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The Long and Short Term Volatility Modeling for Load Series based on ARCH Type Models

Ke Xu, Jianzhong Zhou, Yongchuan Zhang and Ran Gu

College of Hydropower and Information Engineering, Huazhong University of Science and Technology, Wuhan, China

xuke19841125@126.com

Abstract—This paper applies non-classical econometrics models to examine the characteristics of load time series, a flexible autoregressive conditional heteroskedasticity (ARCH) type models is adopted to analyze the typical volatility facts such as clustering volatility, asymmetric impact and long memory volatility among others. Then we have a positive research on the long and short term volatility modeling for load series based on ARCH type models, from the empirical out-of-sample forecasts, it appears that there is no different in the positive-negative direction between the long and short term fluctuation process and the asymmetry between the positive and negative impact is obvious.

Keywords- ARCH; Volatility; Time series; Fluctuation

1. Introduction

Load forecasting is necessary for the reliable and economical operation of power systems, its precision is quite important to each department of national economy, so research on load forecasting method is valuable both on theory and practice. There have been a great number of efficient forecasting methods in this field of load forecasting. But, unfortunately, some problems also exist, for example, some problems also exist the preconditions of some classical methods can hardly be applicated in practice and sometimes the parameters of some models have few definite physical meanings etc.

In this study, we employ the asymmetric power ARCH type models for the conditional volatility modeling. However, only the specific models, which include the asymmetric effect, volatility clustering, long memory, and heavy-tailed innovations, are considered in the load time series,, analyzed the ARCH effect of the load time series, then we aim to address this issue by the long and short term volatility modeling for load series based on ARCH type models.

2. Related Literature

The study of economic determinants of load forecast has been well-documented in the literature and there have been a number of approaches to modeling loads series, it was not until the 1980s that the theoretical study of load forecasting began to occur, and a series of forecasting methods, such as MA algorithm, AR algorithm, ARMA algorithm and ARIMA algorithm, had been successively developed and are widely utilized in the load forecasting of power systems (Chenhui, 1987) [1] [4]. With the development of the manual neural network, grey system, expert system, genetic algorithm and other theories and methods, the method of load forecasting of power systems has continuously improved (Santos, Martins, &Pires, 2007; Ying&Pan, 2008.etc) [6][7]. In general, most of the algorithms above are based on the time series.

As exemplified in the aforementioned studies, the fluctuation of time series and forecasting volatility are the hot topics that have attracted the interests of global related researchers and investors. There are ample studies addressing the accuracy of time series volatility modeling and forecasting. One of the most widely used of these is the ARCH formulation developed and first tested by Robert Engle (1982). In order to resolve the mentioned problems, this study employs the autoregressive moving average (ARMA)/ autoregressive conditional heteroscedasticity (ARCH) prediction model as an excellent approach that can specify and forecast the problem in terms of a variance process over time [8].

Traditional time series models such as ARMA model have been extended to essentially analogous models for the variance. Autoregressive conditional heteroskedasticity (ARCH) was developed (Engle, 2001) in order to model and forecast the variance of economic time series over time. ARCH models have been generalized to become the generalized ARCH or GARCH models. ARCH and GARCH models have become common tools for dealing with time series hetroskedastic models [8] [11] ， considering the moments of a time series as variant. ARCH and GARCH models have been applicated in many fields in the time series analysis [3] [8], GARCH models have proven to be a useful means for empirically capturing the momentum in conditional variance and had been used intensively in academic studies..

3. Methodology and Data

3.1 The Theoretical Model

In time series modeling, the error series usually have the characteristic of fat tail assembly, once it happens, if we still regard the error series as the independent identical distribution (i.i.d) variable according to the classical assumption of least-square method, it is unreasonable. Actually, this time series model can only explain time series fluctuation partly and there is still a part of information exists in the error term of the regression equation, consequently, the error terms do not satisfy the homoscedastic assumption of constant variance. The time-varying variance (i.e., volatility or heteroscedasticity), which depends on the observations of the immediate past, is called conditional variance, the ARCH model can describe this problem well [8].

ARCH model is the newly developed time series model, which reflects the special characteristics of stochastic process: the variance changes with the time changing and the variance is crowd together and fluctuated, for the common linear regression model [8]:

$$\begin{aligned} y_t &= X_t'\beta + \varepsilon_t \\ \text{if} \quad \varepsilon_t &= \sqrt{h_t} \cdot v_t \\ h_t &= \alpha_0 + \sum_{i-1}^{q} \alpha_i \varepsilon_{t-i}^2 \end{aligned} \tag{1}$$

In the formula, v_t~i. i. d. N (0. 1) obeys the normal identically distribution, h_t is the conditional variance of ε_t and $\varepsilon_t|I_{t-1}$~ i.i.N(0, h), I_t is the information set which is known. In order to ensure conditional variance h_t take the positive value, constraint conditions which are nonnegative are added into the model: $a_0 > 0$, $a_i > 0$ (i =1,2,…q).The model which satisfies the conditions above can be called as ARCH (q) process, that is $\{\varepsilon_t\}$ obeys the ARCH (q) process [8].

Since the first ARCH model introduced by Engle (1982), its various extensions keep emerging one after another, such as GARCH, ARCH-M, EGARCH and TARCH models and etc.

When we describe some time series using the ARCH model, sometimes the order of the parameter q is too high, it not only aggravates the burden for excess parameters, but all of i in conditional variance expression should be applied the constraint conditions. In view of this issue, Bollerselev proposed the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, the form of conditional variance is expressed as [3]:

$$h_t = \alpha_0 + \sum_{i-1}^{q} \alpha_1 \varepsilon_{t-i}^2 + \sum_{i-1}^{p} \theta_j h_{t-j} \tag{2}$$

Engle, Lilien, Robins proposed ARCH-Mean(ARCH-M) model in 1985, the influence of conditional variance was added into mean equation, the general form of ARCH-M model is written as[17]:

$$y_t = X_t'\beta + \gamma g(h_t) + \varepsilon_t$$
$$\varepsilon_t = \sqrt{h_t} \cdot v_t \tag{3}$$

Among them, g (h_t) is the monotone function of the conditional variance.

In order to describe the asymmetry in series fluctuation, Zakoizn proposed Threshold ARCH (TARCH) model in 1990, the form of ARCH-M model is expressed as [13]:

$$ht = \alpha_0 + \sum_{i=1}^{q} \alpha_0 \varepsilon_{t-i}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} + \sum_{j=1}^{p} \theta_j h_{t-j}$$
$$d_t = \begin{cases} 1 & \varepsilon_t < 0 \\ 0 & other \end{cases} \tag{4}$$

Among them, d_t is a dummy variable.

Afterwards, Nelson proposed another asymmetrical ARCH model in 1991, Exponential GARCH model, which is EGARCH model, the form of EGARCH model is written as [12]:

$$\ln h_t = \alpha_0 + \sum_{j=1}^{p} \theta_j \log(h_{t-j})$$
$$+ \sum_{i=1}^{q} \left(ai \left| \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right| + \phi i \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} \right) \tag{5}$$

Among them, Φ_i is the asymmetry factor.

Next in order to judge whether there exists ARCH affect in residual series, Lagrange Multipliers method is the most common method of test, that is LM test of ARCH, the concrete steps are expressed as [8][11]:

(1) First we use the ordinary least squares method for regression, the residual series $\hat{u}_t$ are obtained;

(2) Then establish a regression equation $\hat{u}_t^2 = c + \alpha_1 \hat{u}_{t-1}^2 + \cdots + \alpha_p \hat{u}_{t-p}^2 + \varepsilon_t$

(3) Hypothesis testing

$H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_p = 0,\ H_1 : \exists \alpha_i \neq 0 (1 \le i \le p)$ Test statistic is: $\mathrm{LM} = \mathrm{nR}^2 \sim \chi^2(p)$

In the formula, n is sample data number in regression equation in steps 2 and R^2 is the coefficient of determination.

In the condition of a given significant level α and degree of freedom p, if $LM > \chi^2{}_{\alpha}(p)$, then reject hypothesis H_0 and the ARCH effect exists, if $LM > \chi^2{}_{\alpha}(p)$, the result is opposite[8][11].

In order to judge whether there exists GARCH effect in random disturbance term of some model, the LM test is applied similarly, if the test is accepted when the value of p in conditional variance equation is large (p>7), it shows that there exists higher order ARCH(p) effect in residual series, in this condition , GARCH(q,p)model should be adopted [3].

3.2 The Data Samples

Our empirical study uses the daily electricity consumption series for the 2006-2008 period of Zhejiang province in China. Nominal electricity consumption data are obtained from the Electricity Power Information Center of the province. In this paper, electricity consumption is expressed in terms of billion kilowatt hours (KWh) or terawatt hours (TWh).

4. Empirical Results

4.1 The ARCH Effect of the Load

The sample we analyze in this paper is the daily electricity consumption data of Zhejiang province from 2006 to 2008. According to the data analysis, the fluctuation range of seasonal periods term will expand with time increasing, it shows that there exists correlation between seasonal term and trend term, so the model for electricity consumption we adopt in this paper is multiplication model: Y=T*S*I, among them T is the trend term; S is the seasonal term; I is the irregular term in load which can not be explained.

The trend term reflects the trend of electricity consumption which deviates from or close to its regularity under the influence of external factors, this term is fitted by the linear regression, the regression equation is expressed as:

Log (load) =7.756428+0.000641*t

The seasonal term reflects the trend of electricity consumption as the seasonal variation, to eliminate the effects of season, the moving average method is applied, the seasonal factors are extracted as follows:

TABLE I. THE SEASONAL FACTORS OF ELECTRICITY SERIES

Month	Jan	Feb	Mar	Apr	May	June
Factor	1.015	0.927	0.961	0.909	0.916	1.012
Month	July	Aug	Sept	Oct	Nov	Dec
Factor	1.129	1.103	1.011	0.912	0.947	1.077

As the I term isn't independent white noise process, it is modeled by the ARMA method in this paper, the main information indexes of the 8 representatinve models are calculated as follows:

TABLE II. THE MAIN INFORMATION OF THE ARMA MODELS

	AIC	SIC	R-squared
ARMA(1,1)	-3.5781	-3.5679	0.7842
ARMA(1,2)	-3.4537	-3.4352	0.6935
ARMA(1,5)	-3.6128	-3.5741	0.7749
ARMA(1,15)	-3.6072	-3.5774	0.7871
ARMA(7,1)	-3.5798	-3.5543	0.7653
ARMA(7,5)	-3.5904	-3.5619	0.7851
ARMA(8,1)	-3.5861	-3.5707	0.7438
ARMA(7,7)	-3.6373	-3.5854	0.7921

Considering the significance level of the AIC and SIC comprehensively, we adopt ARMA (7, 7) model for the daily electricity consumption., thus the expression of multiplication model is obtained: Load=Trend*S*ARMA

The result of parameter estimation and test for ARMA (7, 7) are shown as follows:

TABLE III. THE ESTIMATION OF PARAMETERS FOR ARMA (7, 7)

	Coefficient	Std. Error	t-Statistic
C	0.9236	0.0042	97.6579
AR (7)	-0.1657	0.0254	-4.2860
MA (7)	-0.1326	0.0175	-3.0947
R-squared	0.7843	Log likelihood	215.8750
Adj. R-squared	0.6542	Akaike AIC	-4.1257
Sum sq. resids	1.2157	Schwarz SC	-4.3768
S.E. equation	0.0253	Mean dependent	1.0962
F-statistic	86.1769	S.D. dependent	0.0637

Indexes of ARMA (7, 7) seem good, but if we analyze the correlogram of squared residual series furtherly, it is found that there exists phenomenon of autocorrelation in the squared residual series term, so we have a LM test for the ARMA(7,7), the results are shown as follows:

According to the adjoint probability of χ^2 distribution, the ARCH effect in residual series of ARMA(7,7) is found remarkable, also, the LM test is processed on other ARMA (p,q) which is not be selected and the similar results are obtained: in the condition of 99% confidence level, there also exists ARCH effect in these models.

TABLE IV. LM TEST FOR ARMA (7, 7)

	LM	Probability
q=1	51.4231	0
q=2	53.7684	0
q=3	54.6945	0
q=4	56.5782	0
q=5	58.0907	0
q=6	58.7965	0

According to the analysis above, the conclusion can be obtained as follows: there is strong appearance of ARCH effect in the residual of the model for the daily electricity consumption series.

4.2 The Volatility Analysis of the Load

When the q value in auxiliary regression equation of LM test is larger, if there still exists significant ARCH effect in the residual, it would result in difficulties in parameter estimation and a series of questions, in this condition the GARCH model is adequate, finally the ARMA (7, 7)-GARCH (1, 1) model is established for the series.

The result of parameter estimation and test for ARMA (7, 7)-GARCH (1, 1) are shown as follows:

TABLE V. THE ESTIMATION OF PARAMETERS FOR ARMA (7, 7)-GARCH (1, 1)

	Coefficient	Std. Error	z-Statistic
C	0.9845	0.0057	34.7659
AR (7)	0.6571	0.0279	7.8103
MA (7)	0.1659	0.0213	2.4785
Variance Equation			
C	0.0013	0.0002	5.7482
ARCH(1)	0.4057	0.0326	9.8713
GARCH(1)	0.5109	0.0237	13.9547
R-squared	0.9640	**Log likelihood**	43.3070
Adj. R-squared	0.9280	**Akaike AIC**	-4.7076
Sum sq. resids	1.0027	**Schwarz SC**	-4.329980
S.E. equation	0.0197	**Mean dependent**	1.1504
F-statistic	26.7798	**S.D. dependent**	0.0735

As shown in the table, the parameters of GARCH: α_1=0.4057, θ_1=0.5109, $\alpha_1+\theta_1$=0.9166< 1, the constraint conditions of parameters in GARCH model are satisfied, then the conditional variance of the model satisfy the conditions as:

$$h_t=0.0013+0.4057\varepsilon_t^2+0.5109\ h_t$$

The parameter a reflects the degree of impact from external impulse to daily electricity consumption series; The parameter θ reflects the characteristic of memory in the electricity consumption series fluctuation. In the formula above, α=0.4057, θ=0.5109, implying that decay rate of the fluctuation impact is faster than that of previous impact.

The α in the GARCH model is the degree of impact from external impulse to daily electricity consumption series which depends on absolute values of the impact and independent of the positive-negative forming, but in fact we are aware that positive and negative returns of the same magnitude do not generate an equal response in volatility, thus in order to whether there exists asymmetry of the positive-negative impact in the electricity consumption series, so the concept of leverage effect in the financial field is introduced, TARCH model is established for the electricity consumption series.

TABLE VI. THE ESTIMATION OF PARAMETERS FOR THE TARCH MODEL

	Coefficient	Std. Error	z-Statistic
C	0.9761	0.0053	97.3612
AR (7)	0.7358	0.0265	23.9658
MA (7)	0.2069	0.0346	4.9052
Variance Equation			
C	0.0025	0.0007	7.3725
ARCH(1)	0.3987	0.0518	8.0953
(RESID<0)*ARCH(1)	-0.1739	0.0741	-3.9865
GARCH(1)	0.6306	0.0453	12.1861

The estimation of parameters show that the estimated value of φ_i is negative and the parameter is unequal to zero significantly, indicating that asymmetry of positive-negative impact is evident. Opposite to the conclusion which the TARCH model is applied in the stock time series modeling, (the negative impulse in the stock series would result in sharper fluctuation usually), the upward movements (shocks) in the electricity consumption series are follow by greater volatilities than downward movements of the same magnitude in this paper, this depends on the own characteristics of the electricity consumption series.

4.3 The Long and Short Term Volatility Modeling

In order to research on the conditional variance of electricity consumption series furtherly, null hypothesis is relaxed and the center of the conditional variance is allowed to be a time-varying variable.the ARCH-M model is established and the result of parameter estimation and test are shown as follows:

TABLE VII. THE ESTIMATION OF PARAMETERS FOR THE ARCH-M MODEL

	Coefficient	Std. Error	z-Statistic
C	0.9861	0.0043	63.1764
AR (7)	-0.0876	0.0219	-8.7865
MA (7)	0.0632	0.0298	2.9879
Variance Equation			
Perm: C	0.0036	0.0007	0.0001
Perm:[Q-C]	0.8675	0.0114	0.0000
Perm: [ARCH-GARCH]	0.0756	0.0218	0.0029
Tran:[ARCH-Q]	0.2459	0.0323	0.0000
Tran:[GARCH-Q]	-0.0247	0.0785	0.6581
R-squared	0.7165	**Log likelihood**	267.6545
Adj. R-squared	0.6908	**Akaike AIC**	-3.2561
Sum sq. resids	0.0378	**Schwarz SC**	-4.8796
S.E. equation	1.5674	**Mean dependent**	1.0987
F-statistic	87.6547	**S.D. dependent**	0.0543

That is:

$$h_t - c_t = 0.2459(\varepsilon_{t-1}^2 - c_{t-1}) - -0.0247(h_{t-1} - c_{t-1})$$

$$c_t = 0.0036 + 0.8675(c_{t-1} - 0.0036) + 0.0756(\varepsilon_{t-1}^2 - h_{t-1})$$

As indicated in the table, the value of long term parameter in the model is 0.8675 which is much lager than short term, it agrees with the theoretical, but it is also can be showed that the value of GARCH parameter in the temporary equation of fluctuation is not significant, so the null hypothesis which the parameters are zero can't be rejected, it can be understood that the high order ARCH effect in the temporary equation of fluctuation is not significant; while the values of all the parameters in the long-term equation of fluctuation are significant, it can be regarded that it is of certain significance to uncondional variance be a time-varying variable.

5. Conclusion

This paper focused on volatility of load time series, provided a systematic study by using the ARCH type models and modeled long and short term volatility for load series based on ARCH type models. We observed that the ARCH type models propose an improvement in modeling to captures the volatility of load time series more efficiently, also it is concluded that there is no different in the positive-negative direction between the long and short term fluctuation process and the asymmetry between the positive and negative impact is evident.

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