

## Load's 2-Degree of Freedom Swing Angle Model and Dynamic Simulation for Overhead Crane or Gantry Crane

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**Abstract.** Exactly positioning of the trolley and promptly eliminating of the load swing for bridge crane are efficient approaches in order to improve the operational efficiency of over-head cranes and gantry cranes. So, it is necessary that model construction for dynamic equations of the trolley-load system and finding out load swing rules and decision factors of load's swing angle while studying anti-swing control scheme. Construct 2-degree of freedom angle's math model of the load-swing through constructing the trolley-load system's dynamic 3-dimension (3-D) equations, and properly simplify those equations. The rope's length of load and trolley-acceleration can obviously affect the swing angle's value of load. Dynamic simulation results show that the rope-length's force is more prominent than the trolley-acceleration's for the swing angle of load and the trolley's running acceleration (deceleration) can obviously affect the value of load's swing angle when the rope length is determined, the value will increase with increasing acceleration (deceleration), and the trolley's running acceleration (deceleration) can not obviously affect the load's swing frequency.

**Keywords:** gantry crane; overhead crane; crane-trolley-load; load's swing angle; dynamic simulation;

### 1. Introduction

Overhead crane or gantry crane (hereinafter referred to as crane for short) is widely applied to load and unload goods in the station, wharf, warehouse, factory's workshop, and so on [1] ~ [7]. The load's 3-dimensional service space area is composed of the crane's running direction, trolley's running direction and load's hoisting direction [8], [9]. Generally, the trolley and load of the crane are connected by the flexible wire rope. So this connection mode can decrease dynamic load, improve the flexibility of lifting and transporting goods, decrease system's power consumption, but also aggravate load's swing and increase positioning's difficulty for trolley and decrease system's working efficiency [10], [11]. In this paper, the crane trolley-load system's nonlinear dynamic model was constructed. The highly nonlinear dynamic model was linearized based on the crane-trolley-load system's simplified geometrical model. The load's swing rules and swing angle's influencing factors were studied. The rope's length of load and trolley's running acceleration can affect the swing angle's value of load. Dynamic simulation results show that the rope length's influencing is more prominent than the trolley's running acceleration's influencing for the load's swing angle. These researches can offer researches on load's anti-swing control methods with some theoretical basis.

### 2. System's 3-Dimensional Dynamic Model Crane-Trolley-Load

The crane connects the load by the flexible wire rope. The load's 3-dimensional space area is composed of crane's running direction ( $Y$ 's direction), trolley's running direction ( $X$ 's direction) and load's hoisting direction ( $Z$ 's direction). Figure 1 is showing the crane-trolley-load system's simplified geometrical model.  $OXYZ$  is inertial coordinate system. The following notations will be used:  $M$  is the trolley's mass;  $m$  is the

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load's mass;  $l$  is the hoisting rope length and  $l$ 's mass is not considered. The load can be hoisted along axis  $Z$ 's direction. The trolley is running along axis  $X$ 's direction. The crane is running along axis  $Y$ 's direction.  $\theta$  is load's 2-degree of freedom swing angle.  $\theta_x$  is  $\theta$ 's projection on the  $XZ$  plane.  $\theta_y$  is  $\theta$ 's projection on the  $YZ$  plane. If the trolley's position coordinate is  $(x, y, 0)$ , the load's position coordinate is  $(x_m, y_m, z_m)$  in this coordinate system. And we have the following relationship between  $(x, y, 0)$  and  $(x_m, y_m, z_m)$ .

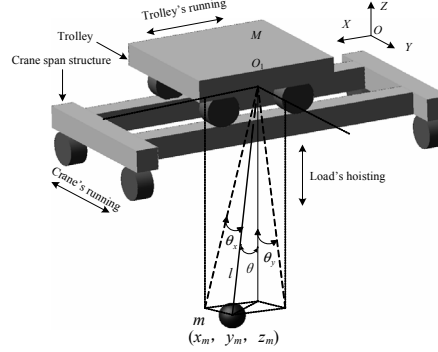


Fig. 1. Crane-trolley-load system's geometrical model

$$\begin{cases} x_m = x + l \sin \theta_x \cos \theta_y \\ y_m = y + l \sin \theta_y \\ z_m = -l \cos \theta_x \cos \theta_y \end{cases} \quad (1)$$

$m$ 's velocity components are expressed as the following expressions along  $X$ 、 $Y$  and  $Z$ , respectively

$$\begin{cases} \dot{x}_m = \dot{x} + \dot{l} \sin \theta_x \cos \theta_y + l \dot{\theta}_x \cos \theta_x \cos \theta_y - l \dot{\theta}_y \sin \theta_x \sin \theta_y \\ \dot{y}_m = \dot{y} + \dot{l} \sin \theta_y + l \dot{\theta}_y \cos \theta_y \\ \dot{z}_m = -\dot{l} \cos \theta_x \cos \theta_y + l \dot{\theta}_x \sin \theta_x \cos \theta_y + l \dot{\theta}_y \cos \theta_x \sin \theta_y \end{cases} \quad (2)$$

System's kinetic energy is

$$\begin{aligned} T = & \frac{1}{2} M (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} m [(\dot{x} + \dot{l} \sin \theta_x \cos \theta_y + l \dot{\theta}_x \cos \theta_x \cos \theta_y - l \dot{\theta}_y \sin \theta_x \sin \theta_y)^2 \\ & + (\dot{y} + \dot{l} \sin \theta_y + l \dot{\theta}_y \cos \theta_y)^2 + (l \dot{\theta}_x \sin \theta_x \cos \theta_y - \dot{l} \cos \theta_x \cos \theta_y + l \dot{\theta}_y \cos \theta_x \sin \theta_y)^2] \end{aligned} \quad (3)$$

We select  $O_1$  as system's zero potential energy position, so system's potential energy is

$$V = -mgl \cos \theta_x \cos \theta_y \quad (4)$$

where:  $g$  is acceleration of gravity, and  $g=9.81\text{m/s}^2$ .

$(x, y, l, \theta_x, \theta_y)$  is system's generalized coordinate, and  $(\dot{x}, \dot{y}, \dot{l}, \dot{\theta}_x, \dot{\theta}_y)$  is system's generalized velocity.  $(\ddot{x}, \ddot{y}, \ddot{l}, \ddot{\theta}_x, \ddot{\theta}_y)$  is system's generalized acceleration. The system's Lagrange equations can be expressed as

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = F_{q_i} \quad (i = 1, 2, \dots, 5) \quad (5)$$

where:  $L$  is Lagrange function,  $L = T - V$ ;  $q_i$  are generalized coordinate,  $q_1 = x$ ,  $q_2 = y$ ,  $q_3 = l$ ,  $q_4 = \theta_x$  and  $q_5 = \theta_y$ ;  $F_{q_i}$  are generalized force,  $F_{q_1} = f_x$ ,  $F_{q_2} = f_y$ ,  $F_{q_3} = f_l$ , and  $F_{q_4} = F_{q_5} = 0$ .  $f_x$  is the trolley's driving force along axis  $X$  and  $f_y$  is the crane's driving force along axis  $Y$ .  $f_l$  is hoisting rope's tensile force.

So we obtain the following crane-trolley-load system's dynamic differential equations about  $x, y, l, \theta_x$  and  $\theta_y$ .

$$\begin{aligned} (M + m)\ddot{x} + m\ddot{l} \sin \theta_x \cos \theta_y + ml\ddot{\theta}_x \cos \theta_x \cos \theta_y - ml\ddot{\theta}_y \sin \theta_x \sin \theta_y + 2ml\dot{\theta}_x \cos \theta_x \cos \theta_y \\ - 2ml\dot{\theta}_y \sin \theta_x \sin \theta_y - ml\dot{\theta}_x^2 \sin \theta_x \cos \theta_y - 2ml\dot{\theta}_x \dot{\theta}_y \cos \theta_x \sin \theta_y - ml\dot{\theta}_y^2 \sin \theta_x \cos \theta_y = f_x \end{aligned} \quad (6)$$

$$(M + m)\ddot{y} + m\ddot{l} \sin \theta_y + ml\ddot{\theta}_y \cos \theta_y + 2ml\dot{\theta}_y \cos \theta_y - ml\dot{\theta}_y^2 \sin \theta_y = f_y \quad (7)$$

$$m\ddot{x} \sin \theta_x \cos \theta_y + m\ddot{y} \sin \theta_y + m\ddot{l} - ml\dot{\theta}_x^2 \cos^2 \theta_y - ml\dot{\theta}_y^2 - mg \cos \theta_x \cos \theta_y = f \quad (8)$$

$$\ddot{x} \cos \theta_x + l \ddot{\theta}_x \cos \theta_y - 2l \dot{\theta}_x \dot{\theta}_y \sin \theta_x + 2l \dot{\theta}_x \cos \theta_x + g \sin \theta_x = 0 \quad (9)$$

$$\ddot{y} \sin \theta_x \sin \theta_y - \ddot{y} \cos \theta_y - l \ddot{\theta}_y - 2l \dot{\theta}_y - l \dot{\theta}_x^2 \cos \theta_x \sin \theta_y - g \cos \theta_x \sin \theta_y = 0 \quad (10)$$

where:  $d_x$  is viscous attenuation coefficient along axis  $X$  and  $d_y$  is viscous attenuation coefficient along axis  $Y$ .  $d_l$  is attenuation coefficient because of rope length's changing. The system's viscous attenuation coefficient  $d$  can be expressed as

$$d = \frac{1}{2} (d_x \dot{x}^2 + d_y \dot{y}^2 + d_l \dot{l}^2) \quad (11)$$

### 3. Simplified Model Load's 2-Degree of Freedom Swing Angle

We obtain the following state equations according to the expression (9) and (10)

$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \dot{x}_3 = -\frac{\cos x_1}{l \cos x_2} \ddot{x} + \frac{2 \sin x_2}{\cos x_2} x_3 x_4 - \frac{g}{l} \frac{\sin x_1}{\cos x_2} \\ \dot{x}_4 = \frac{\sin x_1 \sin x_2}{l} \ddot{x} + \frac{\cos x_2}{l} \ddot{y} - \frac{1}{2} x_3^2 \sin 2x_2 - \frac{g}{l} \cos x_1 \sin x_2 \end{cases} \quad (12)$$

Generally, in order to ensure safety of the load and operation, the operator will operate the crane to control the load's swing in actual operating process. So the load's swing angle amplitude will be less. Then,

$$|\theta_x| \ll 1, |\theta_y| \ll 1, |\dot{\theta}_x| \ll 1 \text{ and } |\dot{\theta}_y| \ll 1$$

are approximately considered because the load's swing angle does not exceed  $5^\circ$ . So, we have the following expressions

$$\begin{cases} \sin \theta_x \approx \theta_x, \sin \theta_y \approx \theta_y \\ \cos \theta_x \approx 1, \cos \theta_y \approx 1 \\ \sin \theta_x \sin \theta_y \approx 0, \cos \theta_x \cos \theta_y \approx 1 \end{cases} \quad (13)$$

Take expressions (13) into expressions (6), (7), (8), (9) and (10). We easily obtain the following load's 2-degree of freedom swing angle  $\theta$ 's linear mathematical model

$$ml \ddot{\theta}_x + m \ddot{l} \theta_x = f_x - (M + m) \ddot{x} - d_x \dot{x} \quad (14)$$

$$ml \ddot{\theta}_y + m \ddot{l} \theta_y = f_y - (M + m) \ddot{y} - d_y \dot{y} \quad (15)$$

$$m \ddot{x} \theta_x + m \dot{y} \theta_y = f_l - m \ddot{l} - d_l \dot{l} + mg \quad (16)$$

$$l \ddot{\theta}_x = -\ddot{x} - g \theta_x \quad (17)$$

$$l \ddot{\theta}_y = -\ddot{y} - g \theta_y \quad (18)$$

According to the load's 2-degree of freedom swing angle model from the expression (17) and (18), the rope length  $l$ , crane's running acceleration (deceleration)  $\ddot{y}$  and trolley's running acceleration (deceleration)  $\ddot{x}$  are main influencing factors of  $\theta_x$  and  $\theta_y$ . So, we can express

$$\theta_x = f(l, \ddot{x}), \theta_y = f(l, \ddot{y}) \quad (19)$$

Figure 2 is showing the load's 2-degree of freedom swing angle's dynamic simulation results. (a) of Figure 2 is showing the dynamic response of  $\theta_x$  relative to  $\ddot{x}$  and  $\theta_y$  relative to  $\ddot{y}$ , respectively. (b) of Figure 2 is showing the linear approximate model's dynamic response for the load's 2-degree of freedom swing angle. From Figure 2, we can draw some conclusions: the amplitude of  $\theta_x$  and  $\theta_y$  is equal but the direction of  $\theta_x$  and  $\theta_y$  is opposite; the curve of the swing angle's nonlinear model and the curve of the swing angle's linear model are basically coincident. So on some definite condition, we can characterize original system's properties with linear approximate model.

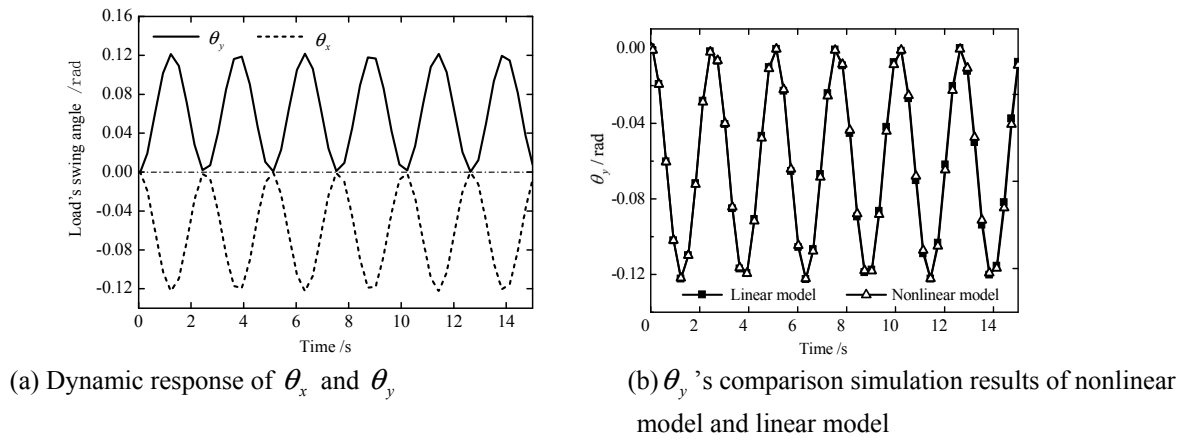


Fig. 2. Dynamic simulation of the load's 2-degree of freedom swing angle.

#### 4. Dynamic Simulation On Load's 2-Degree of Freedom Swing Angle

According to expression (17) and (18), when the rope length  $l$  is determined,  $\theta_x$  is only related to  $\ddot{x}$  and  $\theta_y$  is only related to  $\ddot{y}$ . And expression (17) and expression (18) have the same structure. In order to study conveniently, we take the expression (16) as an example and dynamically simulate. Namely, according to the expression (17), the trolley is running on the  $XZ$  plane. There are two velocity models showing in (a) and (b) of Figure 3 simulating the trolley's moving distance. The trolley's maximum running speed is  $v = 0.85 \text{ m} \cdot \text{s}^{-1}$ . (a) in Figure 3 is the velocity model when the load is transported with long-distance. (b) in Figure 3 is the velocity model when the load is transported with short-distance.

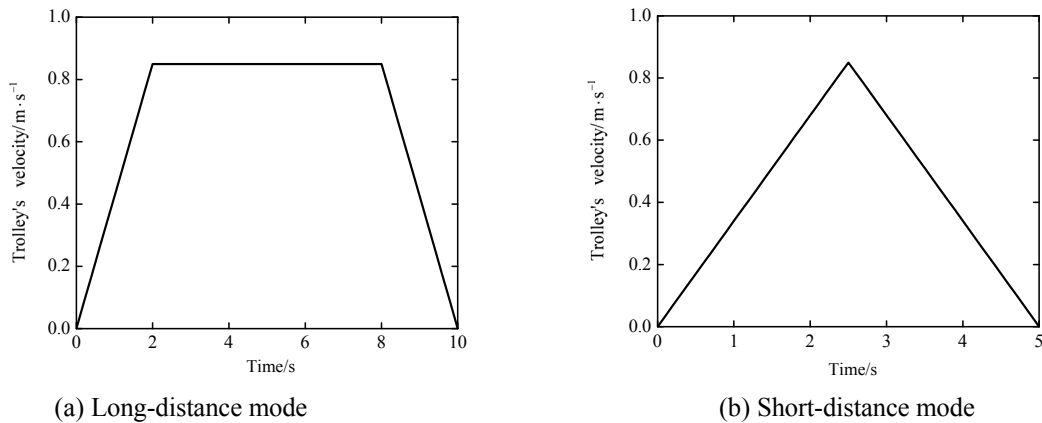


Fig. 3. Trolley's velocity model.

When the rope length  $l$  is 1m, 3m and 5m, respectively, the load's swing angle curves are showing with long-distance in Figure 4, with short-distance in Figure 5. The load's swing angle-velocity curves are showing in Figure 6 and Figure 7. From these curves, the load is forced to swing when the trolley is accelerated or decelerated. The load will freely swing after forced swing when the trolley is running with uniform speed or is in static state after running. Being similar to simple pendulum, the load's swing frequency will decrease with the rope length  $l$ 's increasing. And  $l$ 's influencing is less on the swing angle's value.

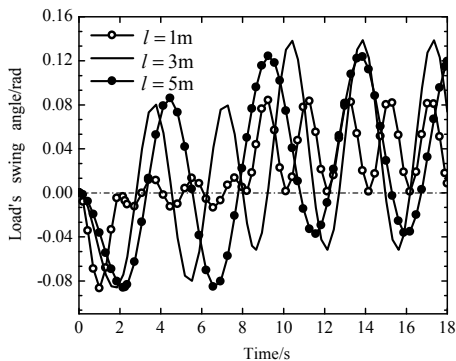


Fig. 4. Curves of load's swing angle for long-distance.

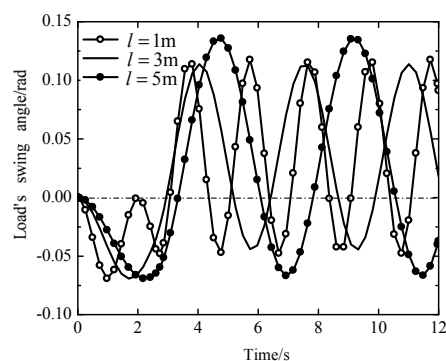


Fig. 5. Curves of load's swing angle for short-distance.

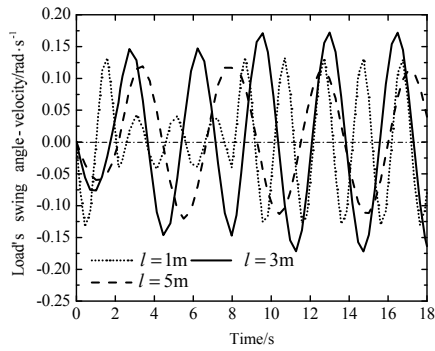


Fig. 6 . Curves of load's swing angle-velocity for long-distance.

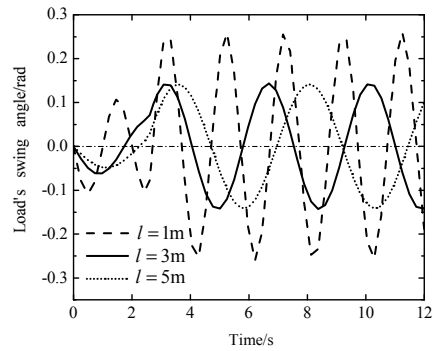


Fig. 7 . Curves of load's swing angle-velocity for short-distance.

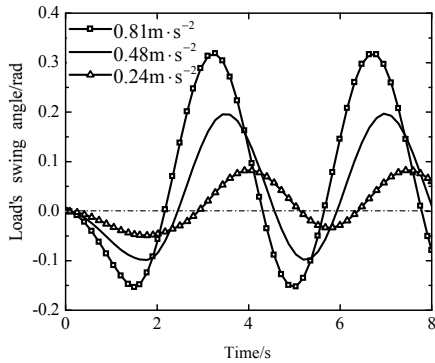


Fig. 8 . Relation curves of trolley's accelerations and angle-velocity.

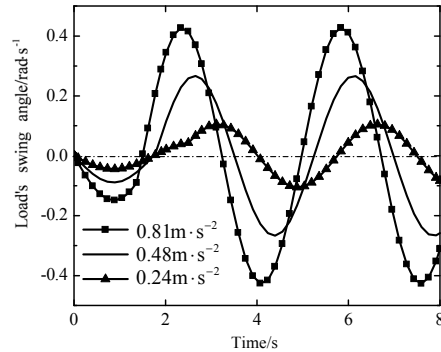


Fig. 9 . Relation curves of trolley-accelerations and load's swing load-swing angle.

When the rope length  $l$  is 3m, trolley's acceleration  $\ddot{x}$  is  $0.24\text{m}\cdot\text{s}^{-2}$ ,  $0.48\text{m}\cdot\text{s}^{-2}$ ,  $0.81\text{m}\cdot\text{s}^{-2}$ , respectively, the load's swing angle curves are showing in Figure 8, and the load's swing angle-velocity curves are showing in Figure 9. From the Figure 8 and 9, we draw the following conclusions: the trolley's running acceleration (deceleration) can obviously affect the value of load's swing angle when the rope length is determined, the value will increase with increasing acceleration (deceleration), and the trolley's running acceleration (deceleration) can not obviously affect the load's swing frequency.

## 5. Simplified Model Load's 2-Degree of Freedom Swing Angle

In this paper, load's 2-degree of freedom swing angle's mathematical model was constructed through constructing the trolley-load system's dynamic 3-dimension (3-D) equations. These equations were properly simplified. The rope's length of load and trolley's acceleration can obviously affect the swing angle's value of load. Dynamic simulation results show that the rope-length's force is more prominent than the trolley-acceleration's for the load's swing angle, the trolley's running acceleration (deceleration) can obviously affect the value of load's swing angle when the rope length is determined, the value will increase with increasing acceleration (deceleration), and the trolley's running acceleration (deceleration) can not obviously affect the load's swing frequency.

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