

Synthesis of a class of general Chebyshev filters with only magnitude

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Abstract. A simple design technique is introduced for general Chebyshev filters with only magnitude requirements. The efficient procedure is given for the determination of transmission zeros and filter order. This technique is based on constructing an objective error function according to given filter magnitude specifications, and then the filter order and transmission zeros (TZs) can be obtained by minimizing the proposed error function using genetic algorithm (GA). Once TZs and filter order are determined, coupling matrix of a given prototype can be obtained using established techniques. The filter with these TZs synthesized by this method has the characteristics of the highest stopband rejection and the least filter order under the premise of meeting given filter magnitude specifications. Application examples illustrate the validity of this method.

Keywords: general Chebyshev filters; transmission zeros (TZs); the least filter order; genetic algorithm (GA)

1. Introduction

Filters play important roles in modern wireless communications systems. Compared with conventional Butterworth and Chebyshev filters, General Chebyshev filters have arbitrarily placed finite-location transmission zeros (TZs), which will lead to higher stopband rejection, sharper selectivity, less filter order, smaller size, lighter weight, and lower cost.

Various techniques are available in the literature for synthesizing coupling matrix of general Chebyshev filters. Cameron proposed general coupling matrix synthesis methods based on analytical technique [1, 2]. Numerical optimization was used in synthesizing coupling matrix of resonator filters [3-6]. But the precondition of these methods mentioned above is that the filter order and TZs must be prescribed. These methods did not discuss how to determine the filter order and TZs to meet the given filter specifications although this is the foundation of the cross-coupled filter design. This question can be solved in [7-9]. TZs can be calculated by solving a system of nonlinear equations based on obtaining frequencies of magnitude characteristic extreme values in the stopband [7-9]. So, the methods in [7-9] requires complicated formula derivation and solving nonlinear equations and the more the number of TZs and filter order are, the more complicated solving nonlinear equations become. There are still few papers to discuss this question.

In this paper, we present a simple and effective method to obtain the filter order and TZs to meet the given filter specifications by minimizing an objective error function using GA. The objective error function for GA is proposed based on the given filter magnitude specifications. This method no longer requires solving nonlinear equations to determine TZs. Once the filter order and the locations of TZs are determined, coupling matrix of the filters can be obtained by using established techniques [1-6]. The next step requires the choice of proper resonator types, for example open-loop resonators [10] or stepped-impedance resonators (SIR) [11], to realize filter design. The validation of the method will be demonstrated by three examples,

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including two numerical synthesized filters and one fabricated filter. The first and second example demonstrates the TZs calculation of general Chebyshev filter, whose specifications are adopted from the corresponding literatures to provide a direct comparison. The last example shows a designed and realized filter.

2. Determination of Filter Order and Transimission Zeros

Genetic algorithm (GA) can be used to solve the minimum value of multivariate function; it is more powerful to search for the global minimum. An objective function is key point for successful optimization using GA.

We start from two-port lossless filter network in the frequency parameter Ω , where the transmission function $S_{21}(\Omega)$ and reflection function $S_{11}(\Omega)$ may be expressed as

$$|S_{21}(\Omega)|^2 = \frac{1}{1 + \varepsilon^2 C_N^2(\Omega)}, \quad |S_{11}(\Omega)|^2 = 1 - |S_{21}(\Omega)|^2. \quad (1)$$

Where Ω is the normalized frequency variable, ε is a ripple constant related to the passband return loss RL by $\varepsilon = [10^{RL/10} - 1]^{-1/2}$. The general Chebyshev function $C_N(\Omega)$ is given by

$$C_N(\Omega) = \cosh\left[\sum_{n=1}^N \cosh^{-1}(x_n)\right], \quad x_n = \frac{\Omega - 1/\Omega_n}{1 - \Omega/\Omega_n}. \quad (2)$$

Here, $s_n = j\Omega_n$ ($n=1, 2 \dots N$) is the location of the n th transmission zero in the complex s -plane. N is the filter order. Cameron has proved that m must satisfy $m \leq N$, where m is the number of TZs with finite locations, and those zeros without finite locations must be placed at infinity. Amari has given a rigorous proof for the maximum number of finite transmission zeros of cross-coupled filters with a given topology [12, 13]. Once the filter order and the finite TZs are determined in the Ω domain, $S_{21}(\Omega)$ and $S_{11}(\Omega)$ (or $S_{21}(f)$ and $S_{11}(f)$ by applying the frequency transformation) are determined. So, determining the filter order and TZs is the first problem of filter synthesis.

Usually, the information obtained from given filter specifications is as follows:

When $\Omega > \Omega_{s1}$ or $\Omega < \Omega_{s1}$ rad/s, the attenuation in the stopband $La \geq L_{s1}$ dB.

When $\Omega > \Omega_{s2}$ or $\Omega < \Omega_{s2}$ rad/s, the attenuation in the stopband $La \geq L_{s2}$ dB.

Where, Ω_{s1} and Ω_{s2} are normalized stopband edge frequency. L_{s1} and L_{s2} are the stopband attenuation values at Ω_{s1} and Ω_{s2} , respectively. So, two functions are constructed as follows:

$$\varphi_1 = \max\left(|S_{21}(\Omega)|^2\right), \quad \Omega > \Omega_{s1} \quad \text{or} \quad \Omega < \Omega_{s1} \quad (3)$$

$$\varphi_2 = \max\left(|S_{21}(\Omega)|^2\right), \quad \Omega < \Omega_{s2} \quad \text{or} \quad \Omega > \Omega_{s2}. \quad (4)$$

Where $\max(\cdot)$ means solving maximum value of the function $|S_{21}(\Omega)|^2$ within the range of Ω . The function φ_1 and φ_2 denote that filter has the highest stopband rejection, when φ_1 and φ_2 reach their minimum, respectively. So, the objective error function is proposed for two requirements in given filter specifications as follow:

$$\varphi = (\varphi_1 - 10^{-L_{s1}/10}) + C \cdot (\varphi_2 - 10^{-L_{s2}/10}). \quad (5)$$

Where C is the weighted constant determined by $C = 10^{(L_{s2} - L_{s1})/10}$. Note that C is key parameter that determines a correct solution. When given filter specifications have only one requirement on the upper or lower side of the stopband, C is equal to zero. In order to meet the given filter specifications, φ must satisfy $\varphi \leq 0$. The proposed error functions in (5) can satisfy most cases of filter synthesis. One can construct the error function similar to (5) for more than two requirements in given filter specifications. By increase the

filter order N and the number of the TZs m gradually, GA is used to search for Ω_n ($n=1, 2, \dots, m$) to minimize (5) until $\varphi < 0$, and then the least filter order and the optimum TZs can be obtained to meet filter specifications.

3. Application Examples

3.1. Filter 1

The given low-pass filter (filter1) specifications are:

The maximum attenuation in the passband is $Lp=0.2803$ dB.

When $\Omega \geq 1.07$ rad/s, the attenuation in the stopband $La \geq 50.6$ dB.

These specifications were taken from [9] to provide a direct comparison. We can get the least filter order $N = 6$ and four TZs at 1.075385, 1.131956, 1.329547 and 2.101861 by minimize error function (5) when $\Omega > 1.07$, and the value of the error function is -5.4976×10^{-6} . The calculated ripple ε is 0.19980. In [9], the filter order $N = 9$ and four TZs at 1.04983, 1.09143, 1.25024 and 1.96773 are given. The comparison of the filter response between the method in [9] and this method is shown in Fig. 1. Compared with the method in [9], this method requires less filter order to meet given specifications of filter 2, which validate the proposed method.

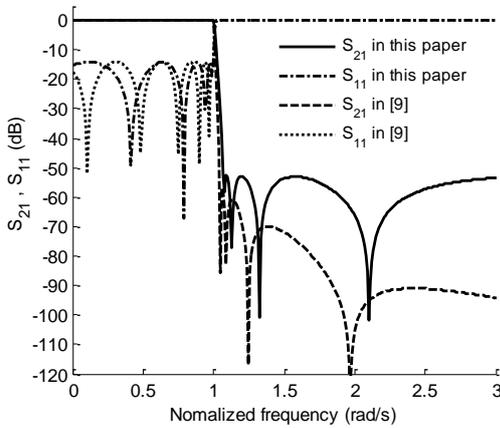


Figure 1. The comparison of S-parameters of filter 1 (synthesized), computed according to TZs obtained from this paper ($N=6$, $m=4$) and [9] ($N=9$, $m=4$), respectively.

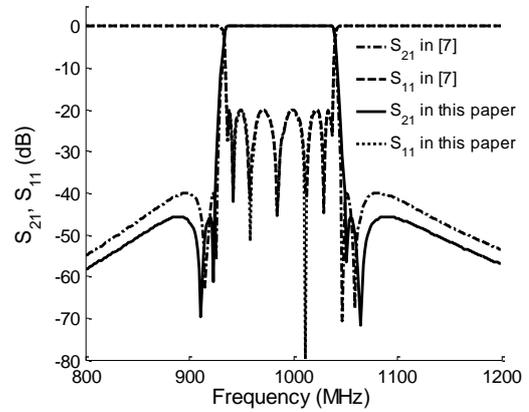


Figure 2. The comparison of S-parameters of filter 2 (synthesized), computed according to TZs obtained from this paper ($N=7$, $m=4$) and [7] ($N=7$, $m=4$), respectively.

3.2. Filter 2

The given bandpass filter (filter 2) specifications are [7]:

The center frequency is 985MHz.

The fractional bandwidth is $FBW=10.359\%$.

The return loss in passband is $RL=20$ dB.

40 dB rejection bandwidth is 125.5 MHz.

By applying the frequency transformation from a practical bandpass to a lowpass prototype, $\Omega_{s1} = 1.230$ rad/s ($L_{s1} = 40$ dB), $\Omega_{s2} = -1.230$ rad/s ($L_{s2} = 40$ dB) are obtained from given filter specifications. The required frequency specification is symmetric with respect to center frequency. In such cases, only half the number of variables (TZs) needs to be optimized. We can get the least filter order $N = 7$ and four TZs at ± 1.498865 and ± 1.251109 by minimize error function (5) when $\Omega > 1.23$ and $\Omega < -1.23$, and the value of the error function is -1.4536×10^{-4} . The calculated ripple ε is 0.1005. In [7], the filter order $N = 7$ and four TZs at ± 1.1892 and ± 1.4030 are given. By applying the frequency transformation from a lowpass prototype to a practical bandpass filter, the comparison of the filter response between the method in [7] and the proposed method is shown in Fig. 2. Compared with [7], the filter 2 with TZs obtained by this paper has higher stopband rejection, which validates the proposed method.

3.3. Filter 3

The given specifications of filter 3 are:

A passband from 2.10 GHz to 2.16 GHz.

The return loss in passband is $RL=20$ dB.

30 dB rejection bandwidth is 108 MHz.

We can get the least filter order $N = 4$ and two TZs at ± 1.9485 to meet the specifications of filter 3 by minimize error function (5) when $\Omega > 1.8$ and $\Omega < -1.8$. The coupling coefficient and external quality factor obtained by the method in [3] as

$$\begin{aligned} K_{12} &= K_{34} = 0.0243 \\ K_{23} &= 0.0216, K_{14} = -0.0051 \\ Q_{ei} &= Q_{eo} = 34.0861. \end{aligned} \quad (6)$$

After the coupling coefficients are determined, the next step requires the choice of proper resonator types to complete the filter design. In this design, SIR [11] was chosen. A full-wave simulator IE3D has been used to extract the parameters in (6). The filter is designed to be fabricated on a Rogers RT/duroid 5880 substrate with a relative dielectric constant of 2.2, a thickness of 0.508 mm, and a loss tangent of 0.0009. Photograph and physical dimensions of the filter 3 are shown in Fig. 3.

The measured and simulated results of the filter are shown in Fig. 4. The simulated response includes the loss factors. Measured response shows a reasonably good agreement with the simulated response.

This example illustrates the general design procedure for general Chebyshev bandpass filters whose TZs are obtained by the proposed method.

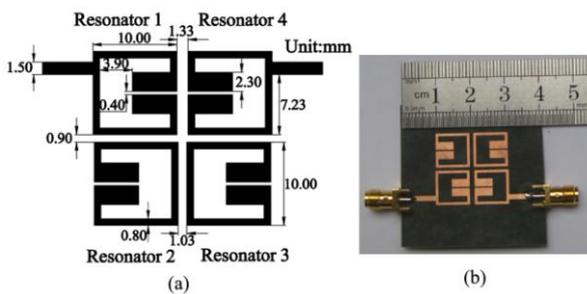


Figure 3. (a) Physical dimensions and (b) photograph of the fabricated filter 3.

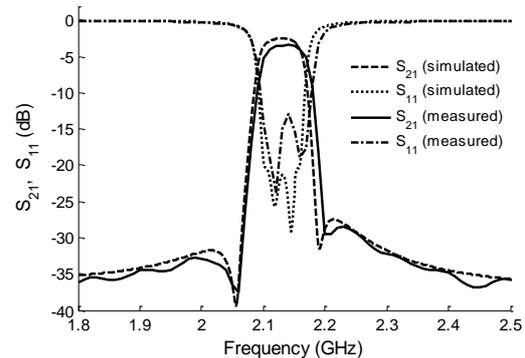


Figure 4. Measured and simulated response of filter 3.

4. Conclusion

A very efficient and simple procedure has been presented for the design of the class of general Chebyshev filters with only magnitude requirements. By directly optimizing the proposed objective error function using GA, the filter order and the locations of TZs can be obtained. Compared with [7-9], the proposed method no longer requires solving complicated nonlinear equations, which simplifies the process of determining the filter order and TZs. The filter synthesized by this method has the least filter order and highest stopband rejection. Three examples have been given to illustrate the validity of this method.

5. References

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