

Application of Linearization Clan in Fan Model

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Abstract. Fan is widely used in the industrial process, but fan usually have complicated nonlinear characteristics. The stronger degree of nonlinearity is, the bigger deviation between the model and actual process is. The performance of conventional linear controller will be significantly stepped down with enhanced nonlinearity of the controlled process. Many practical applications need the model of fan to be linearized. And the result is scribed in the paper which adopts the theory of linearization clan to deal with the relationships of the flow and pressure on nonlinear fan model. First, fan is given as a nonlinear object, and then is processed by using the linearization clan method, to get a series of linear models segments. These linear models are superimposed to describe the nonlinear model. Simulation results show the method of linearization clan can approximately describe the nonlinear system of fan, and it has good operability and value of application.

Keywords: Linearization clan, Fans, Non-linear system, Model, Simulation;

1. Introduction

with the improvement of modern industrial, the requirements of system control performance is continuously improving. Because most of the actual control systems are non-linear, the traditional control of linear feedback is very difficult to meet the actual demand. The use of approximate linear model makes us analyze various characteristics of the system more comprehensive and easily, but it is difficult to characterize the nature of the nonlinear system, the dynamic characteristics of the linear system have been insufficient to explain many common actual non-linear phenomena.

Previously, most study of the fan control model is based on model-by-board. After linearization and model reduction, low-order linear state model or transfer function are got, which describe the dynamic characteristics of non-linear model. However, the features of fan model is a serious non-linear in nature. For operating point moving and a large disturbance, the linear approximation has many limitations. The research and application of the nonlinear control system theory has made encouraging progress in the past 20 years [1]-[3]. Especially, the exact linearization method that uses differential geometry as a tool receives widespread attention, but the exact linearization method is only applicable to very few actual control objects. For majority actual control systems, a variety of approximation methods, such as pseudo-linear method, describing function method, approximate the input / output linearization, linearization clan, and so on, still play an important role in the nonlinear system analysis and synthesis. People attach importance to the approximate treatment of nonlinear systems in recent years [4]. It has been demonstrated that in the field of equilibrium point, the approximate linearization method is valid, and the error is acceptable. As a result, this method is widely used in the actual system. In this paper, we use linearization clan method to analyze and process the object model of non-linear fan system.

2. Linearization Clan

Usually it is different with considering a linear approximation near the equilibrium point only, Linearization clan considers the nonlinear system that is corresponding to the case that has different non-zero

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input and output, and a clan of equilibrium operating points. The linearization model has an adjustable parameter for different operating points. On the basis of the linearization clan, it can easily adjust the control method by the gain [5].

Nonlinear system can be expressed as follows:

$$\begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases} \quad (1)$$

The linear approximation relating to the equilibrium point (0,0) can be expressed as follows:

$$\begin{cases} \dot{x} = \frac{\partial f}{\partial x} \Big|_{(0,0)} x + \frac{\partial f}{\partial u} \Big|_{(0,0)} u \\ y = \frac{\partial h}{\partial x} \Big|_{(0,0)} x + \frac{\partial h}{\partial u} \Big|_{(0,0)} u \end{cases} \quad (2)$$

Where f, h is continuously differentiable vector function, and $f(0,0) = 0$, $h(0,0) = 0$. According to the implicit function theorem, the system has a clan of equilibrium in the (0, 0) neighborhood under the conditions which is non-zero input or non-zero output. The set of the equilibrium point is defined as follows:

$$E = \{(x, u, y) \in R^n \times R^m \times R^p \mid f(x, u) = 0, y = h(x, u)\} \quad (3)$$

The equilibrium point will change when the input or output is changing, so the equilibrium point x is $x(a)$, where the parameters a is the input or output function of operation point. it is called a linearization clan of nonlinear systems. To calculate the approximation of the system with Lyapunov method, its coefficient matrix is a function of the input u and output y . The general expression can be expressed as follows:

$$\begin{cases} \dot{x} = A(a)[x(t) - x(a)] + B(a)[u(t) - u(a)] \\ y(t) - y(a) = C(a)[x(t) - x(a)] + D(a)[u(t) - u(a)] \end{cases} \quad (4)$$

Obviously, Linearization Clan is still on the base of the Lyapunov method, but even in the neighborhood of equilibrium, its coefficient matrix may be adjusted by a corresponding input or output parameters according to the movement of the operating point [6].

3. Mathematical Model of Fan

Differential equation or nonlinear operator is often used to describe the nonlinear systems, and a class of nonlinear systems can be described with the ordinary differential equations as follows [7],[8]:

$$T_{1(x,y)} T_{2(x,y)} \frac{d^2 y}{dt^2} + (T_{1(x,y)} + T_{2(x,y)}) \frac{dy}{dt} + y = K_{(x,y)} x \quad (5)$$

Where T_1, T_2 are time constant, K is the gain. T_1, T_2, K are all related with the state. Processed with method of linearization clan the above nonlinear system can be studied by some linear system theories. On different operating points, the non-linear link can be equivalent to different linear link. In each workspace, the T_1, T_2, K in the linearization ordinary differential equations have nothing to do with the state, and then the ordinary differential equations can be described as follows:

$$T_{1(m)} T_{2(m)} \frac{d^2 y}{dt^2} + (T_{1(m)} + T_{2(m)}) \frac{dy}{dt} + y = K_{(m)} x, x_m \leq x < x_{m+1} \quad m=0,1,2,\dots,N \quad (6)$$

Using fan as the object of nonlinear system, we know that the fan can be described as first-order nonlinear object from the literature and experience, because the T of research object changes little over time, it can be regarded as parameter that has nothing to do with the state, and the parameter K relates with the state. In this case, the ordinary differential equation of nonlinear system can be expressed as follows:

$$T \frac{dy}{dt} + y = k_{(m)} x, x_m \leq x < x_{m+1}, m = 0, 1, 2, \dots, N \quad (7)$$

Refer to (7), K is related to the work point. Nonlinear systems with different regions have different slope, and the gain K of every working range is different. If we know the gain, it will be able to get the differential equations of nonlinear system. The differential equation has the same form with the front described linear

differential equation, so it can use the algorithm of linear object. Therefore, according to the relationship between input output curves and the specific characteristics of object, it is the key to get the gain K of the nonlinear system working range. Using theoretical derivation method and characteristics of the object, K can be got.

By flow and wind pressure index, the flow and pressure relationship curve of the fan can be fitted out. The mathematical model of fan can be obtained from this curve. The mathematical model can be expressed as follows:

$$y = -0.0001x^2 + 0.2x + 2000 \tag{8}$$

Where x is flow, m³/s ,y is wind pressure, pa.

The fan flow- pressure relationship curve is shown in Fig.1:

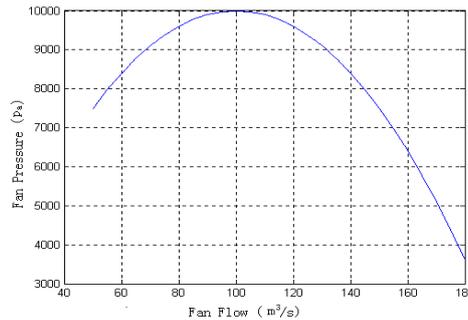


Fig.1. Fan flow-pressure curve

4. Linearization Method

The nonlinear object is linearized with the methods of linearization clan which studies nonlinear object according to certain linear degrees.

Linearity is value of degree which performance or response is closed to linear. In this paper, the nonlinear object is analyzed at the base of different linear degrees and is simulated with Matlab. It can draw conclusion about the non-linear which provides theoretical basis for the simulation.

Three different linearity 2‰, 5‰, 10‰ were used to describe the nonlinear fan. The curves 1,2,3 in Fig. 2 are the dynamic trend curves of fan at the linearity 2 ‰, 5 ‰, 10 ‰, curve 4 is the actual fan dynamic trend curve.

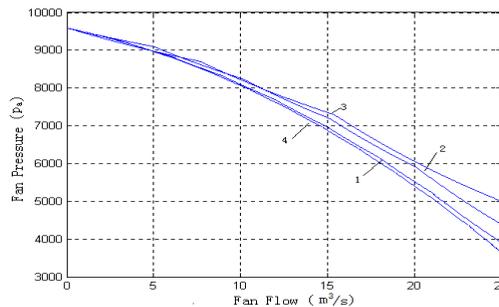


Fig.2. The dynamic trend curves

Obviously, from Fig.2 we can see that the object dynamic trend is closing to the actual trend of with linearity reducing. When the linearity is 2 ‰, the relative error of non-linear objects is 5.6% combining to the actual nonlinear object, and absolute error is 200Pa; when the linearity is 5 ‰, the relative error is 16.7%, and absolute error is 600Pa; when the linearity is 10 ‰, the relative error is 28%, and absolute error is 1000Pa. We can choose a nonlinear object based on the requirements of different precisions. Here, linearity is selected at 2‰ to approximately instead the actual nonlinear object. And the object will be divided into nine line segments. The nonlinear object is simulated with the nine line segments. The corresponding paragraph of the coordinates, the slope of the line segments, the dynamic equation corresponding to each segment and the corresponding transfer function are listed in Table 1. And in Table 1, linearity is 2 ‰, time constant is 20, disturbance is 50, every segment value and its dynamic equations are shown in it.

Table 1. The value of each segment after linear clan

Coordinates	Corresponding interval of y	Slope	Approximately linear transfer function
(120,9600)			
(127,9271)	(9600,9271)	-47	$G(s) = \frac{-47}{20s+1}$
(134,8844)	(9271,8844)	-61	$G(s) = \frac{-61}{20s+1}$
(141,8319)	(8844,8319)	-75	$G(s) = \frac{-75}{20s+1}$
(148,7696)	(8319,7696)	-89	$G(s) = \frac{-89}{20s+1}$
(455,6975)	(7696,6975)	-103	$G(s) = \frac{-103}{20s+1}$
(162,6156)	(6975,6156)	-117	$G(s) = \frac{-117}{20s+1}$
(169,5239)	(6156,5239)	-131	$G(s) = \frac{-131}{20s+1}$
(176,4224)	(5239,4224)	-145	$G(s) = \frac{-145}{20s+1}$

5. Conclusion

The eight segments are the approximate linear object in table1. Given inputs (disturbances), the approximate linear object has the same change trends as the actual nonlinear object by using Simulink software to observe output waveforms. Therefore, the nonlinear objects can be seen as the superposition of several segments in a certain range of error. Conclusion can be obtained from the table1: the nonlinear fan object can be described with the eight segments approximated the linear object in a certain error range. Obviously, by using the linearization clan we can process the fan nonlinear model, the complexity of fan control is reduced. It can reflect the fan operating characteristics truly. So it is a practical and easy approach to control the object model of fan.

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