

Unknown input extended Kalman filter-based fault diagnosis for satellite actuator

Wang Zhi ^{a,*} and Shi Jun^a

Computer Science and Engineering College, Xi'an Technological University, Xi'an 710032, China

Abstract. Unknown input Kalman filter(UIKF) is extended to nonlinear systems, and then applied to detect the early gradual faults of satellite flywheels to improve the timeliness of fault detection, to avoid the occurrence of major accidents. A set of structured residuals are constructed to achieve fault isolation, and Wald sequence detection method is employed to process filter residuals. Finally, the fault diagnosis logic is given. Numerical simulation results show that this method is effective and able to quickly detect faults.

Keywords: satellite actuator; fault diagnosis; UIKF; sequence detection

1. Introduction

The attitude control system is one of the most complex sub-systems of satellite, and has very high probability of failure. According to a survey in literature [1], the faults that occur in this sub-system account for more than 30% of all satellite faults. Since the existence of rotating parts, the flywheels are the highest incidence of fault of attitude control components. In recent years, some achievements have been made in the field of fault diagnosis for satellite actuator. In literature [2], the method of fault detection and diagnosis for satellite reaction wheels is proposed, which uses state space approximation to study neural network fault recognition with nonlinear parameter. The literature [3] employs a set of detection filters to detect the faults of satellite reaction wheels. In addition, fault diagnoses based on expert systems have also been widely studied.

Fault diagnosis of nonlinear systems and robust fault diagnosis are the hot and difficult issues in the current study. As a class of simple construction, high universality of the nonlinear state estimator, the extended Kalman filter (EKF) is subject to a wide range of attention both in theory or practice. For robust fault diagnosis, a well-known method is the unknown input observer (UIO), the basic idea of which is to use the extra degrees of freedom in Luenberg observer design to make the outputs and the unknown inputs (Including disturbances, modeling uncertainties, etc.) decoupling^[4,5]. On the basis of linear unknown input observer, Kitanidis studied the problem of unbiased minimum variance estimation of linear stochastic system with unknown disturbances, and the proposed algorithm was called the unknown Input Kalman filter(UIKF)^[6].

Through the combination of EKF and UIKF, a filtering algorithm applied to nonlinear system with unknown disturbances is proposed, which is called unknown input extended Kalman filter(UIEKF) in this paper. The UIEKF is used to detect the early gradual faults of satellite flywheels, which can improve the timeliness of fault detection, and provide reconstruction information for the attitude control system of satellite to avoid the occurrence of major accidents.

2. Satellite Model and Fault Description

When the servo is three-orthogonal-flywheel, the kinematics and dynamic equation of satellite are described as follows

* Corresponding author. Tel.:13201501617.
E-mail address: zhi_wang_xa@163.com.

$$\begin{aligned}\dot{q}(t) &= \frac{1}{2}(q'(t) + q_0(t)E)\omega(t) \\ \dot{q}_0(t) &= -\frac{1}{2}q^T(t)\omega(t)\end{aligned}\quad (1)$$

$$I\dot{\omega}(t) + \omega(t) \times (I\omega(t) + J\omega_F(t)) = u(t)$$

Where $q \in R^3$, $q_0 \in R$ denote quaternion vector and $q_0^2 + q^T q = 1$; q' is the skew symmetric matrix of q , E is unit matrix; $I \in R^3 \times R^3$ ($Kg \cdot m^2$) is the total moment of inertia matrix of satellite, $\omega \in R^3$ (rad/s) is its inertial angular velocity vector, $u \in R^3$ ($N \cdot m$) is the control torque acting on satellite, $J \in R^3 \times R^3$ ($Kg \cdot m^2$) is the moment of inertia matrix of flywheel, $\omega_F \in R^3$ (rad/s) is its angular velocity vector. Then, the state equation of satellite attitude control system can be denoted as follows

$$\dot{x}(t) = f(x(t)) + Bu(t) \quad (2)$$

As long continuous mechanical movement, the flywheels are the highest incidence of fault of attitude control components. Since the friction torque increases and other reasons, the flywheels may appear gradual faults. Such failures in the early stages are difficult to detect, as time increases, the flywheel failures become clear, it will affect the platform's normal posture. Therefore, this type of failure should be diagnosed as early as possible, and then the control system is reconfigured to avoid a greater impact. For simplicity, this paper only consider the bias-type failure, that is

$$\bar{u}^i(t) = u^i(t) + \rho^i(t) \quad i=1,2,3 \quad (3)$$

where $\rho^i(t)$ is the bias-type fault indicator, $\rho^i(t) \neq 0$ will mean that the bias-type fault occurs. Then, the fault system can be expressed as

$$\dot{x}(t) = f(x(t), u(t)) + B\rho(t) \quad (4)$$

3. UIEKF Algorithm

Considering factors such as disturbances and un-modeled dynamics, the discrete model of the satellite (2) is obtained as follows

$$\begin{cases} x_{k+1} = f(x_k, u_k) + E(x_k)d_k + w_k \\ y_{k+1} = h(x_{k+1}, u_{k+1}) + v_{k+1} \end{cases} \quad (5)$$

Where $y_k \in R^m$ is the measurement vector; $d_k \in R^q$ is the unknown input vector which denotes unknown disturbances and modeling uncertainties; $f(x_k, u_k)$, $E(x_k)$ and $h(x_{k+1}, u_{k+1})$ are formed by smooth nonlinear functions; System noise w_k and measurement noise v_k are zero-mean Gaussian white noises, the covariance matrix of them is respectively denoted as Q_k , R_k .

Assumption: The distribution matrix $E_k \triangleq E(x_k)$ of the unknown input is full column rank, and $\forall H = \partial h / \partial x$, $\forall E_k$ satisfies

$$\text{rank}(H_k E_k) = \text{rank}(E_k) = q$$

Based on the assumption, the UIEKF algorithm for the system (5) is as follows

$$\begin{aligned}\hat{x}_{k+1|k} &= f(\hat{x}_{k|k}, u_k) \\ P_{k+1|k} &= F_k P_{k|k} F_k^T + Q_k \\ \hat{x}_{k+1|k+1} &= \hat{x}_{k+1|k} + L_{k+1} \left(y_{k+1} - h(\hat{x}_{k+1|k}, u_{k+1}) \right) \\ P_{k+1|k+1} &= (I - K_{k+1} H_{k+1}) P_{k+1|k} + \Gamma_{k+1} \Upsilon_{k+1} \Phi_{k+1} \Upsilon_{k+1}^T \Gamma_{k+1}^T\end{aligned}\quad (6)$$

Where

$$\begin{aligned}L_{k+1} &= K_{k+1} + \Gamma_{k+1} \Upsilon_{k+1}; \quad K_{k+1} = P_{k+1|k} H_{k+1}^T \Phi_{k+1}^{-1}; \quad \Gamma_{k+1} = (I - K_{k+1} H_{k+1}) \hat{E}_k; \\ \Upsilon_{k+1} &= \left[\left(H_{k+1} \hat{E}_k \right)^T \Phi_{k+1}^{-1} \left(H_{k+1} \hat{E}_k \right) \right]^{-1} \left(H_{k+1} \hat{E}_k \right)^T \Phi_{k+1}^{-1}; \quad \Phi_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1},\end{aligned}$$

and the matrix F_k , H_{k+1} and \hat{E}_k is respectively determined by

$$F_k = \frac{\partial f}{\partial x} \Big|_{(\hat{x}_{k|k}, u_k)}; H_{k+1} = \frac{\partial h}{\partial x} \Big|_{(\hat{x}_{k+1|k}, u_{k+1})} \text{ and } \hat{E}_k = E(\hat{x}_{k|k}).$$

The difference between the UIEKF and EKF is that, the gain matrix L_{k+1} and the filtered state error covariance matrix $P_{k+1|k+1}$ in UIEKF are amended on the results of EKF, and the correction terms are directly related with the distribution matrix E_k of unknown input. It can be known by the assumption that L_{k+1} meets the constraint of disturbance decoupling

$$L_{k+1}H_{k+1}\hat{E}_k = \hat{E}_k \quad (7)$$

This constraint ensures that the state estimation obtained by the filter (6) is unbiased, and the final state error covariance matrix is large than the EKF.

4. Fault Isolation and Residual Treatment

Based on the UIEKF algorithm, this section first gives the robust fault detection and isolation strategies, and then discusses the treatment of residuals. Considering the equation (5), the fault model of satellite is represented as follows

$$\begin{cases} x_{k+1} = f(x_k, u_k) + E(x_k)d_k + \vartheta(x_k)\rho + w_k \\ y_{k+1} = h(x_{k+1}, u_{k+1}) + v_{k+1} \end{cases} \quad (8)$$

Fault isolation is realized by constructing a set of structured residual, each of which is sensitive to a subset of the faults and robust for the remaining faults. One of the most commonly used structured residual designs is that, there are three UIEKF designed, each of UIEKF is only decoupled for one dimensional fault and all the disturbances and sensitive for the remaining two-dimensional faults^[7]. That is, for the following system (9), there are following three UIEKF designed to achieve fault isolation

$$\begin{cases} x_{k+1}^i = f(x_k^i, u_k) + E(x_k^i)d_k + \vartheta^i(x_k^i)\rho^i + w_k \\ y_{k+1}^i = h(x_{k+1}^i, u_{k+1}) + v_{k+1} \end{cases} \quad i = 1, 2, 3 \quad (9)$$

Clearly, more degrees of freedom are required to achieve fault isolation, that all $[E(x_k^i), \vartheta^i(x_k^i)]$ satisfy assumption 1. Filter residual is respectively defined as

$$\gamma_{k+1}^i = y_{k+1}^i - h(\hat{x}_{k+1|k+1}^i, u_{k+1}), \quad i = 1, 2, 3$$

Next, the Wald sequence detection method will be employed to handle the filtering residuals and detect the flywheel faults. Under the condition that the w_k and v_k are zero mean white noise, if the filtering residual r_{k+1}^i does not contain fault information, it is also a zero mean white noise sequence and obeys the normal distribution $N(0, \sigma_i^2)$. This modal will be expressed as $H_i(0)$. If the filtering residual contains fault information, its White characteristics will be destroyed and it can be supposed to obey the normal distribution $N(\mu_i, \sigma_i^2)$. This modal will be expressed as $H_i(1)$. The optimal decision law of the Wald sequence detection method can be given by the following likelihood ratio function

$$\lambda_i(n) = \ln \frac{p(r^i(1), \dots, r^i(n) | H_1)}{p(r^i(1), \dots, r^i(n) | H_0)} \quad i = 1, 2, 3 \quad (10)$$

According to the independence between $\gamma^j, \gamma^j (i \neq j)$ and the normal distribution, the foregoing decision law can be improved as follows

$$\lambda_i(n) = \sum_{j=1}^n \ln \frac{e^{-\frac{(\gamma^j(i) - \mu_i)^2}{2\sigma_i^2}}}{e^{-\frac{(\gamma^j(i))^2}{2\sigma_i^2}}} \quad (11)$$

Formula (11) can be further simplified as

$$\lambda_i(n) = \lambda_i(n-1) + \frac{\mu_i}{\sigma_i^2} \left[\gamma^j(n) - \frac{1}{2} \mu_i \right] \quad (12)$$

According to the given false alarm rate α and missing report rate β , two thresholds can be calculated as follows

$$L_i^0 = \ln \left[\frac{\beta}{1-\alpha} \right] \quad L_i^1 = \ln \left[\frac{\alpha}{1-\beta} \right]$$

And then the decision rule is

$$\begin{cases} H_i(0) : \lambda_i(n) \leq L_i^0 \\ H_i(1) : \lambda_i(n) \geq L_i^1 \end{cases}$$

If $L_i^0 < \lambda_i(n) < L_i^1$, the testing will continue.

According to all the decisions made by three UIEKF, the result of fault isolation can be obtained by the following logic

$$\begin{cases} H_i(0) \\ H_j(1), \forall j \neq i \end{cases} \rightarrow \text{ith flywheel fault}$$

5. Numerical Simulation and Conclusions

The main physical parameters of a satellite are given as follows

$$I = [5247.97, -230.52, 115.30; -230.52, 5110.05, 41.11; 115.30, 41.11, 4142.48]$$

Two covariance matrices are

$$W = \text{diag}[0.01, 0.02, 0.03]; V = \text{diag}[0.002, 0.002, 0.002].$$

Sampling period is 0.02 seconds. It is set that between the 50th sampling point and 200th sampling point, the 2th actuator appears slowly varying gain type of fault, the fault value is

$$\rho^2(k) = \begin{cases} 0N \cdot m, & k < 50 \\ 0.1k \sin(40k) N \cdot m, & 50 \leq k \leq 200 \end{cases}$$

The simulation results are shown in Figures 1 through 6. In the simulation, Fig 1 and Fig 5 show that the output residuals of 1th UIEKF and 3th UIEKF contain fault information, and the faults are quickly detected by the Wald sequence detection method (Fig 2 and Fig 6); the output residual of 2th UIEKF does not contain fault information. It can be known from the fault diagnosis logic that the 2th satellite flywheel appears fault. The simulation results show that the proposed fault detection and isolation strategies are effective.

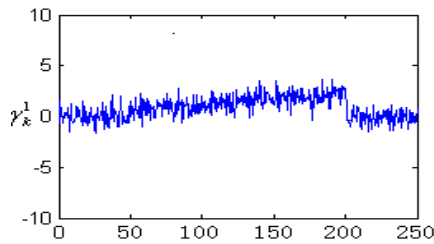


Fig. 1. Residual Curve (1th UIEKF)

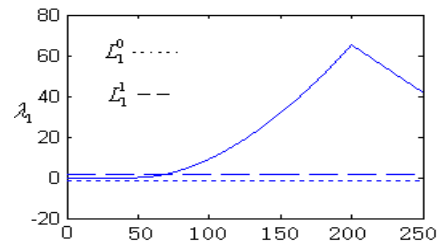


Fig. 2. Two Thresholds and Decision Curve (1th UIEKF)

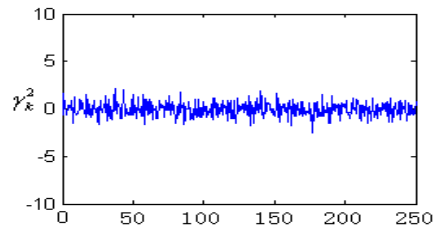


Fig. 3. Residual Curve (2th UIEKF)

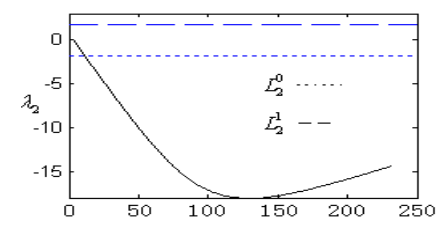


Fig. 4. Two Thresholds and Decision Curve (2th UIEKF)

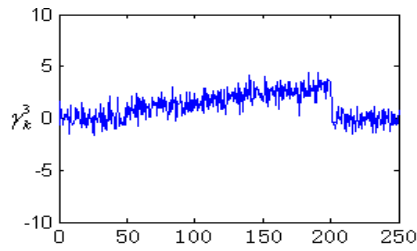


Fig. 5. Residual Curve (3th UIEKF)

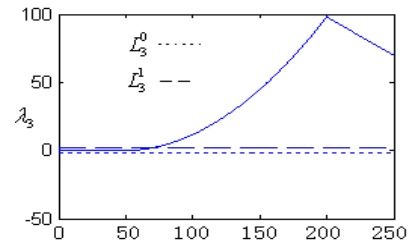


Fig. 6. Two Thresholds and Decision Curve (3th UIEKF)

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