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Federated Cubature Kalman Filter for Multi-sensor Information Fusion

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Abstract—Decentralized cubature Kalman filter (CKF) in federated configuration for multi-sensor information fusion is developed in this paper. Firstly, we use CKF as the sub-filter of the federated filter in order to deal with nonlinear model. Subsequently, master filter adopts reset mode to fuse local estimates of subfilters to generate a global estimate according to the scalar weighted rule. Finally, a simple experiment is carried out to evaluate the new algorithm. Compared with the federated extended Kalman filter (FEKF), the results show that the novel method can further improve the performance in terms of the root mean square error (RMSE).

Keywords- information fusion; nonlinear flter; federated filter; CKF;

1. Introduction

Multi-sensor information fusion is the process of merging outputs from sensors with information from other sensors, information processing blocks, databases or knowledge bases, into one representational form. It has been widely applied in automated target recognition, guidance of autonomous vehicles, remote sensing, battlefield surveillance, automatic threat recognition systems, monitoring of manufacturing processes, robotics and medical etc [1, 2].

Traditional fusion algorithms conclude centralized filter and distributed filter. The latter has been widely studied because of its obvious advantages in assigning the overall computation burden among local processors, and in fault tolerant capability. As one kind of distributed fusion method, federated Kalman filter proposed by Carlson. This filter employs an information sharing principle for the local and master filters and eliminates the correlation between local estimates by using an Upper Bound technique [3]. When comes to nonlinear system, extended Kalman filter (EKF) and Unscented Kalman filter (UKF) are used to design sub-filters [4-8]. However, the limitations of EKF and UKF are as follows. 1) Estimate accuracy of EKF is too low owing to the first-order Taylor series approximation for nonlinear functions. 2) The performance of UKF will decrease obviously when the system state dimension is high comparatively, which is called the curse of dimensionality. Recently, the Cubature Kalman Filter (CKF) was developed in [8]. It adopts the spherical cubature rule and radial rule to optimize the sigma points and weights. So, the ability to deal with high dimension state is greatly enhanced for the nonlinear estimation.

In view of this, this paper discusses federated form of the cubature Kalman filter for multi-sensor information fusion algorithm. Simulation results show that proposed method can effectively improve the estimation accuracy.

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The rest of this paper is organized as follows. Section II presents the formulation of the problem. Federated cubature Kalman filter algorithm was derived in Section III. Computer simulation is illustrated to show the effectiveness of the novel algorithm in Section V. The fina1 section concludes this paper.

2. Problem Formulation

State equation and measurement models of the nonlinear are given by

$$\boldsymbol{x}(k) = \boldsymbol{f}\left(\boldsymbol{x}(k-1)\right) + \boldsymbol{w}(k-1) \tag{1}$$

$$\boldsymbol{y}_j(k) = \boldsymbol{h}_j(\boldsymbol{x}(k)) + \boldsymbol{v}_j(k) \quad j = 1,2,\cdots,N \tag{2}$$

where $\boldsymbol{x}(k)$ is the state vector; $\boldsymbol{f}(\cdot)$ is the nonlinear process model; $\boldsymbol{w}(k)$ is the process noise; $\boldsymbol{y}_j(k)$ is the measurement vector of the sensor *j*; $\boldsymbol{h}_j(\cdot)$ is the *j*th nonlinear measurement model and $\boldsymbol{v}_j(k)$ is the measurement noise.

Assumption 1 $\boldsymbol{w}(k)$ and $\boldsymbol{v}_i(k)$ are mutually uncorrelated Gauss white noises satisfying

$$\begin{cases} E\{\boldsymbol{w}(k)\} = 0, E\{\boldsymbol{v}_i(k)\} = 0 \\ E\{\boldsymbol{w}(k)\boldsymbol{w}^T(s)\} = \boldsymbol{Q}(k)\delta_{ks} \\ E\{\boldsymbol{v}_j(k)\boldsymbol{v}_j^T(s)\} = \boldsymbol{R}_i(k)\delta_{ks} \\ E\{\boldsymbol{v}_j(k)\boldsymbol{v}_k^T(s)\} = 0, \quad j \neq k \end{cases}$$

Assumption 2 The initial state $\boldsymbol{x}(0)$ is independent of $\boldsymbol{w}(k)$ and $\boldsymbol{v}_j(k)$, $j = 1,2,\cdots,N$, and

$$E\{\boldsymbol{x}(0)\} = \boldsymbol{x}_0, E\{[\boldsymbol{x}(0) - \boldsymbol{x}_0][\boldsymbol{x}(0) - \boldsymbol{x}_0]^T\} = \boldsymbol{P}_0$$

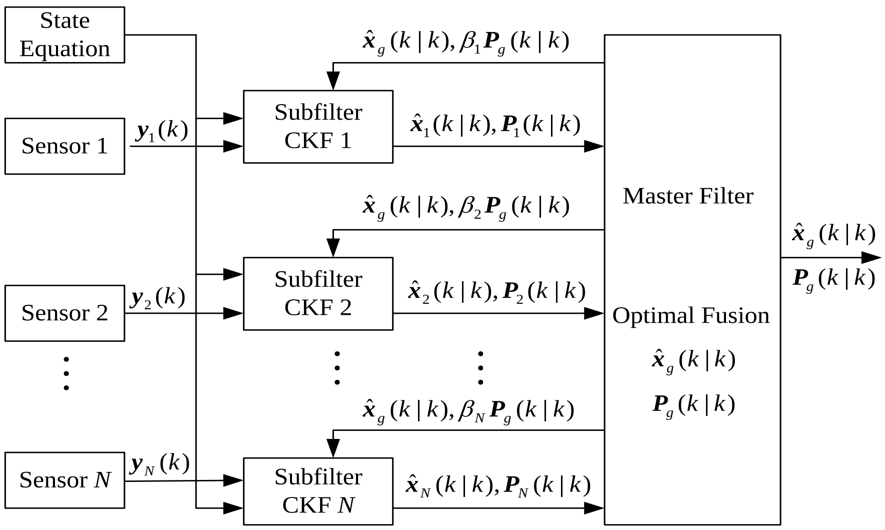

Figure 1. Federated Cubature Kalman Filter Structure (FCKF)

As shown in Figure 1, it wishes to jointly estimate $\boldsymbol{x}(k)$ based on the distributed measurements $\boldsymbol{y}_j(k)$, $j = 1,2,\cdots,N$. This can be accomplished as follows. First, each sensor makes a measurement $\boldsymbol{y}_j(k)$, and sub-filter compute the local estimate $\hat{\boldsymbol{x}}_j(k|k)$ based on CKF. Then the master filter will combine all local estimates to obtain the global estimate $\hat{\boldsymbol{x}}_g(k|k)$ according to the scalar weighted rule.

3. Federated Cubature Kalman Filter

In the Bayesian framework, the nonlinear filter reduces to computing multi-dimensional integrals when all conditional densities are assumed to be Gaussian, whose integrands are all of the form nonlinear function multiply the Gaussian. Similar to UKF, CKF obtains a set of cubature points and its weights according to spherical-radial integration rule.

3.1.Local Cubature Kalman Filter (Subfilter)

Assume the state estimate $\hat{\boldsymbol{x}}_j(k-1|k-1)$ and its error covariance matrix $\boldsymbol{P}_j(k-1|k-1)$ are available at time k. Under Assumption 1 and 2, the CKF can be executed as follows

3.1.1 Time Update

- Factorize

$$\boldsymbol{P}_j(k-1|k-1)=\boldsymbol{S}_j(k-1|k-1)\boldsymbol{S}_j^T(k-1|k-1) \tag{3}$$

- Calculate the cubature points($i=1,2,\cdots,m_x$)

$$\boldsymbol{x}_j^i(k-1|k-1)=\boldsymbol{S}_j(k-1|k-1)\boldsymbol{\xi}_i+\hat{\boldsymbol{x}}_j(k-1|k-1) \tag{4}$$

where, $m_x=2n_x$; And the parameter ξ_i ($\boldsymbol{\varepsilon}_i$ is the *p*-order unit vector) is given by

$$\xi_i=\begin{cases}\sqrt{m_x/2}\cdot\boldsymbol{\varepsilon}_i, & i=1,2,\cdots,n_x\\ -\sqrt{m_x/2}\cdot\boldsymbol{\varepsilon}_{i-p} & ,i=n_x+1,n_x+2,\cdots,2n_x\end{cases}$$

- Compute the propagated cubature points ($i=1,2,\cdots,m_x$)

$$\breve{\boldsymbol{x}}_j^i(k|k-1)=\boldsymbol{f}\left(\boldsymbol{x}_j^i(k-1|k-1)\right) \tag{5}$$

- Evaluate the predicted state and its error covariance

$$\begin{cases}\hat{\boldsymbol{x}}_j(k|k-1)=\dfrac{1}{m_x}\sum_{i=1}^{m_x}\breve{\boldsymbol{x}}_j^i(k|k-1)\\ \boldsymbol{P}_j(k|k-1)=\dfrac{1}{m_x}\sum_{i=1}^{m_x}\breve{\boldsymbol{x}}_j^i(k|k-1)[\breve{\boldsymbol{x}}_j^i(k|k-1)]^T\\ \qquad -\hat{\boldsymbol{x}}_j(k|k-1)\hat{\boldsymbol{x}}_j^T(k|k-1)+\boldsymbol{Q}_j(k)\end{cases} \tag{6}$$

2) Measurement Update

- Factorize

$$\boldsymbol{P}_j(k|k-1)=\boldsymbol{S}_j(k|k-1)\boldsymbol{S}_j^T(k|k-1) \tag{7}$$

- Calculate the cubature points($i=1,2,\cdots,m_x$)

$$\boldsymbol{x}_j^i(k|k-1)=\boldsymbol{S}_j(k|k-1)\boldsymbol{\xi}_i+\hat{\boldsymbol{x}}_j(k|k-1) \tag{8}$$

- Compute the propagated cubature points ($i=1,2,\cdots,m_x$)

$$\breve{\boldsymbol{y}}_j^i(k|k-1)=\boldsymbol{h}_j\left(\boldsymbol{x}_j^i(k|k-1)\right) \tag{9}$$

- Estimate the predicted measurement

$$\hat{\boldsymbol{y}}(k|k-1)=\frac{1}{m_x}\sum_{i=1}^{m_x}\breve{\boldsymbol{y}}_j^i(k|k-1) \tag{10}$$

- Compute the innovation covariance matrix

$$\begin{aligned}\boldsymbol{P}_{yy}^j(k|k-1)=&\frac{1}{m_x}\sum_{i=1}^{m_x}\breve{\boldsymbol{y}}_j^i(k|k-1)[\breve{\boldsymbol{y}}_j^i(k|k-1)]^T\\ &-\hat{\boldsymbol{y}}(k|k-1)\hat{\boldsymbol{y}}^T(k|k-1)+\boldsymbol{R}_j(k)\end{aligned} \tag{11}$$

- Evaluate the cross-covariance matrix

$$\begin{aligned}\mathbf{P}_{xy}^j(k|k-1)=&\frac{1}{m_x}\sum_{i=1}^{m_x}\mathbf{x}_j^i(k|k-1)\breve{\mathbf{y}}_j^i(k|k-1)\\ &-\hat{\mathbf{x}}_j(k|k-1)\hat{\mathbf{y}}_j^T(k|k-1)\end{aligned} \tag{12}$$

- Compute the gain matrix of the KF

$$\boldsymbol{K}_j(k)=\boldsymbol{P}_{xy}^j(k|k-1)[\boldsymbol{P}_{yy}^j(k|k-1)]^{-1} \tag{13}$$

- Estimate the updated weight and its error covariance

$$\begin{cases}\hat{\boldsymbol{x}}_j(k|k)=\hat{\boldsymbol{x}}_j(k|k-1)+\boldsymbol{K}_j(k)[\boldsymbol{y}_j(k)-\hat{\boldsymbol{y}}_j(k|k-1)]\\ \boldsymbol{P}_j(k|k)=\boldsymbol{P}_j(k|k-1)-\boldsymbol{K}_j(k)\boldsymbol{P}_{yy}^j(k|k-1)\boldsymbol{K}_j^T(k)\end{cases} \tag{14}$$

3.2.Global Fusion Algorithm (Master filter)

In order to efficiently recombine and fuse the subfilter outputs and obtain an enhanced estimation result, the scalar weighted optimal fusion method is used. The federated filter can be presented with the following steps:

Information distribution process ($j=1,2,\cdots N$)

$$\begin{cases} \boldsymbol{Q}_j(k) = \beta_j(k)\boldsymbol{Q}(k) \\ \boldsymbol{P}_j(k|k) = \beta_j(k)\boldsymbol{P}(k|k) \\ \hat{\boldsymbol{x}}_j(k|k) = \hat{\boldsymbol{x}}_g(k|k) \end{cases} \tag{15}$$

where $\beta_j(k)$ is the information distributing parameter at time k, and satisfies the principle as below

$$\sum_{j=1}^{N} \beta_j(k) = 1 \tag{16}$$

Those parameters play an important role in performance of the federated filter. In order to improve fusion accuracy, we adopt dynamic information-sharing algorithm based on matrix norm [6]. Denote

$$\beta_j(k) = \frac{\|\boldsymbol{P}_j(k|k)\|_F^{-1}}{\sum_{j=1}^{N} \|\boldsymbol{P}_j(k|k)\|_F^{-1}}, \quad j = 1,2,\cdots N \tag{17}$$

where, $\|\boldsymbol{P}_j(k|k)\|_F$ indicates Frobenius norm of the covariance matrix $\boldsymbol{P}_j(k|k)$.

- Information update

In this materiel, master filter is considered as a fusion estimator. Time update and measurement update are performed in each subfilter. Concrete steps are described in above subsection A.

- Information fusion

Master filter fuse the subfilter outputs and calculate the global estimation and error covariance matrix at time *k* according to the scalar weighted optimal fusion rule as

$$\begin{cases} \boldsymbol{P}_g(k|k) = [\sum_{j=1}^{N} \boldsymbol{P}_j^{-1}(k|k)]^{-1} \\ \hat{\boldsymbol{x}}_g(k|k) = \boldsymbol{P}_g(k|k) \sum_{j=1}^{N} \boldsymbol{P}_j^{-1}(k|k)\hat{\boldsymbol{x}}_j(k|k) \end{cases} \tag{18}$$

4. Computer Simulation

In this section, a simulation example of target tracking is illustrated to compare the estimate accuracies between FEKF (federated extended Kalman filter) and FCKF. The results from both algorithms are compared in terms of the root mean square error (RMSE) in position and velocity. Results are averaged over 50 Monte Carlo runs.

Consider a target with a constant-velocity motion, whose dynamic model is described in (1). The state is denoted by $\boldsymbol{x}(k) = [x(k)\ \dot{x}(k)\ y(k)\ \dot{y}(k)]^T$,where $x(k)$ and $y(k)$ are the position components of east and north, respectively. $\dot{x}(k)$ and $\dot{y}(k)$ are the corresponding velocity components respectively. We choose the parameters as follows:

$$\boldsymbol{f}(\boldsymbol{x}(k)) = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \boldsymbol{x}(k)$$

where sampling interval $T = 0.5\,s$. The parameters of the target is given by initial state: $[100 \quad 10 \quad 100 \quad 20]^T$. Here, the covariance of initial state and process noise are chosen

$$\boldsymbol{P}_0 = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\boldsymbol{Q}(k) = \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix} \times 0.25$$

The measurement system consists of two phased array radars that measure the range and the direction cosines of the target. The measurement matrix are given by

$$\boldsymbol{h}_j(\boldsymbol{x}(k)) = \begin{bmatrix} \sqrt{x^2(k)+y^2(k)} \\ \arccos\left(\dfrac{x(k)}{\sqrt{x^2(k)+y^2(k)}}\right) \end{bmatrix}, \quad j=1,2$$

The covariance matrices of measurement noises are chosen

$$\boldsymbol{R}_1(k) = \begin{bmatrix} 10m & 0 \\ 0 & 0.2° \end{bmatrix}, \ \boldsymbol{R}_2(k) = \begin{bmatrix} 12m & 0 \\ 0 & 0.5° \end{bmatrix}$$

The simulation results are given by Figure 2 to Figure 5, synchronously, the fusion accuracy improvement results of two algorithms are given by Table 1.

Table 1. TARGET TRACKING RESULTS

Algorithm	**Root Mean Square Error**			
	Position (East)	***Position(North)***	***Velocity(East)***	***Velocity(North)***
FEKF	47.0565	49.6056	2.5652	2.7485
FCKF	9.3414	13.3629	1.6588	1.6692
Improved Percentage	80.15%	73.06%	35.34%	39.27%

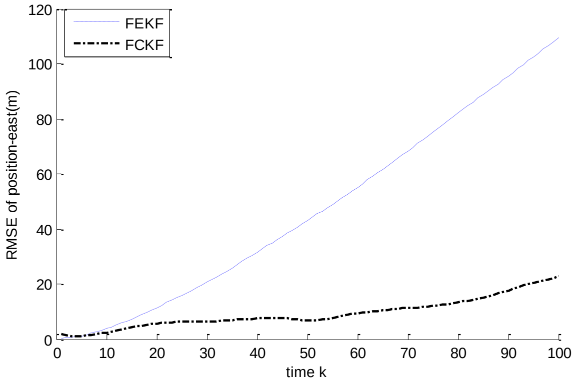

Figure 2. RMSE of position-east

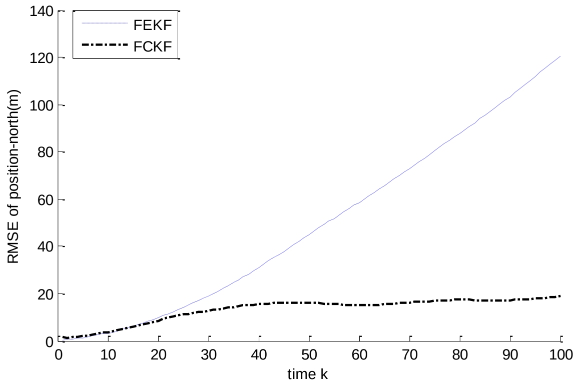

Figure 3. RMSE of position-north

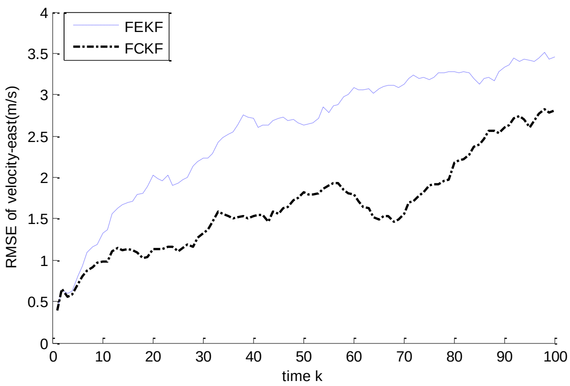

Figure 4. RMSE of velocity-east

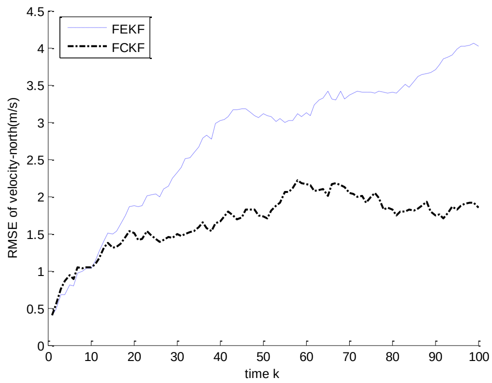

Figure 5. RMSE of velocity-north

From Table 1, Figure 2 to Figure 5, it is easy to see that the RMSE performance of FCKF is far lower FEKF. That means the fusion accuracy of FCKF is better than FEKF. Compared with the latter, the former increases by 80.15%, 73.06%, 35.34% and 39.27% for position-east, position-north, velocity-east and velocity-north, respectively. This is because that EKF only approximate the nonlinear system by rough first order linearization.

5. Conclusion

The complexity of the multi-sensor data fusion problem has given rise to a considerable interest in the development of suitable architectures for multi-sensor systems. In this material, we adopt federated filter architecture to design fusion algorithm and develop a novel method called federated cubature Kalman filter (FCKF). First of all, each subfilter computes the local estimate based on CKF. Secondly, the master filter combines all local estimates to generate the global estimate according to the scalar weighted rule. The simulation results validate the efficiency of the presented algorithm in light of RMSE in position and velocity.

Further work will emphasize solving the problem of correlations between local sensors or the measurements, which will be conductive to further perfection and development in the field of multi-sensor information fusion.

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