

# Design and Simulation of Multiuser Detection Algorithm Based on Momentum Factor

Tiantian Li, Guanghui Cai, Jun Chang , Ruizhe Shang

School of information Science and Engineering

Yunnan University, Kunming, China

e-mail: Selina\_li8013@126.com

**Abstract**—Blind multiuser detection is the key technique of 3G mobile communication. In this paper, the performance of MOE blind multiuser detection algorithm in DS-CDMA communication system is analyzed, and some improvements on the algorithm are proposed by adding a momentum factor, the iteration formulas based on momentum factor multiuser detection algorithm is derived and simulated. The simulations show that the convergence rate of the improved algorithm is increased and the bit error rate is reduced.

**Keywords-** Blind Multiuser Detection; Minimum Output Energy Blind Multiuser Detection; LMS Algorithm; Momentum Factor

## 1. Introduction

CDMA commutation system is widely used in third-generation mobile system due to its high capacity, soft capacity, soft handoff, high frequency utilization ratio and the resistance to multipath fading. However, multi-access interference seriously impacts on the capacity and performance of CDMA commutation system. Multiuser detection technology is the major measure to eliminate interference and improve system capacity. The prior information of blind multiuser detection is the same as those of traditional receivers, which user can be detected by only knowing the characteristic waveform and timing information of the desired user but without sending training sequence and interference user information. Therefore, blind multiuser detection has broad prospect, and is becoming a hot research topic in communication, as well as in signal and information processing. Current blind multiuser detection algorithms are mainly concentrated in three areas: subspace decomposition algorithm<sup>[1,2]</sup>, constrained modulus algorithm<sup>[3,4]</sup> and minimum output energy criterion algorithm<sup>[5-10]</sup>. In 1995, M.Honig first proposed blind adaptive multiuser detection algorithm based on minimum output energy criterion(MOE)<sup>[5]</sup>, which chose the multiuser detector output energy as a cost function, found an optimized weight vector to minimize the cost function under one or more constraint conditions, then used an adaptive algorithm to approximate the optimal weight vector. In order to overcome the effect of multipath interference mentioned in [5], M.K.Tsatsani<sup>[6]</sup> produced the multipath channel constraint MOE blind multiuser detection algorithm. Z.Tian<sup>[7]</sup> put forward robust constraint MOE blind multiuser detection algorithm that enhances the system robustness by adding a quadratic inequality constraint, and also reduces the system mismatch caused by other forms of calibration error and channel disturbance. In classic MOE blind multiuser detector, HuiKui<sup>[8]</sup> designed a new blind space-time multiuser detector by combining space-time technology with MOE, which can be applied to the overload system, and the capacity has been improved due to the introduction of spatial information. By combining MOE with sato

---

E-mail address: Selina\_li8013@126.com

algorithm, ShaohuaZeng<sup>[9]</sup> invented a decision feedback MOE blind detector with variable step size which ensures the MOE detector's global convergence and illustrates high output Signal-to-Noise Ratio (SNR). In [10], an improved space-time MOE blind multiuser detection algorithm based on RLS was proposed, which is got through expanding the time-only MOE blind multiuser detection algorithm to space-time domain. By analyzing the MOE blind multiuser detection algorithm, an improved multiuser detection algorithm based on momentum factor is put forward and simulated in this paper.

## 2. SYSTEM MODEL

Considering a synchronous DS/CDMA communication system with  $K$ -users, in Gaussian channel, the received baseband signal at the receiver can be expressed as

$$x(t) = \sum_{k=1}^K A_k b_k(j) s_k(t - jT) + \sigma v(t), \quad t \in [jT, jT + T] \quad (1)$$

where  $A_k$  is the signal amplitude of the  $k$  th user,  $b_k(j) \in \{-1, +1\}$  is the bit set of  $k$  th user,  $T$  is symbol interval,  $v(t)$  is additive white Gaussian noise with unit spectral density,  $\sigma$  is noise variance,  $s_k(t)$  is the signature waveform with unit energy of the  $k$  th user, which satisfies

$$s_k(t) = \begin{cases} 0 & t \notin [0, T] \\ \sum_{i=0}^{J-1} c_k(i) \psi(t - iT_c) & t \in [0, T] \end{cases} \quad (2)$$

$$\|s_k(t)\|^2 = \int_0^T s_k(t) dt = 1 \quad (3)$$

where  $J$  is processing gain,  $\{c_k(i)\}_{i=0}^{J-1}$  is address code (spread spectrum code) assigned to the  $k$  th user, whose value is  $\pm 1$ .  $\psi(t) \in [0, T_c]$  is the normalized chip waveform,  $T_c = T/J$  is the chip interval.

Assume that all the information sequence are independently and equally transmit, indicating  $j = 0$  in (1), so the model can be simplified

$$x(t) = \sum_{k=1}^K A_k b_k s_k(t) + \sigma v(t), \quad t \in [0, T] \quad (4)$$

Sampling the received signal by chip rate, the output sampling vector of  $N$  chips matching filter can be obtained in one symbol interval  $T$ . The model of synchronous baseband systems received signal can be expressed as a  $N \times 1$  vector.

$$\mathbf{x} = \sum_{k=1}^K A_k b_k \mathbf{s}_k + \sigma \mathbf{v} \quad (5)$$

where  $\mathbf{v}$  is the white Gaussian noise vector whose means is 0 and covariance matrix is  $I_N$ ,  $A_k$  is the amplitude of the signal received by  $k$  th user,  $b_k$  is the information bits of signal received by  $k$  th user,  $\mathbf{s}_k = \frac{1}{\sqrt{N}} [\beta_1^k, \beta_2^k, \dots, \beta_N^k]^T$  is normalized as the spread spectrum vector of  $k$  th user,  $N$  is the spread spectrum gain,  $[\beta_1^k, \beta_2^k, \dots, \beta_N^k]^T$  is spread spectrum signal sequence with value  $\pm 1$ .

If user 1 is the desired user, the received signal vector can be presented as

$$\mathbf{x} = A_1 b_1 \mathbf{s}_1 + \underbrace{\sum_{k=2}^K A_k b_k \mathbf{s}_k}_{\text{Multi-Access Interference}} + \underbrace{\sigma \mathbf{v}}_{\text{Background noise}} \quad (6)$$

where  $A_1 b_1 s_1$  is the desired user's signal,  $\sum_{k=2}^K A_k b_k s_k$  is the sum of interfering signal from other users,  $\sigma$  is background noise.

If the blind multiuser detector for the desired user 1 is  $\mathbf{w}_1$ , the output is

$$y_1(n) = \langle \mathbf{w}_1, \mathbf{x} \rangle = \mathbf{w}_1(n)^T \mathbf{x}(n) \quad (7)$$

Though a judger we yield

$$\hat{b}_1(n) = \text{sgn}(\langle \mathbf{w}_1, \mathbf{x} \rangle) = \text{sgn}(\mathbf{w}_1(n)^T \mathbf{x}(n)) \quad (8)$$

The desired user1's detector can be expressed as

$$\mathbf{w}_1 = \mathbf{c}_1 + \mathbf{d}_1 \quad (9)$$

which satisfies

$$\langle \mathbf{c}_1, \mathbf{d}_1 \rangle = 0 \quad (10)$$

Indicating  $\mathbf{w}_1$  is the sum of two quadrature components. Component  $\mathbf{c}_1$  is the spread spectrum sequence for users 1, which is not changed in adaptive training, while component  $\mathbf{d}_1$  needs to be updated. Therefore, the update of the detector's weight vector  $\mathbf{w}_1$  equals to that of  $\mathbf{x}_1$ . It is clear that linear transformation sets  $\{\mathbf{w}_1\}$  which obey (9) and (10) satisfy the following relation

$$\langle \mathbf{w}_1, \mathbf{c}_1 \rangle = \langle \mathbf{c}_1 + \mathbf{d}_1, \mathbf{c}_1 \rangle = \|\mathbf{c}_1\|^2 = 1 \quad (11)$$

The inner products of linear transformation and desired user's spread spectrum sequence can be normalized as 1. Therefore, the canonical expressed linear multiuser detector can avoid using linear transformation orthogonal with the desired signal as linear multiuser detector, which makes the bit error rate equals to 1 / 2.

### 3. MOE BLIND MUTIUSER DETECTION ALGORITHM

M.Honig first proposed Gaussian channel MOE blind multiuser detection technology, which the essence is to satisfy a single constraint, adaptively adjust  $\mathbf{w}$ , and minimize the output energy of detector.

Single constraint MOE blind multiuser detector can be expressed as

$$\begin{cases} \min & E[\langle \mathbf{w}, \mathbf{x}(n) \rangle^2] \\ \text{s.t.} & \langle \mathbf{c}_1, \mathbf{w} \rangle = \mathbf{c}_1^T \mathbf{w} = 1 \end{cases} \quad (12)$$

As long as the constraint  $\langle \mathbf{c}_1, \mathbf{w} \rangle = 1$  is satisfied, the desired signal cannot be negated on the output side. Therefore, MOE blind multiuser detector can restrain the energy of interferential signal and noise.

Set the average output energy of blind multiuser detector as cost function, then yields

$$J(\mathbf{w}) = E[\langle \mathbf{w}, \mathbf{x} \rangle^2] = E[|\mathbf{w}^T \mathbf{x}|^2] = \mathbf{w}^T \mathbf{R} \mathbf{w} \quad (13)$$

where  $\mathbf{R} = E[\mathbf{x}(n)\mathbf{x}^T(n)]$  is the autocorrelation matrix of  $\mathbf{x}(n)$ .

Build a Lagrange function, yields

$$L(\mathbf{w}) = J(\mathbf{w}) + \lambda(1 - \mathbf{c}_1^T \mathbf{w}) \quad (14)$$

where  $\lambda$  is Lagrange multiplier.

Using LMS algorithm to update  $\mathbf{w}$ ,  $\lambda$  of  $L(\mathbf{w})$  yields,

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu_1 \frac{\partial L(n)}{\partial \mathbf{w}(n)} \quad (15)$$

$$\lambda(n+1) = \lambda(n) - \mu_2 \frac{\partial L(n)}{\partial \lambda} \quad (16)$$

where  $\mu_1$  and  $\mu_2$  are iteration step size.

The variations of  $\mathbf{w}$  and  $\lambda$  are

$$\Delta \mathbf{w}(n) = \mathbf{w}(n+1) - \mathbf{w}(n) = -\mu_1 \frac{\partial L(n)}{\partial \mathbf{w}(n)} \quad (17)$$

$$\Delta \lambda(n) = \lambda(n+1) - \lambda(n) = -\mu_2 \frac{\partial L(n)}{\partial \lambda(n)} \quad (18)$$

From (14)  $\frac{\partial L(n)}{\partial \mathbf{w}(n)}$  and  $\frac{\partial L(n)}{\partial \lambda}$  can be concluded,

$$\begin{aligned} \frac{\partial L(n)}{\partial \mathbf{w}(n)} &= \frac{\partial J(n)}{\partial \mathbf{w}(n)} - \lambda \mathbf{c}_1 \\ &= 2\mathbf{w}^T(n)\mathbf{x}(n)\mathbf{x}(n) - \lambda \mathbf{c}_1 \end{aligned} \quad (19)$$

$$\frac{\partial L(n)}{\partial \lambda(n)} = 1 - \mathbf{c}_1^T \mathbf{w} \quad (20)$$

Substituting (19) into (15) then yields iteration formulas of  $\mathbf{w}(n)$

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu_1 [2\mathbf{w}^T(n)\mathbf{x}(n)\mathbf{x}(n) - \lambda \mathbf{c}_1] \quad (21)$$

Substituting (20) into (16) then yields iteration formulas of  $\lambda(n)$

$$\lambda(n+1) = \lambda(n) - \mu_2 [1 - \mathbf{c}_1^T \mathbf{w}(n)] \quad (22)$$

Because the step sizes related to the convergence speed and offset errors, they are especially important. Generally,  $0 < \mu_1 < \frac{1}{\lambda_{\max}}$ ,  $\lambda_{\max}$  is maximal eigenvalues of  $\mathbf{R}$ ,  $\mu_2$  has the same value range.

LMS algorithm is based on the steepest descent algorithm, which searches along the negative direction of the gradient valuation to find the optimal weights, and achieves an adaptive filter with minimum square error.

Steepest descent algorithm is an error descent algorithm based on gradient, which is clear, simple and easy to implement. However the convergence speed is slow and it is easy to fall into the local minimum. Using gradient descent algorithm, training initiates from a starting point along the gradual slope of the objective function to search the minimum error. For a complex objective function, the multi-dimensional curve surface likes a bowl with uneven surface and bottom is the minimum point. Thus it is easy to fall into a small valley area and produce a local minimum point. Then, changes on all directions will increase error and inevitably fall into local minimum.

#### 4. Improved Algorithm By Adding Momentum Factor

Momentum factor can be used to overcome abovementioned shortcomings. The essence is to transfer the impact of last weight changes to the current weight changes by using a momentum factor. When modifying the weight, the error impact to gradient and the trend of the objective function on a curved surface are both taken into consideration, which decreases the sensitivity to the details of error curved surface in training, efficiently restrains from falling into the local minimum and accelerates the convergence speed. When the momentum factor is added, the variation of weight is

$$\Delta \mathbf{w}(n) = \gamma_1 \Delta \mathbf{w}(n-1) - \mu_1 \frac{\partial L(n)}{\partial \mathbf{w}(n)} \quad (23)$$

$$\Delta \lambda(n) = \gamma_2 \Delta \lambda(n-1) - \mu_2 \frac{\partial L(n)}{\partial \lambda(n)} \quad (24)$$

where  $0 \leq \gamma_1 < 1$  and  $0 \leq \gamma_2 < 1$  are the added momentum factors,  $\gamma_1 \Delta \mathbf{w}(n-1)$  is the modification direction storing the previous weight. The weight modification direction at time  $n+1$  is a combination of time  $n-1$  and  $n$ . When the sign of current  $-\mu_1 \frac{\partial L(n)}{\partial \mathbf{w}(n)}$  is the same as previous one, the sum of current and previous increases,  $\Delta \mathbf{w}(n)$  increases, thus the adjusting speed is accelerated, and the convergence speed is accelerated. When the sign of current  $-\mu_1 \frac{\partial L(n)}{\partial \mathbf{w}(n)}$  is the opposite, indicating oscillations exist, and  $\Delta \mathbf{w}(n)$  decrease, which restrains from falling into the local minimum and makes stable. The function of  $\gamma_2$  is similar to that of  $\gamma_1$ .

Using a recursive formula, (23) and (24) can be represented as

$$\begin{aligned} \Delta \mathbf{w}(n) &= -\mu_1 \sum_{m=0}^n \gamma_1^{n-m} \frac{\partial L(m)}{\partial \mathbf{w}(m)} \\ &= -\mu_1 \sum_{m=0}^n \gamma_1^{n-m} [2\mathbf{w}^T(m)\mathbf{x}(m)\mathbf{x}(m) - \lambda \mathbf{c}_1] \end{aligned} \quad (25)$$

$$\begin{aligned} \Delta \lambda(n) &= -\mu_2 \sum_{m=0}^n \gamma_2^{n-m} \frac{\partial L(m)}{\partial \lambda(m)} \\ &= -\mu_2 \sum_{m=0}^n \gamma_2^{n-m} [1 - \mathbf{c}_1^T \mathbf{w}(m)] \end{aligned} \quad (26)$$

Substituting (25) and (26) into (15) and (16) then yields the iteration formulas of  $\mathbf{w}(n)$  and  $\lambda(n)$

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu_1 \sum_{m=0}^n \gamma_1^{n-m} [2\mathbf{w}^T(m)\mathbf{x}(m)\mathbf{x}(m) - \lambda \mathbf{c}_1] \quad (27)$$

$$\lambda(n+1) = \lambda(n) - \mu_2 \sum_{m=0}^n \gamma_2^{n-m} [1 - \mathbf{c}_1^T \mathbf{w}(m)] \quad (28)$$

Momentum factor greatly impacts on the performance of the algorithm. From (25), it can be concluded that momentum factor increases learning rate (i.e. iteration step size), and accelerates the convergence speed.

## 5. Simulation Results And Analysis

*Experiment 1:* Performance comparison on eliminating multiple access interference between traditional MOE blind multiuser detection algorithm and momentum factor multiuser detection algorithm.

In this experiment, the number of user is 6, that is  $K=6$ , user 1 is set as the desire user, the other 5 users as interference users, the multi-access interference strength of interference users relative to desire user is  $10\lg(A_k^2/A_1^2)=10\text{dB}$ , total iterative digit is  $N=5000$ , step sizes are  $5 \times 10^{-7}$ , momentum factor are  $(\gamma_1, \gamma_2) = \{(0,0), (0.3,0.3), (0.5,0.5)\}$ , the Signal-to-Noise Ratio(SNR) and Bit Error Rate(BER) relation curve is obtained from the average of 20 times of experiments.

Figure 1 is the SNR and BER relational curves of traditional MOE blind multiuser detection algorithm ( $\gamma_1=0, \gamma_2=0$ ) and two momentum factors multiuser detection algorithm ( $(\gamma_1, \gamma_2) = \{(0.3,0.3), (0.5,0.5)\}$ ). From fig.1, it is concluded that when  $\gamma_1=0.5, \gamma_2=0.5$ , BER declines slightly, yet the performance on eliminating multiple access interference of momentum factors multiuser detection algorithm is not obviously improved.

*Experiment 2:* Performance comparison on near-far resistance between traditional MOE blind multiuser detection algorithm and momentum factor multiuser detection algorithm.

In this experiment, the number of user is 11, that is  $K = 11$ , user 1 is the desired user, the other 10 users are interference users, the multi-access interference strength of interference users relative to the desired user is  $10\lg(A_k^2/A_1^2)=10dB$ , total iterative digit is  $N=10000$ , step sizes are  $5 \times 10^{-7}$ , momentum factor are  $(\gamma_1, \gamma_2) = \{(0, 0), (0.3, 0.3), (0.5, 0.5)\}$ , SNR of the system is 15 dB.

Figure 2 is the SIR and iterative digit relational curves of traditional MOE blind multiuser detection algorithm ( $\gamma_1=0, \gamma_2=0$ ) and two momentum factors multiuser detection algorithm ( $(\gamma_1, \gamma_2) = \{(0.3, 0.3), (0.5, 0.5)\}$ ). From fig.2, it is obvious that the convergence speeds and the resistant to near-far effect is improved along with the increase of momentum factor.  $\gamma_1=0.3, \gamma_2=0.3$  are the optimal values when considering the convergence speed and stability.

## 6. Conclusion

This paper analyzes the performance of momentum factor on the MOE blind multiuser detection algorithm. Because momentum factor uses the previous weight regulating variable to modify current weight, the sensitivity to details of error curved surface in training process is decreased, which efficiently restrains from falling into the local minimum, accelerates the convergence speed and prevents, oscillation, reduces the BER, and improve the convergence.

## 7. References

- [1] Kikuchi, S. Sano, A. Ottersten. Accelerated subspace tracking scheme and its applications to multiuser systems [J]. IEEE International Symposium Mobile Radio Communications, 2005,(4):2425.
- [2] Cacciapuoti.A.S, Gelli.G, Paura.L. Widely Linear Versus Linera Blind Multiuser Detection With Subspace-Based Channel Estimation: Finite Sample-Size Effects [J]. IEEE Trans Signal processing, 2009, 57(4):1426 – 1443.
- [3] Liu Jiabin, Liang Zhonghua. Linearly Constrained Modulus Algorithm Based Blind MultiUser Detection for DS-UWB Systems [J]. IEEE International Conference Wireless Communications , Networking and Mobile Computing, 2007.
- [4] Weijuan Xia, Lidong Zhu, Xingzhong Xiong. Fractional lower order statistics based generalized constant modulus algorithmfor blind multiuser detection in DS/CDMA systems [J]. IEEE International Conference Wireless Communications and signal Processing, 2010, (4): 1-6.
- [5] M.Honig, U.Mahow, S.Verdu. Blind multiuser detection[J]. IEEE Trans. Inform. Theory, 1995, 41(4): 944-960.
- [6] M.K.Tsatsanis, Z.Xu. Performance analysis of minimum variance CDMA receivers[J]. IEEE Trans.S.P, 1998, 46(11): 3014-3022.
- [7] Z.Tian, K.L.Bell, H.L.Van Trees. Robust constrained linear receiver for CDMA wireless systems [J]. IEEE Trans.S.P, 2001, 49(7): 1510-1522.
- [8] HuiKui, HengYang. Blind adaptive multiuser detector with space-time coding for CDMA system [J]. Tsinghua university journal(Natural science edition), 2002, 42(3): 403-406.
- [9] ShaohuaZeng, GuizongLiu. An equivalent blind multiuser detector algorithm with a variable step-size [J]. Xian university of electronic science and technology journal(Natural science edition), 2003, 30(5):713-716.
- [10] TingLiu, YunshanSun, LiyiZhang. The improved space-time MOE blind multiuser detection algorithm [J]. IEEE International Conference Wireless Mobile and Sensor Networks, 2006.

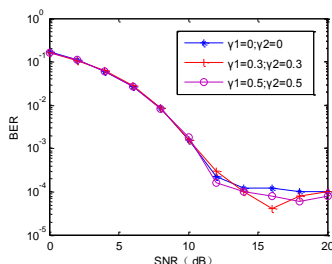


Fig.1 BER versus SNR of traditional MOE and momentum factor ( $\gamma$ ) MOE

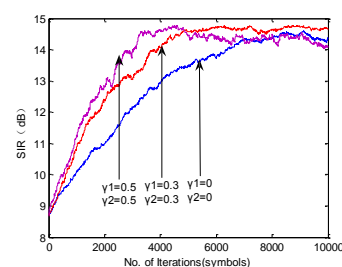


Fig.2 SIR curves of traditional MOE and momentum factor ( $\gamma$ ) MOE