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New Analytical Solutions For (3+1) Dimensional Kaup-Kupershmidt Equation

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Abstract. In this paper, we derive exact traveling wave solutions of Kaup-Kupershmidt equation by a proposed Bernoulli sub-ODE method. The method appears to be efficient in seeking exact solutions of nonlinear equations. We also make a comparison between the present method and the known (G'/G) expansion method.

Keywords: Bernoulli sub-ODE method, traveling wave solutions, exact solution, evolution equation, Kaup-Kupershmidt equation

1. Introduction

Research on nonlinear equations is a hot topic. The powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far. During the past four decades or so searching for explicit solutions of nonlinear evolution equations by using various different methods have been the main goal for many researchers, and many powerful methods for constructing exact solutions of nonlinear evolution equations have been established and developed such as the homogeneous balance method, the hyperbolic tangent expansion method, the trial function method, the tanh method, the nonlinear transform method, the inverse scattering transform, the Backlund transform, the Hirotas bilinear method, the generalized Riccati equation, the theta function method, the sine-Ccosine method, the Jacobi elliptic function expansion, the complex hyperbolic function method [1-25], and so on. In this paper, we proposed a Bernoulli sub-ODE method to construct exact traveling wave solution-ns for NLEES.

The rest of the paper is organized as follows. In Section 2, we describe the Bernoulli sub-ODE method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the Bernoulli Sub-ODE method and the known (G'/G) expansion method to find exact traveling wave solutions of the Kaup-Kupershmidt equation. In the last Section, some conclusions are presented.

2. Description of the Bernoulli Sub-ODE method

In this section we present the solutions of the following ODE:

$$G'+\lambda G=\mu G^2, \tag{2.1}$$

where $\lambda \neq 0, G=G(\xi)$

When $\mu \neq 0$, Eq. (2.1) is the type of Bernoulli equation, and we can obtain the solution as

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$$G = \frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}, \tag{2.2}$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t , is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy} ......) = 0 \tag{2.3}$$

where u = u(x, y, t) is an unknown function, P is a polynomial in u = u(x, y, t) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. By using the solutions of Eq. (2.1), we can construct a serials of exact solutions of nonlinear equations:.

*Step 1.*We suppose that

$$u(x, y, t) = u(\xi), \xi = \xi(x, y, t) \tag{2.4}$$

the traveling wave variable (2.4) permits us reducing Eq. (2.3) to an ODE for $u = u(\xi)$

$$P(u, u', u'', ...) = 0 \tag{2.5}$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in G as follows:

$$u(\xi) = \alpha_m G^m + \alpha_{m-1} G^{m-1} + ... \tag{2.6}$$

where $G = G(\xi)$ satisfies Eq. (2.1), and $\alpha_m, \alpha_{m-1}...$ are constants to be determined later, $\alpha_m \neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the same order of G together, the left-hand side of Eq. (2.5) is converted into another polynomial in G . Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \alpha_{m-1}, ...\lambda, \mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (2.1), we can construct the traveling wave solutions of the nonlinear evolution equation (2.5).

In the subsequent sections we will illustrate the propo-sed method in detail by applying it to Kaup-Kupershmidt equation.

3. Application Of the Bernoulli Sub-ODE Method For Kaup-Kupershmidt Equation

In this section, we will consider the following Kaup-Kupershmidt equation:

$$u_{xxxxx} + u_t + 45u_x u^2 - \frac{75}{2} u_{xx} u_x - 15uu_{xxx} = 0 \tag{3.1}$$

Suppose that

$$u(x, y, t) = u(\xi), \xi = kx + \omega t \tag{3.2}$$

where k, ω are constants that to be determined later.

By (3.2), (3.1) is converted into an ODE

$$k^5 u^{(5)} + \omega u' + 45ku'u^2 - \frac{75}{2} k^3 u'u'' - 15k^3 uu''' = 0 \tag{3.3}$$

Integrating (3.3) once we obtain

$$k^5 u^{(4)} + \omega u + 15ku^3 - \frac{45}{4} k^3 u'^2 - 15k^3 uu'' = g \tag{3.4}$$

where g is the integration constant.

Suppose that the solution of (3.4) can be expressed by a polynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i \tag{3.5}$$

where a_i are constants, and $G = G(\xi)$ satisfies Eq. (2.1).

Balancing the order of u^3 and $u^{(4)}$ in Eq.(3.4), we have $3m+2 = m+4 \Rightarrow m=2$ .So Eq.(3.5) can be rewritten as

$$u(\xi) = a_2G^2 + a_1G + a_0, a_2 \neq 0 \tag{3.6}$$

where a_2, a_1, a_0 are constants to be determined later.

Substituting (3.6) into (3.4) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0 : -g + \omega a_0 + 15ka_0^{\ 3} = 0$$

$$G^1 : -15a_1k^3a_0\lambda^2 + \omega a_1 + k^5a_1\lambda^4 + 45ka_0^{\ 2}a_1 = 0$$

$$G^2 : 45ka_0^{\ 2}a_2 - 60k^3a_0a_2\lambda^2 + 45ka_0a_1^{\ 2} + 45a_0a_1\mu\lambda k^3 - \frac{105}{4}k^3a_1^{\ 2}\lambda^2 + 15a_2k^5\lambda^4 + \omega a_2 - 15a_1\mu k^5\lambda^3 = 0$$

$$G^3 : 90ka_0a_1a_2 + \frac{135}{2}\lambda\mu k^3a_1^{\ 2} - 30k^3\mu^2a_0a_1 - 130a_2\mu\lambda^3k^5 + 15ka_1^{\ 3} + 50k^5\mu^2a_1\lambda^2 - 120a_1a_2\lambda^2k^3 + 150a_0a_2\lambda\mu = 0$$

$$G^4 : 285k^3a_1a_2\mu\lambda + 45ka_1^{\ 2}a_2 - 60k^5\mu^3a_1\lambda - 90a_0a_2k^3\mu^2 - 105k^3a_2^{\ 2}\lambda^2 + 330k^5a_2\mu^2\lambda^2 + 45ka_0a_2^{\ 2} - \frac{165}{4}k^3a_1^{\ 2}\mu^2 = 0$$

$$G^5 : 240k^3a_2^{\ 2}\mu\lambda + 24k^5\mu^4a_1 - 336k^5a_2\mu^3\lambda + 45ka_0a_2^{\ 2} - 165k^3a_1a_2\mu^2 = 0$$

$$G^6 : 120k^5a_2\mu^4 + 15ka_2^{\ 3} - 135k^3a_2^{\ 2}\mu^2 = 0$$

Solving the algebraic equations above, yields:

Case 1:

$$a_2 = k^2\mu^2, a_1 = -k^2\mu\lambda, a_0 = \frac{1}{12}k^2\lambda^2,\ k = k, \omega = -\frac{1}{16}k^5\lambda^4, g = \frac{1}{288}k^7\lambda^6 \tag{3.7}$$

where $k \neq 0$ is an arbitrary constant.

Substituting (3.7) into (3.6), we obtain

$$u_1(\xi) = k^2\mu^2G^2 - k^2\mu\lambda G + \frac{1}{12}k^2\lambda^2,\ \xi = kx - \frac{1}{16}k^5\lambda^4 t \tag{3.8}$$

Combining with Eq. (2.2), we can obtain the traveling wave solutions of (3.1) as follows:

$$u_1(x,t) = k^2\mu^2[\frac{1}{\frac{\mu}{\lambda} + de^{\lambda(kx - \frac{1}{16}k^5\lambda^4 t)}}]^2 - k^2\mu\lambda[\frac{1}{\frac{\mu}{\lambda} + de^{\lambda(kx - \frac{1}{16}k^5\lambda^4 t)}}] + \frac{1}{12}k^2\lambda^2 \tag{3.9}$$

Case 2:

$$a_2 = 8k^2\mu^2, a_1 = -8k^2\mu\lambda, a_0 = \frac{2}{3}k^2\lambda^2,\ k = k, \omega = -11k^5\lambda^4, g = -\frac{29}{9}k^7\lambda^6 \tag{3.10}$$

where $k \neq 0$ is an arbitrary constant.

Substituting (3.7) into (3.6), we obtain

$$u_2(\xi) = 8k^2\mu^2G^2 - 8k^2\mu\lambda G + \frac{2}{3}k^2\lambda^2,\ \xi = kx - 11k^5\lambda^4 t \tag{3.11}$$

Combining with Eq. (2.2), we can obtain the traveling wave solutions of (3.1) as follows:

$$u_2(x,t) = k^2\mu^2[\frac{1}{\frac{\mu}{\lambda} + de^{\lambda(kx - 11k^5\lambda^4 t)}}]^2 - k^2\mu\lambda[\frac{1}{\frac{\mu}{\lambda} + de^{\lambda(kx - 11k^5\lambda^4 t)}}] + \frac{1}{12}k^2\lambda^2 \tag{3.12}$$

4. Application Of (G'/G) expansion Method For Kaup-Kupershmidt Equation

In this section, we apply the (G'/G) expansion method to obtain the traveling wave solutions of Kaup-Kupershmidt equation (3.1).

Suppose that the solution of (3.4) can be expressed by a polynomial in G as follows:

$$u(\xi)=a_2(\frac{G'}{G})^2+a_1\frac{G'}{G}+a_0, a_2\neq 0 \tag{4.1}$$

where a_i are constants, and $G=G(\xi)$ satisfies

$$G''+\lambda G'+\mu G=0 \tag{4.2}$$

Substituting (4.2) into (3.4) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations. Solving these equations we obtain

$$a_2=8k^2, a_1=8k^2\lambda, a_0=\frac{2}{3}k^2(\lambda^2+8\mu), k=k, \omega=-11k^5(-8\lambda^2\mu+16\mu^2+\lambda^4),$$
$$g=\frac{1664}{9}k^7\mu^3+\frac{104}{3}k^7\mu\lambda^4-\frac{416}{3}k^7\mu^2\lambda^2 \tag{4.3}$$

where $k\neq 0$ is an arbitrary constant.

Substituting (4.3) into (4.2), we obtain

$$u(\xi)=8k^2(\frac{G'}{G})^2+8k^2\lambda\frac{G'}{G}+\frac{2}{3}k^2(\lambda^2+8\mu),\ \xi=kx-11k^5(-8\lambda^2\mu+16\mu^2+\lambda^4)t \tag{4.4}$$

Combining (4.2) and (4.4) we obtain the following solutions.

When $\lambda^2-4\mu>0$

$$u_1(\xi)=\frac{2}{3}k^2(\lambda^2+8\mu)-2k^2\lambda^2+2k^2(\lambda^2-4\mu)\ (\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi})^2$$

When $\lambda^2-4\mu<0$

$$u_2(\xi)=\frac{2}{3}k^2(\lambda^2+8\mu)-2k^2\lambda^2+2k^2(4\mu-\lambda^2)\ (\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi})^2$$

When $\lambda^2-4\mu=0$

$$u_3(\xi)=\frac{2}{3}k^2(\lambda^2+8\mu)-2k^2\lambda^2+\frac{8k^2C_2^{\ 2}}{(C_1+C_2\xi)^2}$$

Remark: As one can see from Section III and Section IV, the traveling wave solutions obtained by the Bernoulli Sub-ODE method are different from those by the known (G'/G) expansion method.

5. Conclusions

We have seen that some new traveling wave solutions of Kaup-Kupershmidt equation are successfully found by using the Bernoulli sub-ODE method. The main points of the method are that assuming the solution of the ODE reduced by using the traveling wave variable as well as integrating can be expressed by an m-th degree polynomial in G, where $G=G(\xi)$ is the general solutions of a Bernoulli sub-ODE equation. The positive integer m can be determined by the general homogeneous balance method, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations. Also we make a comparison between the proposed method and the known (G'/G) expansion method. The Bernoulli Sub-ODE method method can be applied to many other nonlinear problems.

6. References

[1] M. Wang, Solitary wave solutions for variant Boussinesq equations, Phys. Lett. A 199 (1995) 169-172.

[2] E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, On the solitary wave solutions for nonlinear Hirota-Satsuma coupled KdV equations, Chaos, Solitons and Fractals 22 (2004) 285-303.

[3] L. Yang, J. Liu, K. Yang, Exact solutions of nonlinear PDE nonlinear transformations and reduction of nonlinear PDE to a quadrature, Phys. Lett. A 278 (2001) 267-270.

[4] E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, Group analysis. and modified tanh-function to find the invariant solutions and soliton solution for nonlinear Euler equations, Int. J. Nonlinear Sci. Numer. Simul. 5 (2004) 221-234

[5] M. Inc, D.J. Evans, On traveling wave solutions of some nonlinear evolution equations, Int. J. Comput. Math. 81 (2004) 191-202

[6] M.A. Abdou, The extended tanh-method and its applications for solving nonlinear physical models, Appl. Math. Comput. 190 (2007) 988-996.

[7] E.G. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (2000) 212-218.

[8] W. Malfliet, Solitary wave solutions of nonlinear wave equations, Am. J. Phys. 60 (1992) 650-654.

[9] J.L. Hu, A new method of exact traveling wave solution for coupled nonlinear differential equations, Phys. Lett. A 322 (2004) 211-216.

[10] M.J. Ablowitz, P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform, Cambridge University Press, Cambridge, 1991.

[11] M.R. Miura, Backlund Transformation, Springer-Verlag, Berlin, 1978.

[12] C. Rogers, W.F. Shadwick, Backlund Transformations, Academic Press, New York, 1982.

[13] R. Hirota, Exact envelope soliton solutions of a nonlinear wave equation, J. Math. Phys. 14 (1973) 805-810.

[14] R. Hirota, J. Satsuma, Soliton solution of a coupled KdV equation, Phys. Lett. A 85 (1981) 407-408.

[15] Z.Y. Yan, H.Q. Zhang, New explicit solitary wave solutions and periodic wave solutions for Whitham-Broer-Kaup equation in shallow water, Phys. Lett. A 285 (2001) 355-362.

[16] A.V. Porubov, Periodical solution to the nonlinear dissipative equation for surface waves in a convecting liquid layer, Phys. Lett. A 221 (1996) 391-394

[17] E.G. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (2000) 212-218.

[18] E.G. Fan, Multiple traveling wave solutions of nonlinear evolution equations using a unifiex algebraic method, J. Phys. A, Math. Gen. 35 (2002) 6853-6872.

[19] Z.Y. Yan, H.Q. Zhang, New explicit and exact traveling wave solutions for a system of variant Boussinesq equations in mathematical physics, Phys. Lett. A 252 (1999) 291-296.

[20] S.K. Liu, Z.T. Fu, S.D. Liu, Q. Zhao, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Phys. Lett. A 289 (2001) 69-74.

[21] Z. Yan, Abundant families of Jacobi elliptic functions of the (2+1)-dimensional integrable Davey-Stawartson-type equation via a new method, Chaos, Solitons and Fractals 18 (2003) 299-309.

[22] C. Bai, H. Zhao, Complex hyperbolic-function method and its applications to nonlinear equations, Phys. Lett. A 355 (2006) 22-30..

[23] E.M.E. Zayed, A.M. Abourabia, K.A. Gepreel, M.M. Horbaty, On the rational solitary wave solutions for the nonlinear Hirota-Satsuma coupled KdV system, Appl. Anal. 85 (2006) 751- 768.

[24] K.W. Chow, A class of exact periodic solutions of nonlinear envelope equation, J. Math. Phys. 36 (1995) 4125-4137.

[25] M.L.Wang, Y.B. Zhou, The periodic wave equations for the Klein-Gordon-Schordinger equations, Phys. Lett. A 318 (2003) 84-92.