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Application of Bispectral Analysis in the Nonlinear Systems

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Abstract. Traditional signal processing techniques, based on the first and second order statistics, is suitable for the signal which is coming from the gaussian and a minimum phase system, but for non-Gaussian and nonlinear processes, it has lost phase information. In this article, the information of nonlinear system which had generated non-Gaussian signal was expressed by bispectrum, and the second order phase-coupling existed in nonlinear system was quantitative estimated by bicoherence coefficient. The bispectrum structures of the typical non-Gaussian signals(EEG signals) were analyzed, effective bicoherence coefficient features were extracted, and the classification accuracy achieved significant improvement.

Keywords: bicoherence coefficient, bispectrum, EEG, feature extraction, nonlinearsystem

1. Introduction

Over the years, several digital signal processing techniques, based on the first and second order statistics (for, e.g., Power spectrum and autocorrelation), have been developed for signal analysis. The information provided by these methods is sufficient if the signal under study is coming from the gaussian and a minimum phase system [1]. However, many signals is a highly non-gaussian, on-linear process, to characterize the internal regularity of a nonlinear system need many integrated information such as amplitude, frequency, phase and so on.

In addition the original frequency component, a new frequency component is generated in the output of the system because of the role of the nonlinear phase coupling, when a non-Gaussian signal passes a nonlinear system[2]. The nonlinear phase coupling is a nonlinear phenomenon, which reflects the change of the energy distribution due to the nonlinear coupling. Generally, traditional power spectrum estimation method, based on the first and second order statisticsis , is to estimate the frequency and power of a signal, while power spectrum is phase blind, therefore it does not recognize the relationship of nonlinear phase coupling.

Higher Order Spectral Analysis （HOS） is a powerful tool of handling non-linear, non-Gaussian signals, it characterizes random signals from higher order probability structure, which have make up the defects of the second-order statistics, so not only it can reflect the signal energy, but also retain the phase information of the signal. Furthermore, HOS can suppress Gaussian noise completely in theory, therefore, analyzing non-Gaussian signal with HOS can extract more effective and useful information.

Bispectrum, is also known as third-order spectrum, has been shown to have the ability to detect second order nonlinear phase coupling information of the nonlinear system [2]. In this paper, bispectrum analysis of typical non-Gaussian signal generated by nonlinear system is carried, and the diagonal slice and the slices parallel to the diagonal slice of the bicoherence coefficient were seen as the feature extraction objects, so the feature vector were constructed, finally a significant classification results was achieved, where the support vector machine classifier was adopted.

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2. Method

EEG signals essentially is a typical non-Gaussian signal, and the underlying system generating the EEG signal is a typical nonlinear system [2]. Bispectrum analysis of EEG signal in different brain function states was carried in this paper, the experimental data is taken from a data set in BCI competition 2003 which is provided by university of technology Graz, Austria[3]. In the experiments, the subject imagined left or right hand movements following a random prompt, and the corresponding motor imagery EEG data were recorded in the scalp electrode position (C3, C4 and Cz). The experiment consists of 7 runs with 40 trials each. Given are 280 trials of 9s length. Each typical data record process is as follows: The first 2s was quite, at t=2s an acoustic stimulus indicates the beginning of the trial, and a cross "+" was displayed for 1s, then at t=3s, an arrow (left or right) was displayed as cue, and the subject began to imagine left or right hand movements for 6s. Because the Cz channel has nothing to do with the left or right hand imagination, only EEG data of the C3 and C4 channels were used to analyze in this paper.

2.1. HOS-based analysis of EEG

Let $x^i(k)$, $i = 1,2,3,\cdots,N$, denote the k-th sample of the i-th segment of digitized EEG data where N is the total number of segments in a recording. In this article, we model $x^i(k)$ as:

$$x^i(k) = h^i(k) * e^i(k) + w^i(k) \tag{1}$$

where $e^i(k)$ is a white non-Gaussian process, and $h^i(k)$ is a stable, possibly nonlinear kernel representing the underlying system generating the EEG segment $x^i(k)$ .The term $w^i(k)$ represents measurement noise within the frequency band of interest, which is traditionally modeled as a white Gaussian process.

2.2. Filtering of EEG segments and zero mean processing

EEG is a low-frequency signal, thus we first filtered $x^i(k)$ using a digital band-pass filter and contained the frequency band of interest to us within 0.5~30Hz to remove out-of-band noise, including the ubiquitous power line interference at 50Hz. Then, $x^i(k)$ was zero mean processed and seen as a third-order stationary process. Let the filtered and zero mean processed segments $x^i(k)$ be denoted by $y^i(k)$.

2.3. The bispectrum estimation

If higher order cumulants $c_{kx}(\tau_1,\cdots\tau_{k-1})$ is absolutely summable:

$$\sum_{\tau_1=-\infty}^{\infty}\cdots\sum_{\tau_{k-1}=-\infty}^{\infty}\left|c_{kx}(\tau_1,\cdots\tau_{k-1})\right| < \infty \tag{2}$$

The k-order cumulants spectrum can be defined as $(k-1)$-D Fourier transform of k-order cumulants:

$$s_{kx}(\omega_1,\cdots\omega_{k-1}) = \sum_{\tau_1=-\infty}^{\infty}\cdots\sum_{\tau_{k-1}=-\infty}^{\infty} c_{kx}(\tau_1,\cdots\tau_{k-1})\exp[-j\sum_{i=1}^{k-1}\omega_i\tau_i] \tag{3}$$

The third-order spectrum (also known as the bispectrum) can be expressed as:

$$B_x(\omega_1,\omega_2) = \sum_{\tau_1=-\infty}^{\infty}\sum_{\tau_2=-\infty}^{\infty} c_{3x}(\tau_1,\tau_2)\mathrm{e}^{-j(\omega_1\tau_1+\omega_2\tau_2)} \tag{4}$$

(4) reveals the physical meaning of the bispectrum, that is, bispectrum can reflect the distribution of signal third-order moment in dual-frequency plane .It can be seen that bispectrum have both amplitude information and phase information, so it can be used to describe quadratic phase coupling of nonlinear system. In this article, the bispectrum of EEG signal can be estimated via estimating the 3rd order cumulant of EEG segments $y^i(k)$, and then taking a 2D-Fourier transform (4).

2.4. Effective bicoherence coefficient features

In practical application, the result is unsatisfactory when the estimated bispectrum is used as a feature directly, this is because the estimated variance depends not only on the length of the data sample, but also is closely related to the power spectrum of signal in ω_1, ω_2, $\omega_1+\omega_2$. In order to eliminate the influence, the modes of bispectrum are often be normalized by power spectrums in ω_1, ω_2, and $\omega_1+\omega_2$, and the normalized bispectrum (bicoherence coefficient) is obtained. Bicoherence coefficient can describe quantitatively the coupling degree of quadratic phase coupling, and its expression is as follows:

$$b^2(\omega_1,\omega_2)=\frac{|B(\omega_1,\omega_2)|^2}{P(\omega_1)P(\omega_2)P(\omega_1+\omega_2)} \tag{5}$$

By the Cauchy inequality, $0 \le b^2(\omega_1,\omega_2) \le 1$, when $b^2(\omega_1,\omega_2)$ is closer to 1, the degree of coupling is stronger, on the contrary, the degree of coupling is weaker.

Bicoherence coefficient is still a two-dimensional function, two-dimensional template matching will be caused when bicoherence coefficient is used directly as signal feature, and the computation amount is considerable, which can't meet the requirement of real-time calssification. One solution to overcome this problem is to extract 1-dimension slice of the bispectrum. In [4], we considered the problem of signal reconstruction from the bispectrum, and proved that any 1-dimensional slice of the bispectrum carries sufficient information to estimate a system response within a timeshift, as long as the chosen slice is not parallel to any one of the frequency axes or to the diagonal at 135°. In this article, we rely on that result to reduce the computational complexity of the HOS techniques. The flexibility offered by the choosing arbitrarily oriented and shifted oblique slices also give us the advantage of avoiding unfavorable regions in the bispectrum.

3. Results

3.1. The bispectrum graphics analysis

The data set provided by Graz University of Science and Technology contains a total of 280 experiments, 140 for the training sample set, and another 140 for the test sample set. Each experiment with 9 seconds each, and the EEG was sampled with 128Hz, 1152 data points for each EEG segment. First, the bispectrums of zero mean processed EEG segments were caculated according to (4), and the bispectrum graphics of imagining left hand movement and right hand movement were drawn, which actually is 3-D graphics of two-dimensional array. Fig.1 and Fig.2 show the bispectrum 3-D graphics and contour graphics of imagining left hand movement and right hand movement respectively. EEG data in Fig.1 is from channel C3, epoch #39—imagining left hand movement, and EEG data in Fig.2 is from channel C3, epoch #53—imagining right hand movement, both in the training sample set. The bispectrum characteristic reflected by Fig.1 and Fig.2 was seen in amost all the EEG data records.

From these two graphs, the following conclusions can be obtained:

(a) In the 3-D graphics of bispectrum, whether the imagination of left handmovement or the imagination of right hand movement, magnitude of the bispectrum is not zero and there are apparent peak. This shows that the Gaussian deviation of the EEG signal is obviously, so we can see the EEG signal is a typical non-Gaussian signal.

(b) EEG signal have different bispectrum structures in different brain function status, both shape and energy of the bispectrum peak have some certain differences. Therefore, it can be further considered to use the bispectrum in mental tasks classification.

(c) For a nonlinear systems, there is a strong correlation between some two frequency components, that is, quadratic phase coupling generates between two frequency components and bispectrum magnitude. This can be seen from the bispectrum contour graphics, there are several bispectrum peaks at every two frequencies, which reveals the location of the quadratic phase coupling. This further illustrates that analzing nonlinear systems with bispectrum may receive more higher order information.

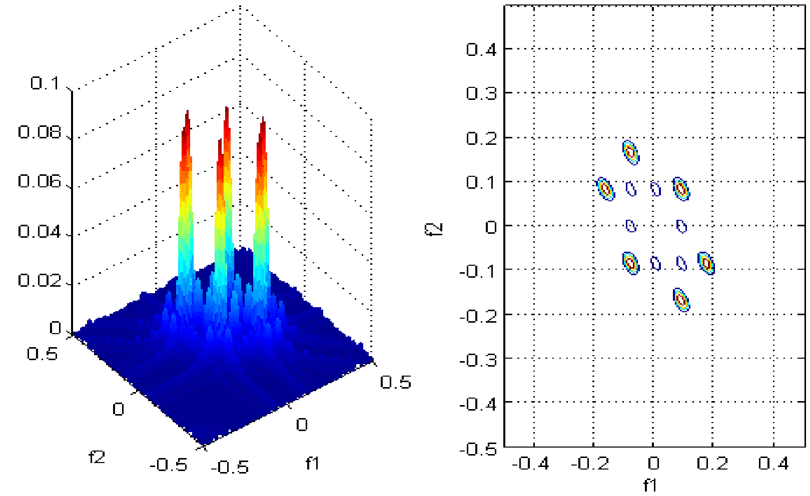

Fig. 1: The 3-D graphics and contour graphics of bispectrum imagining left hand movement.

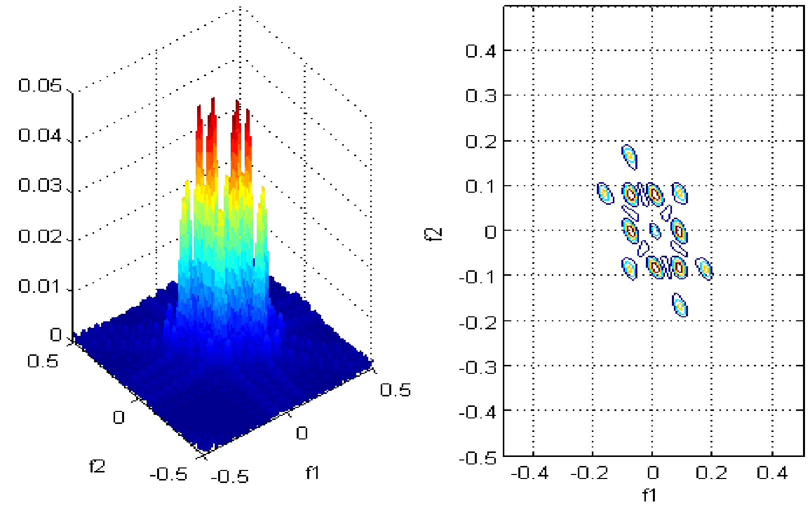

Fig. 2: The 3-D graphics and contour graphics of bispectrum imagining right hand movement.

3.2. Effective bicoherence coefficient features extraction

The common method of bispectrum feature extraction is to form the feature vectors basing on bispectrum diagonal slice (such as mean square deviation , maximum of the bispectrum amplitude, sum, etc[5, 6].). However, in the process of experimental simulation, we found that those slices parallelling to the diagonal slice also contain useful information of the dual-frequency domain.Furthermore, when feature vector was formed with each slice, a relatively higher average classification rate was obtained via summation method, it is because the summation method is equivalent to the energy accumulation, which can effectively reduce the classification rate influence causing by signal disturbance. In summary, the final feature extraction scheme is, for the bicoherence coefficient of every EEG segment $y^i(k)$, let its diagonal slice and other slices parallelling to the diagonal slice as the feature extraction objects, and then sum processing was carried on each slice, so the feature vector was formed. Where, FFT points is 512 points, in addition, taking into account the symmetry of the bispectrum, we extracted features only in the triangular area of $\omega 1 \geq \omega 2 \geq 0$ and $\omega 1 + \omega 2 \leq \pi$ which had included all information of bispectrum.

3.3. Indentification

Support vector machine classifier was used to classify in this article. First, the training feature vector was obtained from the effective bicoherence coefficient of the 140 training sample set. In training process, optimal classifier model will be obtained with training samples. Here, chosen kernel function was redial basis kernel function, and kernel parameters (for redial basis kernel function, there are two parameters: width of kernel function arg and error penalty parameter C) were chosen by cross validation, the final optimal parameters of model are arg=5.4, C=260. Then, the test feature vector was obtained from the effective bicoherence coefficient of the 140 test sample set. Finally, according to the test sample category provided by the competition, the average recognition rate reached 91.36%. Compared to the results of conventional power spectrum feature (such as 6-order AR model power spectrum estimation 86.14%) and the best result of BCI competition 2003 (89.29%), the classification accuracy using effective bicoherence coefficient as feature achieved significantly improvement.

4. Summary

The information of a nonlinear system which had generated non-Gaussian signal was expressed by bispectrum in this article, and the non-Gaussian and nonlinear characteristics of EEG signal was analyzed via bispectrum technology, which revealed some valuable higer order information implicited in the typical non-Gaussian signal (EEG signal), and realized the nonlinear feature extraction under different brain function status. This shows that analyzing nonlinear system with bispectrum technology can effectively suppress the influence of Gaussian noise and reveal nonlinear structural characteristics implicited in the non-Gaussian signal. Therefore, higher order spectral analysis methods can provide more effective information and approach for non-Gaussian signal analysis.

5. References

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