2\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right)^2 - \frac{2}{3}k^2\mu\lambda\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) + \frac{b_1^2}{2k^2\mu^2} \tag{3.15}$$

$$v(\xi) = b_1\left(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}\right) - \frac{1}{2}\frac{b_1\lambda}{\mu} \tag{3.16}$$

where b_1, k, μ, λ are arbitrary constants.

4. Application Of (G’/G) expansion Method For nonlinear D-S Equation

In this section, we apply the (G'/G) expansion method to obtain the traveling wave solutions of nonlinear D-S equations (3.1)-(3.2).

Suppose that the solution of (3.6) and (3.7) can be exp-ressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi)=\sum_{i=0}^{m} a_i (\tfrac{G'}{G})^i \tag{4.1}$$

$$v(\xi)=\sum_{i=0}^{n} b_i (\tfrac{G'}{G})^i \tag{4.2}$$

where a_i , b_i are constants, $G=G(\xi)$ satisfies the second order LODE in the form:

$$G''+\lambda G'+\mu G=0 \tag{4.3}$$

where λ and μ are constants. Balancing the order of u and v^2 in Eq.(4.6), the order of v'' and uv in Eq.(4.7), we can obtain $m=2n, m+2=m+n \Rightarrow m=2, n=1$. So Eq.(4.1) and (4.2) can be rewritten as

$$u(\xi)=a_2(\tfrac{G'}{G})^2+a_1(\tfrac{G'}{G})^1+a_0, a_2\neq 0 \tag{4.4}$$

$$v(\xi)=b_1(\tfrac{G'}{G})^1+b_0, b_1\neq 0 \tag{4.5}$$

where a_2, a_1, a_0, b_1, b_0 are constants to be determined later.

Substituting (4.4) and (4.5) into (3.6) and (3.7) and co-llecting all the terms with the same power of $(\frac{G'}{G})$ together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

For Eq.(4.6):

$$(\frac{G'}{G})^0 : -\omega a_0 - g_1 + kb_0^{\,2} = 0$$

$$(\frac{G'}{G})^1 : -\omega a_1 + 2kb_1b_0 = 0$$

$$(\frac{G'}{G})^2 : kb_1^{\,2} - \omega a_2 = 0$$

For Eq.(4.7):

$$(\frac{G'}{G})^0 : -\omega b_0 - g_2 - k^3 b_1\lambda\mu + 3ka_0b_0 = 0$$

$$(\frac{G'}{G})^1 : -k^3 b_1\lambda^2 + 3ka_0b_1 - \omega b_1 - 2k^3 b_1\mu + 3kb_0a_1 = 0$$

$$(\frac{G'}{G})^2 : 3ka_1b_1 - 3k^3 b_1\lambda + 3kb_0a_2 = 0$$

$$(\frac{G'}{G})^3 : -2k^3 b_1 + 3kb_1a_2 = 0$$

Solving the algebraic equations above, yields:

$$a_2=\frac{2}{3}k^2, a_1=\frac{2}{3}k^2\lambda, a_0=\frac{1}{6}\frac{3b_1^{\,2}+4k^4\mu}{k^2}, b_1=b_1, b_0=\frac{1}{2}b_1\lambda,\ k=k, \omega=\frac{3b_1^{\,2}}{2k}, g_1=\frac{b_1^{\,2}(-3b_1^{\,2}-4k^4\mu+k^4\lambda)}{4k^3}, g_2=0 \tag{4.6}$$

where b_1, k are arbitrary constants. Substituting (4.6) into (4.4) and (4.5), yields:

$$u(\xi)=\frac{2}{3}k^2(\tfrac{G'}{G})^2+\frac{2}{3}k^2\lambda(\tfrac{G'}{G})^1+\frac{1}{6}\frac{3b_1^2+4k^4\mu}{k^2} \quad (4.7)$$

$$v(\xi)=b_1(\tfrac{G'}{G})^1+\frac{1}{2}b_1\lambda \quad (4.8)$$

where $\xi=kx-\frac{3b_1^2}{2k}t$.

Substituting the general solutions of (4.3) into (4.7) and (4.8), we have:

When $\lambda^2-4\mu>0$

$$u_1(\xi)=-\frac{k^2\lambda^2}{6}+\frac{k^2}{6}(\lambda^2-4\mu).\left(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)^2+\frac{1}{6}\frac{3b_1^2+4k^4\mu}{k^2}$$

$$v_1(\xi)=-\frac{1}{2}b_1\lambda+\frac{b_1\sqrt{\lambda^2-4\mu}}{2}\cdot\left(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)^2+\frac{1}{2}b_1\lambda$$

where $\xi=kx-\frac{3b_1^2}{2k}t$, b_1,k are arbitrary constants.

When $\lambda^2-4\mu<0$

$$u_2(\xi)=-\frac{k^2\lambda^2}{6}+\frac{k^2}{6}(4\mu-\lambda^2).\left(\frac{C_1\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2+\frac{1}{6}\frac{3b_1^2+4k^4\mu}{k^2}$$

$$v_2(\xi)=-\frac{1}{2}b_1\lambda+\frac{b_1\sqrt{4\mu-\lambda^2}}{2}\cdot\left(\frac{C_1\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cosh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sinh\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2+\frac{1}{2}b_1\lambda$$

where $\xi=kx-\frac{3b_1^2}{2k}t$, b_1,k are arbitrary constants.

When $\lambda^2-4\mu=0$

$$u_3(\xi)=-\frac{k^2\lambda^2}{6}+\frac{k^2C_2^2}{3(C_1+C_2\xi)^2}+\frac{1}{6}\frac{3b_1^2+4k^4\mu}{k^2}$$

$$v_3(\xi)=\frac{b_1(2C_2-C_1\lambda-C_2\lambda\xi)}{2(C_1+C_2\xi)}+\frac{1}{2}b_1\lambda$$

where $\xi=kx-\frac{3b_1^2}{2k}t$, b_1,k are arbitrary constants.

Remark: As one can see from Section III and Section IV, the traveling wave solutions obtained by the Bernoulli Sub-ODE method are different from those by the known (G'/G) expansion method

5. Conclusions

We have seen that some new traveling wave solutions of nonlinear D-S equation are successfully found by using the Bernoulli sub-ODE method. The main points of the method are that assuming the solution of the ODE reduced by using the traveling wave variable as well as integrating can be expressed by an m-th degree polynomial in G , where $G = G(\xi)$ is the general solutions of a Bernoulli sub-ODE equation. The positive integer m can be determined by the general homogeneous balance method, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations. Also we make a comparison between the proposed method and the known (G'/G) expansion method. The Bernoulli Sub-ODE method method can be applied to many other nonlinear problems.

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