

Complex Network Model Based on Node Attraction with Tunable Parameters

SUN Rui ⁺, LUO Wanbo, MU Ali, LI Lin and ZHONG Mi

School of Computer Science, Sichuan University, Chengdu, China

Abstract. According to the evolution rule of real-world networks and aiming at the problems in BA model and initial attractive model, a complex network model based on node attraction with tunable parameters is proposed. The concept of node attraction is introduced based on BA model, and common affect preferential attachment existed in networks with degree. Theory research and simulation analysis show the fact that we can more flexible adjust the evolution process of network by adjusting model parameters, and the degree distribution follows power-law distribution, while the clustering coefficient and the average path length are high.

Keywords: Complex network, Node attraction, Tunable parameters, Degree distribution, Clustering coefficient, Average path length

1. Introduction

Many real-world complex systems were found to show in a form of networks, such as population relationship network, scientists co-authorship network and epidemiology network in social systems, neural network, metabolic network and protein-protein interaction network in the ecosystem, citations network, telephone network, Internet and World Wide Web in information systems etc. After a lot of research on real-world networks, Watts and Stregatz as well as Barabasi and Albert published papers respectively, they proposed Small-World network model (WS model) [1] and Scale-Free network model (BA model) [2]. Discovery and research about small-world effect, scale-free property, community structure [3] and other important properties of the network topology, making the study of complex networks spanning from random graph theory [4] into a new era. The study about the microscopic mechanism which leads to all kinds of macro properties and a series of evolution law of the networks become the focus of research. WS model and BA model had a very concise formation rules, but at the same time they inevitable to ignore some factors impact the evolution of actual networks, and then make some statistical properties have large deviation compared with actual networks [5]. Such as WS model has high clustering coefficient like the actual networks, but it is subject to Poisson distribution; the degree distribution of BA model is power-law which consistent with the reality characteristics, but it did not have obvious clustering effect, as well as the power index fixed for 3, this don't match the power index of actual networks usually in [1,3].

In order to solve the above problems, academics put forward many corresponding improvement models in recent years. Dorogovtsev [6] first proposed the original node attraction model. Dorogovtsev model rules that all nodes in network have same initial attraction, the growth of network affected by node attraction. This model is a good way to solve the problem about that isolated nodes in real-world network will also be connected. Reference [7] proposed a complex networks evolution model based on node attraction (NAM model), this model defined node attraction is the number of connection get in the unit time, and supposed that each nodes have equal opportunity to connect. Node attraction and degree decided together the

⁺ Corresponding author. Tel.: 13488908522.
E-mail address: cug123456@126.com.

probability for new node connected to old nodes. Reference [8] found the original node attraction model and its extension model with smaller clustering coefficient, therefore proposed a complex network model based on node attraction with clustering effect. The new model aimed at the local feature of preferential attachments in real-world networks, defined node attraction is function as changing with time. Experimental results show that the model has a high clustering coefficient, so it can better reflect the network topology and statistical characteristics of real-world networks. This paper proposed a complex network model based on node attraction with tunable parameters on the basis of above research.

2. Node Attraction Model With Tunable Parameters

2.1. Node Attraction

Many real-world networks expressed the preferential attachments, this shows that the connected probability between nodes is concerned with degrees, but the choice standard is not only simple to select the nodes with high degree as description by BA model. For example, in the social network, not everyone make friends with the same rate, some quite charming people will be more easy to make friends, but sometimes, this people often are not big nodes in network; in the scientists co-authorship network, a new one will select cooperator not only if he is a great successful and prestigious scholar but also if he is more active and have contribution in the interested field recent; in the public opinion network, if a node is a sponsors of opinion or a supporter of distinctive opinion, then it will be paid attention and be concerned by others with a higher rate in the short term. The above examples show that the preferential attachments is universal in the evolution of complex network, but when we choice connection, we have to think about other attractive factors in addition to degree.

We define attractive factor β denoted the attraction of nodes for new nodes, each node has attractive factor β_i . $\beta_i = \frac{n_i}{\Delta T}$ denoted the number of connection got in the unit time, where $\Delta T = t - t_i$, t_i denoted node i join to the network on t_i , n_i denoted the number of connection i got in ΔT . The degree of nodes describes the topological position in network, it is a static measurement in moment time, while node attraction describes the changes in a period of time, it is a dynamic measurement.

2.2. Algorithm

The description of algorithm of complex network model based on node attraction with tunable parameters as follows:

Initial conditions: the initial network is an undirected and unweighted network which be composed of m_0 nodes and e_0 edges;

Growth mechanism: In each time step, a new node j joins in the network, this node will connect with m ($m \leq m_0$) nodes existing in the network;

Preferential attachments: Each new node j connected to node i obey the following rules

$$\Pi_i = \frac{\alpha k_i + \gamma \beta_i}{\sum_l (\alpha k_l + \gamma \beta_l)}. \quad (1)$$

Where k_i denoted degree of node i ; β_i denoted attractive factor of node i ; $\sum_l (\alpha k_l + \gamma \beta_l)$ denoted the weighted sum of degree and attractive factor of the rest nodes in network; α and γ are tunable parameters, and

$$\alpha + \gamma = 1. \quad (2)$$

2.3. Node Distribution of Node Attraction Model

A New node j is connected with the original node i , node i will increase it's degree k_i in proportion according to (1). According to the mean-field theory [9], it is:

$$\frac{\partial k_i}{\partial t} = m \frac{\alpha k_i + \gamma \beta_i}{\sum_l (\alpha k_l + \gamma \beta_l)}. \quad (3)$$

Considering $\sum_l k_l = 2mt$, we make it to go into (3)

$$\frac{\partial k_i}{\partial t} = m \frac{\alpha k_i + \gamma \beta_i}{\sum_l (\alpha k_l + \gamma \beta_l)} = m \frac{\alpha k_i + \gamma \beta_i}{2\alpha mt + \gamma \sum_l \beta_l}. \quad (4)$$

The initial condition is node i join into network in the moment t_i , the degree is $k_i(t_i) = m$. So the solution of (4) meet initial conditions is:

$$k_i(t) = \frac{(2mt + \frac{\gamma}{\alpha} \sum_l \beta_l)^{\frac{1}{2}} (m + \frac{\gamma}{\alpha} \beta_i)}{(2mt_i + \frac{\gamma}{\alpha} \sum_l \beta_l)^{\frac{1}{2}}} - \frac{\gamma}{\alpha} \beta_i. \quad (5)$$

In the part $k_i \geq 0$, the probability of $k_i(t)$ can be written

$$\begin{aligned} p(k_i(t) < k) &= p \left(t < \frac{2mt_i + \frac{\gamma}{\alpha} \sum_l \beta_l}{2m(m + \frac{\gamma}{\alpha} \beta_i)^2} (k + \frac{\gamma}{\alpha} \beta_i)^2 - \frac{\gamma}{\alpha} \frac{\sum_l \beta_l}{2m} \right) \\ &= p \left(t_i > \left(t + \frac{\gamma}{\alpha} \frac{\sum_l \beta_l}{2m} \right) \left(\frac{\alpha m + \gamma \beta_i}{\alpha k + \gamma \beta_i} \right)^2 - \frac{\gamma}{\alpha} \frac{\sum_l \beta_l}{2m} \right) \\ &= 1 - p \left(t_i \leq \left(t + \frac{\gamma}{\alpha} \frac{\sum_l \beta_l}{2m} \right) \left(\frac{\alpha m + \gamma \beta_i}{\alpha k + \gamma \beta_i} \right)^2 - \frac{\gamma}{\alpha} \frac{\sum_l \beta_l}{2m} \right). \end{aligned} \quad (6)$$

Because time t obey the uniform distribution,

$$p(t_i) = \frac{1}{m_0 + t}. \quad (7)$$

Get it into (6)

$$p(k_i(t) < k) = 1 - \frac{1}{m_0 + t} \left[\left(t + \frac{\gamma}{\alpha} \frac{\sum_l \beta_l}{2m} \right) \left(\frac{\alpha m + \gamma \beta_i}{\alpha k + \gamma \beta_i} \right)^2 - \frac{\gamma}{\alpha} \frac{\sum_l \beta_l}{2m} \right]. \quad (8)$$

Therefore, the degree distribution is

$$\begin{aligned} p(k_i(t) = k) &\approx \frac{\partial p(k_i(t) < k)}{\partial k} \\ &= -\frac{1}{m_0 + t} \left(t + \frac{\gamma}{\alpha} \frac{\sum_l \beta_l}{2m} \right) (\alpha m + \gamma \beta_i)^2 (-2\alpha (\alpha k + \gamma \beta_i)^{-3}) \\ &= \frac{2\alpha}{m_0 + t} \left(t + \frac{\gamma}{\alpha} \frac{\sum_l \beta_l}{2m} \right) (\alpha m + \gamma \beta_i)^2 (\alpha k + \gamma \beta_i)^{-3}. \end{aligned} \quad (9)$$

$$\begin{aligned} \text{When } t \rightarrow +\infty, p(k) &\approx \lim_{t \rightarrow +\infty} \frac{2\alpha}{m_0 + t} \left(t + \frac{\gamma \sum \beta_i}{\alpha} \right) (\alpha m + \gamma \beta_i)^2 (\alpha k + \gamma \beta_i)^{-3} \\ &= 2\alpha (\alpha m + \gamma \beta_i)^2 (\alpha k + \gamma \beta_i)^{-3}. \end{aligned} \quad (10)$$

Therefore, when $\alpha=1, \gamma=0$, the model have degenerated into BA model, $p(k) \approx 2m^2 k^{-3}$; when $\alpha=\gamma=\frac{1}{2}$, $p(k) \approx 2(m+\beta_i)^2 (k+\beta_i)^{-3}$, this is like what described in [7]; when $\alpha=0, \gamma=1$, the preferential attachments completely according to node attraction, at this time the model is similar to nodes have the same initial attraction in [8].

3. Experiment and Analysis

According to the proposed complex network model based on node attraction with tunable parameters, we use computer simulation to compare with BA model, Dorogovtsev model and NAM model from degree distribution, clustering coefficient and average path length respectively.

3.1. Degree Distribution

The degree of node is the number of edges which connect with this node. Degree distribution $P(k)$ is the probability distribution function of degrees, it mean the probability of a node exactly right have k edges. The empirical research shows that the degree distribution of most complex networks obeys power-law distribution, that is $P(k) \sim k^{-\gamma}$, where $2 < \gamma \leq 3$.

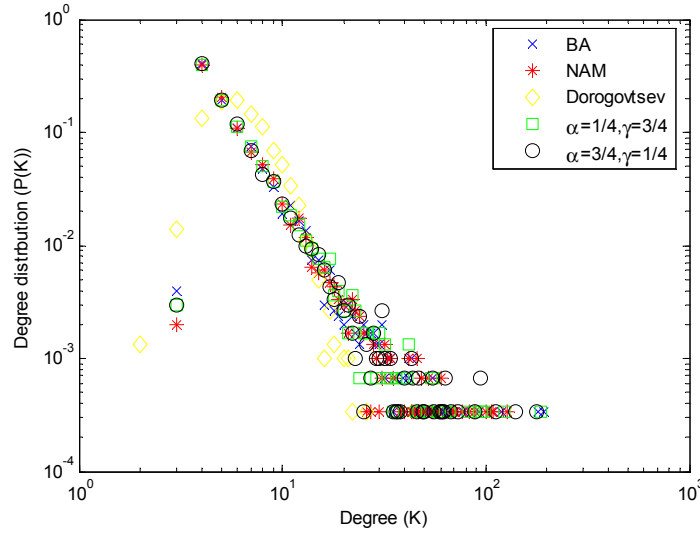


Fig. 1: Degree distribution

The degree distribution of models are shown in Fig. 1, where $N=3000$, $m_0=5$, $m=3$, $\alpha=\frac{1}{4}, \gamma=\frac{3}{4}$ and $\alpha=\frac{3}{4}, \gamma=\frac{1}{4}$ respectively. From Fig. 1, we can reach a conclusion that the degree distribution of the model based on node attraction with tunable parameters and other models all obeys power-law distribution. Therefore, although we can flexibility to adjust the parameters of model, and make to change its formation mechanism, the final network topology did not change too much. This suggests that preferential attachments considering the node degree and node attraction will effective produce stable complex networks, in addition they more close to real-world.

3.2. Clustering Coefficient

The clustering coefficient C of network is used to depict the aggregation degree of network. For a node i in complex network, it's clustering coefficient C_i denoted the probability of any two nodes in all nodes

connect with i are connected to each other, that is $C_i = \frac{E_i}{E_m} = \frac{2E_i}{k_i(k_i-1)}$, where k_i denoted degree of i , $E_m = \frac{k_i(k_i-1)}{2}$ denoted the maximum of edges existing between k_i nodes, and E_i denoted the number of real edges between k_i nodes. Therefore, the clustering coefficient C is defined as the average value of all nodes' clustering coefficient in network, that is $C = \frac{1}{N} \sum_i C_i$, where N denoted the total number of nodes in network.

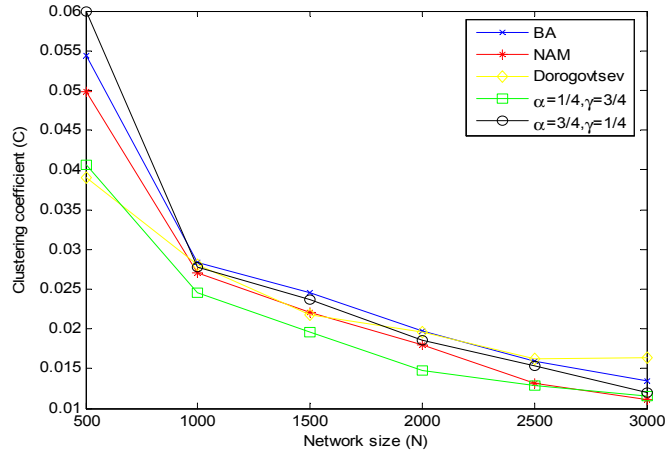


Fig. 2: Clustering coefficient

As shown in Fig. 2, the clustering coefficient of proposed model in this paper and other models have basic consistent change trend, that is, with the expanding of network size, clustering coefficient reducing. In the same network size, several models are little difference, but we can change the clustering coefficient to some extent by adjusting the parameters of model.

3.3. Average Path Length

Average path length is an important statistical characteristic in the complex networks, it used to measure the small-world characteristics. For an undirected and unweighted network, average path length L is defined as the average value of the shortest distance between any two nodes, that is $L = \frac{1}{\frac{1}{2}N(N+1)} \sum_{i \geq j} d_{ij}$, where

d_{ij} denoted the shortest distance between node i and node j ; N denoted the total number of nodes in network.

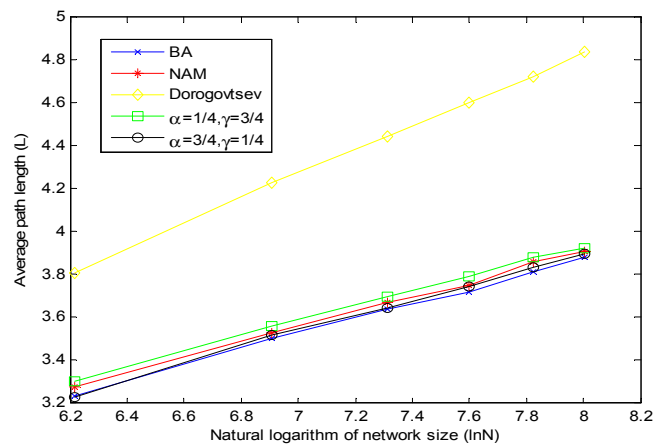


Fig. 3: Average path length

As shown in Fig. 3, the increase speed of average path length of models basic to remain consistent, they are proportion to the logarithm of network size. This is consistent with most real-world networks. In the same network size, the average path length of the proposed complex network model is very close to BA model and NAM model, by adjusting the parameters of model to corresponding change the average path length. But the average path length of Dorogovtsev model significantly greater than other models. This is mainly because when the network grow, Dorogovtsev model only consider node attraction and ignore the node degree, result in network not easy to gather.

4. Conclusion

This paper studies the evolution law of the real-world networks, and then proposes a complex network model based on node attraction with tunable parameters in order to solve the problems existing in BA model and the original node attraction model. The model considers the effects of preferential attachments by the changes of degree and node attraction in the evolution process of networks. Theory research and simulation analysis show that we can more flexible adjust the evolution process of network through adjusting model parameters, therefore make it more accord with the network topology and statistical characteristics of real-world networks.

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6. References

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