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Variable Structure Control Continuous Traffic flow in Automatic Highway System

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Abstract. In order to get stable and orderly road traffic flow, the speed controller was designed based on the variable structure control theory. The density of each section of road was the control objective, firstly, the target speed got from the first step Lyapunov function, then, control item got from the second step Lyapunov function. The simulation results show that the density stabilized within a certain range, thus avoided road congestion.

Keywords: traffic flow; Lyapunov function; variable structure control; speed controller

1. Introduction

Internal and external factors cause fluctuation of traffic flow, and unstable traffic flow may cause traffic congestion and lower efficiency of the system. Automatic highway system[1-7] provides appropriate feedback control command for vehicles on road based on road traffic condition. Because computer control instead of subjective control, traffic flow can be ordered and stabilized. According to the difference of control objects, there are two main control types including micro control [1-3]and macro control [4-7], and the macro control is the main research area.

In [4], ramp control was adopted with variable structure theory. Chien C C, Zhang Y and Ioannou P A [5] designed a speed controller derived from inverse integration, but the process needs solving equations, and perhaps they have no solution. Xie J S *et al*[6] improved method of [5] by Tayor expanding the coefficient matrix. Yang X H *et al* [7] introduced a indirect control, which avoid congestion by controlling the speed of each road section.

This paper presents a new control method for continuous macro traffic flow, which derived from variable structure control theory. The aim is getting stable traffic flow, and the speed controller is designed, under the control the density approaches to specified value, then avoided traffic congestion.

The remainder of this paper is organized as follows: traffic model is given in Section2. Section 3 introduces the variable structure control, and based on Lyapunov function, the control item is deduced in Section 4. In Section 5, numeric simulation proves the effectiveness of our method.

2. Macro Traffic Model

Traffic models include into macro models and micro models, the study target in this paper is the macro traffic model, which proposed by Pageorgiou M [8,9]. In Fig 1, the road divided into many sections, and all vehicles in the same section have the same speed and acceleration rule.

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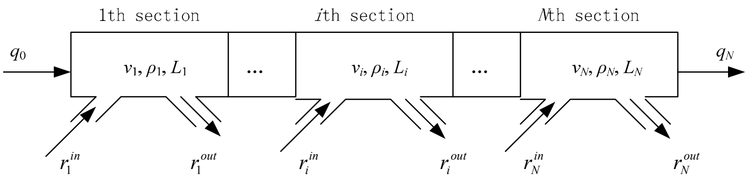

Fig. 1. Diagram of highway

For the ith section, let v_i, ρ_i, q_i, L_i denote speed, density, volume, length of section, respectively. Variable r_i^{in} and r_i^{out} are the inflow and outflow of the ramp of the ith section.

The macro traffic model consists of three equations [8, 9]:

$$\dot{\rho}_i = \frac{1}{L_i}(q_{i-1} - q_i + r_i^{\text{in}} - r_i^{\text{out}}); \tag{1}$$

$$\dot{v}_i = \frac{1}{\tau}(v_e(\rho_i) - v_i) + \frac{v_i}{L_i}(v_{i-1} - v_i) - \frac{\gamma}{\tau L_i} \cdot \frac{\rho_{i+1} - \rho_i}{\rho_i + \lambda}; \tag{2}$$

$$q_i = \alpha \rho_i v_i + (1-\alpha)\rho_{i+1} v_{i+1}. \tag{3}$$

The drivers within each section accelerate or decelerate according to the conditions of where they placed in and the adjacent sections. In (2), τ is the delay time, γ and λ are constants, and the function $v_e(\rho_i)$ is defined as

$$v_e(\rho) = v_f\left[1 - (\rho/\rho_{cr})^l\right]^m, \tag{4}$$

where l and m are constants, and the variable ρ_{cr} and v_{f} are the jam density and the maximum speed. In (3), the variable α is a momentum factor.

From (2), the equation added control item is

$$\dot{v}_i = \frac{1}{\tau}(v_e(\rho_i) - v_i) + \frac{v_i}{L_i}(v_{i-1} - v_i) - \frac{\gamma}{\tau L_i} \cdot \frac{\rho_{i+1} - \rho_i}{\rho_i + \lambda} + u_i. \tag{5}$$

Defining

$$f_i = \frac{1}{\tau}(v_e(\rho_i) - v_i) + \frac{v_i}{L_i}(v_{i-1} - v_i) - \frac{\gamma}{\tau L_i} \cdot \frac{\rho_{i+1} - \rho_i}{\rho_i + \lambda}, \tag{6}$$

then (5) can be transformed as follows

$$\dot{v}_i = f_i + u_i. \tag{7}$$

According to (3), it is easy to get

$$\dot{\rho}_i = \frac{\alpha}{L_i}\rho_{i-1}v_{i-1} - \frac{2\alpha - 1}{L_i}\rho_i v_i + \frac{1}{L_i}\left[-(1-\alpha)\rho_{i+1}v_{i+1} + r_i^{\text{in}} - r_i^{\text{out}}\right]. \tag{8}$$

For the sake of simplicity,

$$\dot{\rho}_i = g_i + e_i, \tag{9}$$

where

$$g_i = \frac{\alpha}{L_i}\rho_{i-1}v_{i-1} - \frac{2\alpha - 1}{L_i}\rho_i v_i, \tag{10}$$

and

$$e_i = \frac{1}{L_i}\left[-(1-\alpha)\rho_{i+1}v_{i+1} + r_i^{\text{in}} - r_i^{\text{out}}\right]. \tag{11}$$

Equations (7) and (9) are the station variation of the ith section under the control. The variable e_i in (9) can be considered as disturbance item, and assuming it is bounded, such that

$$|e_i| \le \bar{e}_i, \tag{12}$$

where $\bar{e}_i$ is upper bound of the absolute value of e_i.

3. Variable Structure Control

The variable structure control theory is an important part of control theory [10]. In variable structure control, the approaching law is usually used to solving control item in many practice applications. For a nonlinear system

$$\dot{x} = f(x,u,t), \tag{13}$$

where x is state vector, u is control item. Suppose s is the switching variable of x, the approaching law has many types, for example

$$\dot{s} = -\varepsilon \operatorname{sgn} s - ks, \varepsilon > 0, k > 0 . \tag{14}$$

It is easy to see that, if s satisfies (14), $s\dot{s} < 0$, thus $s \to 0$. Because symbols function sgn (s) is not continuous, in many situations sgn(s) substituted by function

$$[\operatorname{sgn} s]_{cd} = \begin{cases} \operatorname{sgn} s, & |s| \geq \dfrac{\pi}{2\omega} \\ \sin \omega s, & |s| < \dfrac{\pi}{2\omega} \end{cases}, \tag{15}$$

where ω is a constant. The derivative of $[\operatorname{sgn}(s)]_{cd}$ is

$$[\operatorname{sgn} s]'_{cd} = \begin{cases} \operatorname{sgn} s, & |s| \geq \dfrac{\pi}{2\omega} \\ \omega\dot{s} \cos \omega s, & |s| < \dfrac{\pi}{2\omega} \end{cases} . \tag{16}$$

4. Solving Control Item

In this paper, the control strategy is using speed controller to stabilize the density within certain range.

Firstly, suppose s_{i_1} is switching function of density

$$s_{i_1} = \rho_i - \rho_i^*, \tag{17}$$

where ρ_i^* is a constant, and the first step Lyapunov function is as follows

$$V_{i_1} = \frac{1}{2} s_{i_1}^2 . \tag{18}$$

Then,

$$\dot{V}_{i_1} = s_{i_1}\dot{s}_{i_1} = s_{i_1}\dot{\rho}_i = s_{i_1}\left(\frac{\alpha}{L_i}\rho_{i-1}v_{i-1} - \frac{2\alpha - 1}{L_i}\rho_i v_i + e_i\right) . \tag{19}$$

If

$$v_i^* = \frac{1}{(2\alpha - 1)\rho_i}\left\{\alpha\rho_{i-1}v_{i-1} + kL_i s_{i_1} + \bar{e}_i L_i [\operatorname{sgn}(s_{i_1})]_{cd}\right\} . \tag{20}$$

Substituting (16) to (15), we can get

$$\dot{V}_{i_1} = -ks_{i_1}^2 + e_i s_{i_1} - \bar{e}_i [\operatorname{sgn}(s_{i_1})]_{cd}\, s_{i_1} . \tag{21}$$

According to(16), if $|s_{i_1}| \geq \frac{\pi}{2\omega}$, then $[\operatorname{sgn}(s_{i_1})]_{cd}\, s_{i_1} = |s_{i_1}|$, from (12), we get $e_i s_{i_1} \leq \bar{e}_i [\operatorname{sgn}(s_{i_1})]_{cd}\, s_{i_1}$, so $\dot{V}_{i_1} \leq -ks_{i_1}^2$.Then, if (16) is satisfied, s_{i_1} is decreasing until $|s_{i_1}| < \frac{\pi}{2\omega}$, and then $\rho_i^* - \frac{\pi}{2\omega} < \rho_i < \rho_i^* - \frac{\pi}{2\omega}$.

In (16), v_i^* can be regarded as target speed. For the speed approaching to the target speed, let

$$V_{i_2} = V_{i_1} + \frac{1}{2}s_{i_2}^2 = \frac{1}{2}s_{i_1}^2 + \frac{1}{2}s_{i_2}^2 , \tag{22}$$

where

$$s_{i_2} = v_i - v_i^* . \tag{23}$$

Then

$$\dot{V}_{i_2} = \dot{V}_{i_1} + s_{i_2}(\dot{v}_i - \dot{v}_i^*) . \tag{24}$$

Where $\dot{v}_i$ can be decided by (7), and from (20), $\dot{v}_i$ decided as follows

$$\dot{v}_i^* = \frac{\alpha\dot{\rho}_{i-1}v_{i-1} + \alpha\rho_{i-1}\dot{v}_{i-1} + kL_i\dot{\rho}_i + \bar{e}_i L_i[\mathrm{sgn}(s_{i_1})]'_{cd}}{(2\alpha-1)\rho_i} - \frac{\{\alpha\rho_{i-1}v_{i-1} + kL_i s_{i_1} + \bar{e}_i L_i[\mathrm{sgn}(s_{i_1})]_{cd}\}\dot{\rho}_i}{(2\alpha-1)\rho_i^2}. \quad (25)$$

Form (7) and (9), substituting $\dot{\rho}_i$, $\dot{\rho}_{i-1}$, $\dot{v}_{i-1}$ into (25), we get

$$\dot{v}_i^* = \frac{\alpha\rho_{i-1}}{(2\alpha-1)\rho_i}\dot{v}_{i-1} + h_i, \quad (26)$$

Where

$$h_i = \{\alpha\dot{\rho}_{i-1}v_{i-1} + kL_i\dot{\rho}_i + \bar{e}_i L_i[\mathrm{sgn}(s_{i_1})]'_{cd}\}/[(2\alpha-1)\rho_i] - \{\alpha\rho_{i-1}v_{i-1} + kL_i s_{i_1} + \bar{e}_i L_i[\mathrm{sgn}(s_{i_1})]_{cd}\}\dot{\rho}_i/[(2\alpha-1)\rho_i^2]. \quad (27)$$

If

$$\dot{s}_{i_2} = \dot{v}_i - \dot{v}_i^* = -ks_{i_2}, \quad (28)$$

s_{i_2} →0. So, in order to get control item, substituting (26) into (28) and considering (7), we have

$$f_i + u_i = \frac{\alpha\rho_{i-1}}{(2\alpha-1)\rho_i}(f_{i-1} + u_{i-1}) + h_i - ks_{i_2}, \quad (29)$$

so

$$u_i = -f_i + \frac{\alpha\rho_{i-1}}{(2\alpha-1)\rho_i}(f_{i-1} + u_{i-1}) + h_i - k(v_i - v_i^*). \quad (30)$$

Where f_i, h_i and v_i^* are decided by (6), (27) and (20), respectively.

The bound condition is treated as follows: assuming ρ_0, v_0 are constants, the Lth and (L+1)th section have the same density and speed $(\rho_{L+1}, v_{L+1}) = (\rho_L, v_L)$. According to (7),

$$\dot{v}_0 = f_0 + u_0 = 0, \quad (31)$$

so

$$u_1 = -f_1 + h_1 - k(v_1 - v_1^*). \quad (32)$$

Equations (30) and (32) are the control items.

5. Simulation

In order to prove proposed control strategy, simulation was carried out with Matlab. The parameters are L=3、L_i=0.5、τ=0.008、λ=5、γ=35、ρ_{cr}=80、v_f=110、l=1.7、m=1.8、α=0.99、k=10、ω=1、$r_i^{in}=r_i^{out}=0$ (i=1,2,3), $\bar{e}_i = 176$. The specified density was $\rho_i^* = 30$, and the initial conditions are (ρ_i, v_i)=(30, 50)(i=1, 2, 3). The controller added to the system until t>1. Fig 1, 2, 3 are shown the variation of the density and speed of each road section. The speed and density fluctuate of the period 0<t<1, finally within a certain range under the control. It proves the effectiveness of the control method.

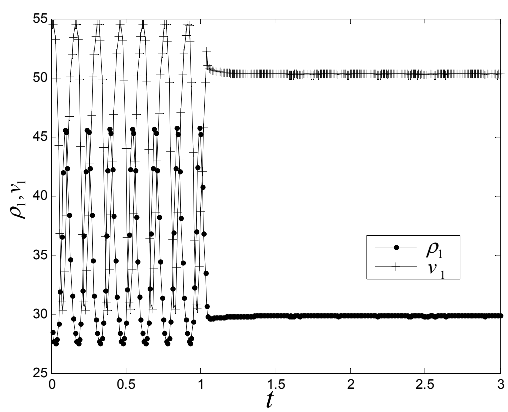

Fig. 2. Density and speed curves of the first segment

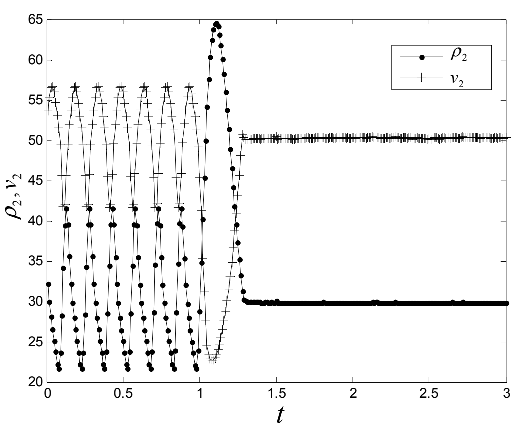

Fig. 3. Density and speed curves of the second segment

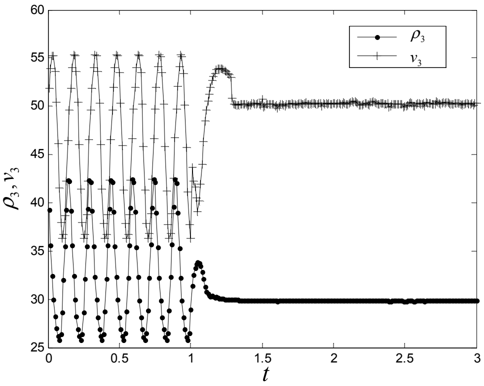

Fig. 4. Density and speed curves of the third segment

6. Conclusion

This paper discussed a control method of continuous macro traffic model based on variable structure control theory. The speed controller of the road section was designed to stabilize the density of traffic flow. Under the control, variations of density and speed were reduced, and the traffic congestion can be avoided.

7. Acknowledgment

This paper discussed a control method of continuous macro traffic model based on variable structure control theory. The speed controller of the road section was designed to stabilize the density of traffic flow. Under the control, variations of density and speed were reduced, and the traffic congestion can be avoided.

8. References

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