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Accurate Representation of Switching in Multi-scale Simulation of Power Systems and Circuits

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Abstract. A new approach for accurate representation of switching in multi-scale simulation of power systems and circuits is proposed. The switching instant is pinpointed by the linear interpolation on analytic signals. Following the switching, flexible changes of the time-step sizes are performed. These changes are achieved through adjustment of the shift frequency as well as the characteristics of numerical integration. The proposed approach is validated through studies of the transients recovery voltage in an electrical network.

Keywords: algorithms, discontinuities, power system simulation, electromagnetic transients, electromechanical transients.

1. Introduction

Multi-scale simulation of electromagnetic and electromechanical transients can be achieved through the shifting of Fourier spectra of analytic signals that describe the transients in power systems and circuits. This leads to a frequency-adaptive simulation of transients (FAST) [1]. Depending on the types of transients to be studied, time-step sizes in a wide range can be chosen. For simulating the switching event which is a representative of electromagnetic transients, a time step in the range of microseconds is required.

In [2], the method flexible integration for readjustment in simulation of transients (FIRST) allows the use of a flexible time-step size in the electromagnetic transients simulation. Through this method, efficient representation of switching events in real time electromagnetic transients simulation is obtained. In the context of multi-scale simulation, electromechanical transients, however, may appear prior to the switching instant, and a time-step in the range of milliseconds is used. Hence, a precise and efficient representation of switching is desirable.

In Section 2 of this paper, techniques related to the multi-scale simulation are reviewed. In Section 3, a basic approach for simulating switching event is presented, and it is followed by an efficient approach based on adjustment of a shift frequency as well as flexible modulation of characteristics of numerical integration. A test case is performed in Section 4. Conclusions are drawn in Section 5.

2. Multi-scale Simulation

From a naturally generated real signal $s(t)$, the corresponding analytic signal is obtained by adding an imaginary part using the Hilbert transform, i.e. $\underline{x}(t) = x(t) + \mathrm{jH}\left[x(t)\right]$. The analytic signal is complex and marked by an underscore. If the original real signal $s(t)$ is with bandpass characteristic about carrier frequency f_c as the ac voltage and current signals behave during electromechanical transients, then the Fourier spectrum of $s(t)$ extends to negative frequencies. Nevertheless, the Fourier spectrum of the corresponding analytic signal $\underline{s}(t)$ contains no negative frequencies as depicted on the middle of Fig.1. As a result, the latter can be shifted by the shift frequency f_s:

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$$\varphi\left[\underline{s}(t)\right]=\underline{s}(t)\mathrm{e}^{-\mathrm{j}2\pi f_s t}\,. \tag{1}$$

In terms of the angular frequency, $\omega_s = 2\pi f_s$. Since $\left|\mathrm{e}^{-\mathrm{j}\omega_s t}\right| = 1$, the magnitude is not changed through the shift operation, i.e. $|\varphi[\underline{s}(t)]| = |\underline{s}(t)|$.

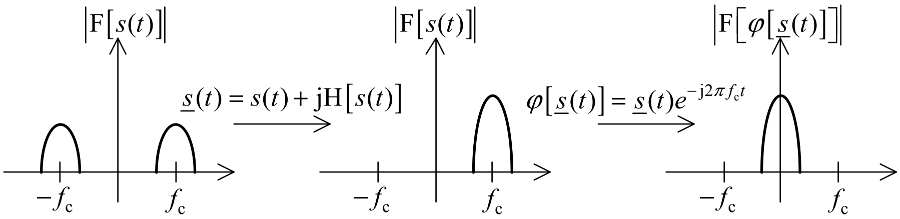


Fig. 1: Application of Hilbert transform and frequency shift.

The shift frequency is adjusted according to the types of transients to be simulated [1] [3]. During electromagnetic transients, the shift frequency is set to zero and real part of the analytic signal tracks the natural waveform. During electromechanical transients, the shift frequency is set equal to the carrier frequency, i.e. $f_s = f_c$, and then the complex envelope is obtained. As the spectrum $\mathrm{F}[\varphi[\underline{s}(t)]]$ on the right of Fig.1 shows that the complex envelope is a lowpass signal whose maximum frequency is lower than the one of the original real bandpass signal. In accordance with Shannon's sampling theorem, a lower sampling rate can be chosen when tracking the complex envelope rather than the original bandpass signal.

3. Representation of Switching in Multi-scale Simulation

Depending on the types of transients to be studied, the multi-scale simulation uses the time steps with different sizes. In the case when the electromechanical transients are of interest, then the simulation employs the time-step size of τ_e in the range of milliseconds. If the electromagnetic transients appear, then the time-step size of τ_d in the range of microseconds is chosen.

It is possible that results obtained with the multi-scale simulation are output in synchronism with the real time clock. A specification of time-step sizes in multi-scale simulation is illustrated by means of Fig.2. The variables $t_{re}(k)$ and $t_{si}(k)$ give the real time and simulation time after the k th time step, respectively. In real time simulation, exchange of information is performed at the equidistantly spaced points separated by the time step τ. For this purpose, the large time-step τ_e is chosen as an integer multiple of the time step τ. During the simulation time steps $t_{si}(k-1) < t < t_{si}(k)$, no solution output of the simulation is provided so that the data exchange is idle. This implies that there is enough time to perform all necessary computations. The small time-step τ_d is chosen for electromagnetic transients simulation, which equals to the time step τ.

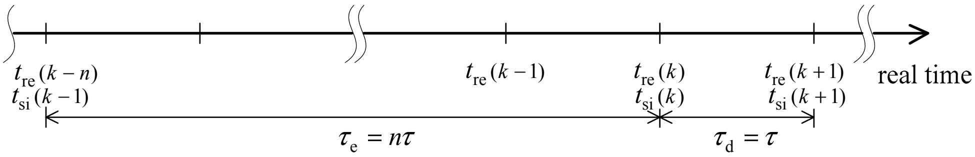


Fig. 2: A specification of time-step sizes for real time and simulation time.

3.1. A basic approach

For the purpose of comparison considered hereafter, a basic approach for the representation of switching is explained in Fig. 3. For the sake of clarity, it is assumed that $n = 4$, i.e. τ_e is four times the size of τ_d. To start the analysis, the electromechanical transients are simulated in which τ_e is employed and the shift frequency is set equal to the carrier frequency f_c. At time point $t_{si}(k-1)$ the switch current is positive whereas at time point $t_{si}(k)$ a negative switch current is obtained. Due to the change of sign of the switch current with respect to the solution obtained at $t_{si}(k-1)$, a change of switch status is required in time step k. Such a change is recognized as a discontinuity. Since discontinuities can trigger transients with frequencies much in excess of the carrier frequency, and the electromagnetic transients do appear. The shift frequency is set to zero, and a sufficiently small time-step is used to simulate the fast transients accurately. As a consequence, the network model needs to be modified to consider the updating of the simulation parameters.

However, this action is only accounted for in the solution of the following time steps. This implies that in time step k, the solution is not correct by neglecting the effects of the network changes. Furthermore, right after zero crossing of the switch current, detailed representation is required to perform the fast transients.

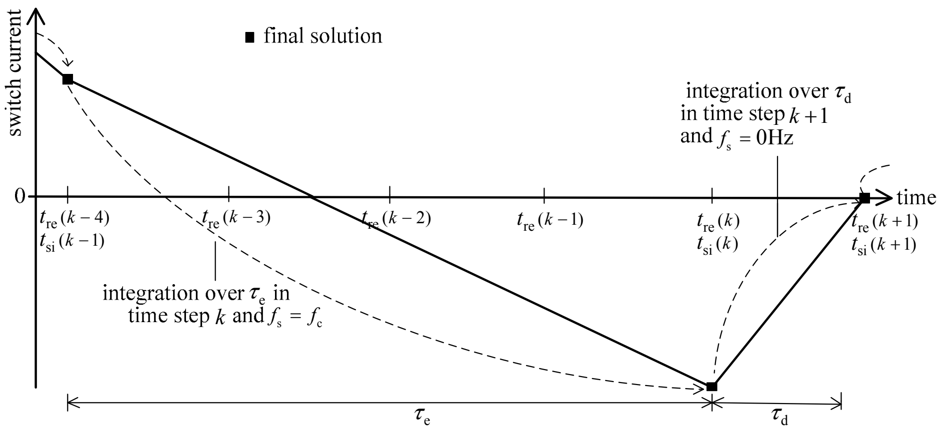


Fig. 3: Switching-off event simulated with the basic approach.

3.2. Flexible numerical integration

In multi-scale simulation, network branches are represented as frequency-adaptive companion models [1] [3]. This is illustrated through the modelling of an inductance. Using analytic signals, the behaviour of the inductor is described through the differential $\mathrm{d}\underline{i}_L(t) = \underline{v}_L(t)/L$. Applying frequency shifting to $\underline{i}_L$ in accordance with (1) yields:

$$\frac{\mathrm{d}\varphi[\underline{i}_L(t)]}{\mathrm{d}t} = \mathrm{e}^{-\mathrm{j}\omega_s t}\left(-\mathrm{j}\omega_s \underline{i}_L(t) + \frac{\underline{v}_L(t)}{L}\right) \quad . \tag{2}$$

where the differential quotient applies to the shifted signal. The weight-averaged integration method [4] is used to discretize the differential equation in (2), and a final result is given as:

$$\underline{i}_L(k) = \underline{G}_L \underline{v}_L(k) + \underline{\eta}_L(k)\,, \tag{3}$$

with

$$\underline{G}_L = \frac{\tau(2-w)}{L\left(2+\mathrm{j}\omega_s\tau(2-w)\right)}, \tag{4}$$

$$\underline{\eta}_L(k) = \mathrm{e}^{\mathrm{j}\omega_s\tau}\left(\frac{2-\mathrm{j}\omega_s\tau w}{2+\mathrm{j}\omega_s\tau(2-w)}\underline{i}_L(k-1) + \frac{\tau w}{L\left(2+\mathrm{j}\omega_s\tau(2-w)\right)}\underline{v}_L(k-1)\right), \tag{5}$$

where w is the weighting factor, and can be adjusted over $0 \le w \le 1$. For $w=1$, the differential equation is discretized by the trapezoidal method while for $w=0$, it is discretized by the backward-Euler rule.

In order to make the flexible time-step size available in multi-scale transients simulation, the variable x is introduced to the frequency-adaptive companion models. τ in (4) and (5) is substituted through $\tau(1+x)$:

$$\underline{G}_L = \frac{\tau(1+x)(2-w)}{L\left(2+\mathrm{j}\omega_s\tau(1+x)(2-w)\right)}, \tag{6}$$

$$\underline{\eta}_L(k) = \mathrm{e}^{\mathrm{j}\omega_s\tau(1+x)}\left(\frac{2-\mathrm{j}\omega_s\tau(1+x)w}{2+\mathrm{j}\omega_s\tau(1+x)(2-w)}\underline{i}_L(k-1) + \frac{w}{2-w}\underline{G}_L\underline{v}_L(k-1)\right) \quad . \tag{7}$$

It can be recognized that just changing x leads to a change of $\underline{G}_L$. It is given that the term $(1+x)(2-w)$ is constant. When the time-step size is changed, the value of the weighting factor is adjusted accordingly. Thus, $\underline{G}_L$ remains constant. In order to maintain $\underline{G}_L$ at the value obtained with the backward-Euler rule, the condition $(1+x)(2-w)=2$ is set. Hence,

$$\underline{G}_L = \frac{\tau}{L\left(1+\mathrm{j}\omega_s\tau\right)} \quad , \tag{8}$$

$$\underline{\eta}_L(k) = \mathrm{e}^{\mathrm{j}\omega_s\tau(1+x)}\left(\frac{1-\mathrm{j}\omega_s\tau x}{1+\mathrm{j}\omega_s\tau}\underline{i}_L(k-1) + x\underline{G}_L\underline{v}_L(k-1)\right) \quad . \tag{9}$$

3.3. Efficient representation of switching in multi-scale simulation

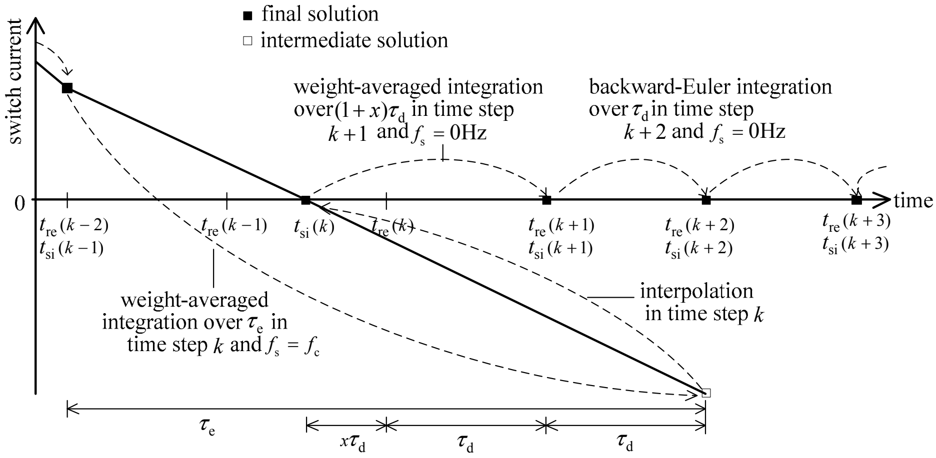


Fig. 4: Efficient representation of switching in multi-scale simulation.

When a switching event is detected, then the sequence of actions to be performed for accurate simulation of switching is shown in Fig.4. It is assumed that τ_e is four times the value of τ_d. At time point $t_{si}(k-1)$, the switch current is positive. In the following time step k, the large time-step simulation continues. The solution at $t_{si}(k-1)+\tau_e$ is obtained through the weight-averaged integration method. In this step, the zero-crossing of the switch current is detected, and the request for change of status of the switching is activated. Then, a backward linear interpolation to the instant of zero-crossing of the switch current is performed. Herein, the nodal voltages, the state variables, the simulator time variable, and the output variables are all interpolated.

Due to the switching event which is a representative of fast transients, τ_d is used and the shift frequency is set equal to zero. Starting from the switching instant, the simulation time is incremented by the time-step $(1+x)\tau_d$. The parameter x is calculated as:

$$x = \text{mod}\left(\frac{t_{si}(k-1)+\tau_e - t_{si}(k)}{\tau_d}\right) \quad , \tag{10}$$

where mod is the modus after the division.

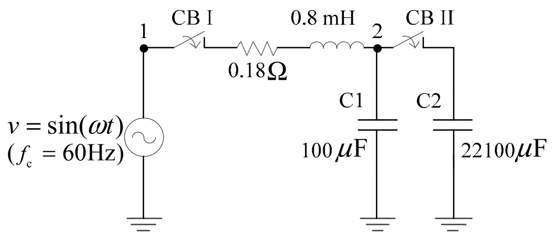
In a special case of $x=0$, w equals to zero due to the condition $(1+x)(2-w)=2$, and the backward-Euler method is used in time step $k+1$. The weighting factor w increases as x is increased. Therefore, higher the value of x, the more the weighting is shifted toward the trapezoidal method. The flexible time-step $(1+x)\tau_d$ results in that the simulation time coincides with the real time. In this step, in addition to the change of the shift frequency f_s, no further changes of other parameters affect the change of the nodal admittance matrix. In the following time steps, if no further switching events are detected, then x is equal to zero in order to retain the nodal admittance matrix unchanged, and the corresponding backward-Euler method is used.

4. Test Case

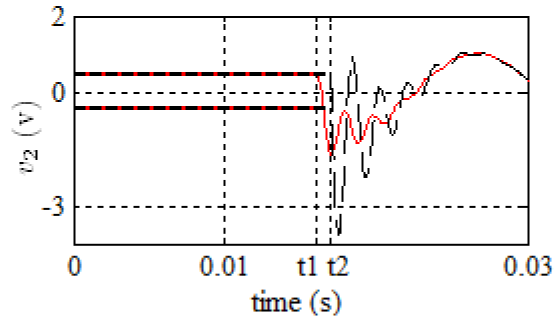
In order to test the accuracy of simulating the switching event in multi-scale simulation, a simple example is performed. The electrical network is shown in Fig. 5 (a) where the parameters can be found in [5]. Both circuit breakers (CB) are closed to charge the capacitors. CB II opens when the capacitor C2 is charged up and when the current through CB II is more or less zero.

The circuit in Fig.5 is simulated using the basic approach in Section 3.1 and the proposed approach in Section 3.3. The results on the voltage at node 2 are depicted in Fig.5 (b). During the first half of the simulation up to t1 (the proposed approach) or t2 (the basic approach), envelope waveforms are tracked in which electromechanical transients and steady state are simulated. A time-step size of 1ms is used, and the shift frequency is set equal to the carrier frequency at f_c =60 Hz. Then, a switching event occurs. The transients recovery voltage (TRV) caused by the switching is of more interest. In the second half, small time-step is used and the shift frequency is set zero. The TRV obtained with the proposed approach would no longer have unrealistic jump as that obtained with the basic approach. This is a consequence of the different instants of the switching using the two approaches. The differences are explained in Fig.5 (c) and (d).

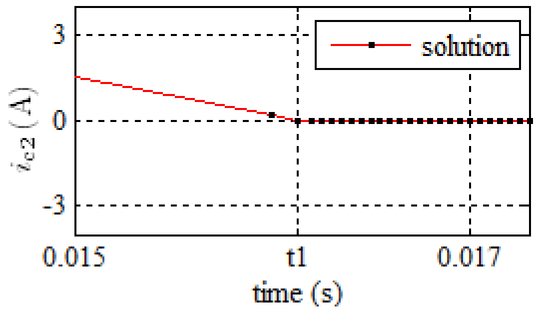
In the proposed approach, the actions involving the switching are dealt with right after the occurrence of zero-crossing of the CB II current. For example, the interpolation to the zero-crossing instant is performed, and the use of flexible time-step sizes is valid. Throughout the simulation in Fig.5 (c), the used time-step sizes are 1ms, 125.8 μs, 74.2 μs and 50 μs. However, using the basic approach the process of the switching is postponed up to t2. The solution at t2 of Fig.5 (d) is incorrect, and thus leads to the erroneous simulation. Consequently, the proposed method in the multi-scale simulation presents the accurate solution.



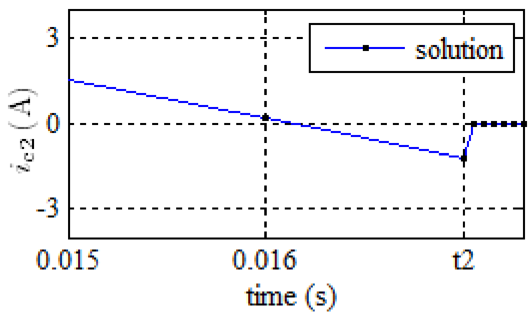
(a) Network



(b) Voltage at node 2; solid bold: envelope; solid light: natural waveform (solid: the proposed approach; dash dot: the basic method)



(c) Switching current in proposed approach



(d) Switching current with the basic approach

Fig. 5: Test network and corresponding results.

5. Conclusions

In the multi-scale simulation of electromagnetic and electromechanical transients, an approach for accurate representation of switching was proposed. In the context of electromechanical transients, if a switching event is detected, then it is accounted for at the instant at which it actually appears. This is realized through linear interpolation on the shifted analytic signals. Following the switching instant, efficient changes of the time-step sizes enable the accurate simulation. This is achieved through adjustment of the shift frequency as well as the characteristics of the numerical integration. The theoretical considerations were substantiated through the simulations in which a comparison is carried out.

6. References

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