

The Distinguishing Skills of Curve Convex Position and Function Extreme Value

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Abstract—As far as curve convex position and function extremum are concerned, this paper proposes a new distinguishing method. According to the sign of the function's second derivative, using the method of clockwise rotating symbol to distinguish curve convex and concave, the function's maximum and minimum values.

Keywords- convex position; function extreme value; distinguishing skills;

1. Introduction

Curve convex position and function extremum are the important properties of function; function with such properties plays an important role in modern analysis and optimization these two major areas. In recent years, New or republished textbooks, such as "mathematical analysis", "Higher Mathematics", have strengthened the introduction of the curve convex and function extreme value's concept and distinguishing method. Students in studying of curve convex and function extreme value's distinguishing method, Such as the use of second derivative to judge curve convex or function extreme value, Students are often able to more easily find whether second derivative of $f(x)$ is exist, and the positive and negative of second derivative. However, they often forget when $f''(x) > 0$ (or < 0), the function $f(x)$ in Interval I is convex or concave, or the function $f(x)$ in Interval I obtains maximum value or minimum value. In order to facilitate students remembering and improve the accuracy of distinguishing, this paper proposes a new distinguishing skill for convex curve and function extreme value. According to the sign of the function's second derivative, using the method of clockwise rotating symbol to distinguish curve convex position and functional extreme value. Such Intuitive identification method is not only convenient for students to remember, but also makes accurate identification results.

2. The Definition of Curve Convex Position and Function Extreme Value

2.1 The Definition of Curve Convex Position

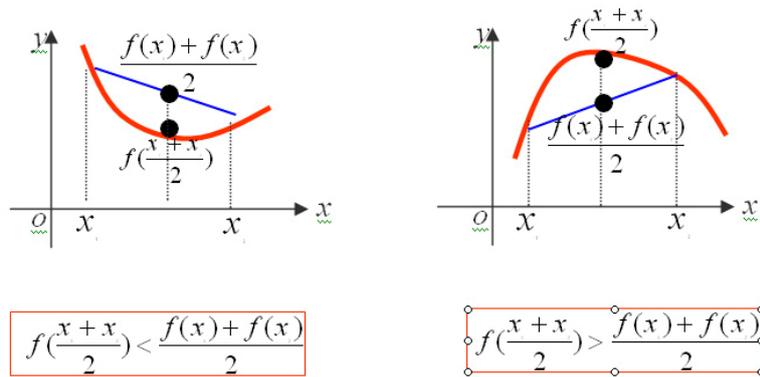
Suppose $f(x)$ is continuous on the interval I, if $(x_1 \neq x_2)$, and $f\left(\frac{x_1+x_2}{2}\right) < \frac{f(x_1)+f(x_2)}{2}$, We call the graph of function $f(x)$ in interval I is concave;

If for any $x_1, x_2 \in I$, $f\left(\frac{x_1+x_2}{2}\right) > \frac{f(x_1)+f(x_2)}{2}$,

We call the graph of function $f(x)$ in interval I is convex;

Seen from the geometry, on the curve arcs, If you take any two points, the connection between these two points usually above this arcs, While other curve arcs, maybe the opposite. Curve's this nature is the

Convexity. Therefore the curve graph can be described using midpoint of the chord which connecting any two points on the curve arcs and corresponding point's location on the curve arcs.

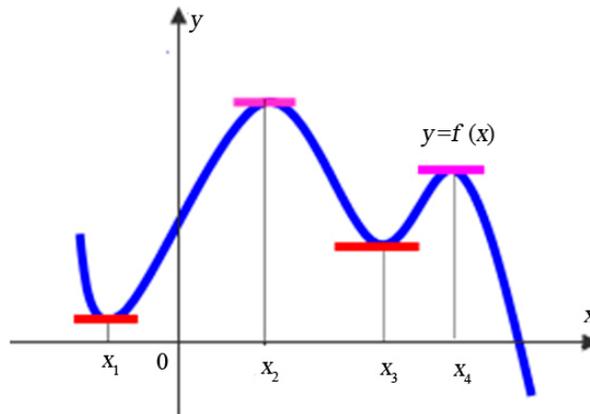


Convex function Graphics convex Concave function Graphics concave

2.2 The Definition of Function Extreme Value

Suppose $f(x)$ is defined within the interval (a, b) , $x_0 \in (a, b)$, if exists $U(x_0, \delta)$, $\forall x \in U(x_0, \delta)$, has

- $f(x) < f(x_0)$, we called $f(x_0)$ as the maximum value of function $f(x)$, x_0 as the maximum point;
- $f(x) > f(x_0)$, we called $f(x_0)$ as the minimum value of function $f(x)$, x_0 as the minimum point;



$f(x_2), f(x_4)$ is the maximum value of function $f(x)$

$f(x_1), f(x_3)$ is the minimum value of function $f(x)$

Function's concept of maximum and minimum are localized. If $f(x_0)$ is a maximum value of function $f(x)$, it is only to a local range near x_0 , $f(x_0)$ is a maximum value of $f(x)$; If for $f(x)$'s entire domain, $f(x_0)$ is not necessarily a maximum. Similarly, the minimum value is also similar.

As shown above, the function $f(x)$ has two maxima: $f(x_2), f(x_4)$, two minima: $f(x_1), f(x_3)$. From the figure we can see, at the position to obtain the extreme value, the curve tangent is horizontal, but where the horizontal tangent on the curve, the function does not always get extreme.

3. The Distinguishing of Curve'S Convex

3.1 Curve Convexity of Criterion of the First Derivative

Suppose $f(x)$ is derivable on the interval (a, b) , thus the necessary and sufficient condition that function $f(x)$ in interval (a, b) is concave(convex)is, that its derivative $f'(x)$ in (a, b) is increase (or decrease).

3.2 The Second Derivative Distinguishing Method of Curve Convex

Suppose function $f(x)$ is continuous on the interval $[a, b]$, has the second derivative on the interval (a, b) ,

- if $f'(x) > 0$ on the interval (a,b) , then $f(x)$ is curve concave on the interval $[a,b]$;
- if $f'(x) < 0$ on the interval (a,b) , then $f(x)$ is curve convex on the interval $[a,b]$.

4. The distinguishing of function extreme value

4.1 Function Extreme Value'S Criterion of the First Derivative

If the function $f(x)$ is derivable at the point x_0 , and obtains extreme value at the point x_0 , then this function's derivative at the point x_0 is zero, that is,

$$f'(x_0) = 0.$$

Suppose the function $f(x)$ at the point of a neighborhood of x_0 can be derivable, and $f'(x_0) = 0$. then.

- if x takes the value of x_0 's left neighboring, $f'(x)$ is constant positive, and if x takes the value of x_0 's right neighboring, $f'(x)$ is constant negative, then the function $f(x)$ obtains maximum at the point x_0 .
- if x takes the value of x_0 's right neighboring, $f'(x)$ is constant positive, then the function $f(x)$ obtains minimum at the point x_0 .
- if x takes the value of x_0 's left or right neighboring, $f'(x)$ is constant positive or negative, then the function $f(x)$ has no extreme at the point x_0 .

4.2 The Second Derivative Distinguishing Method of Function Extreme Value

Suppose function $f(x)$ has second derivative on the point x_0 , and $f'(x_0) = 0$, $f''(x_0) \neq 0$, then if $f''(x_0) < 0$, $f(x_0)$ is the maximum; if $f''(x_0) > 0$, $f(x_0)$ is the minimum.

5. The Distinguishing Skills of Curve Convex position and Function Extreme Value

In the teaching process of applying derivative to study the function and curve's some behavior, especially in the teaching of using the second derivative to determine the curve convex position and function extreme value problems, Common problem found is that after students find the second derivative is positive (or negative), students is easy to forget the function curve is convex or concave, the function extreme value is maximum or minimum. According to the problems exist between the students, this paper sum up a memorable and less error-prone distinguishing skill of convex curve position and function extreme value. By teaching visually distinguishing skills, students not only memory solid, problem-solving fast and accurate, but also simulate their interest in learning.

5.1 The Distinguishing Skills of Curve Convex Position

Suppose function $f(x)$ is continuous on the interval $[a,b]$, has a second derivative within (a,b) , if $f''(x) > 0$ on the interval (a,b) , this time rotate " $>$ " 90° in accordance with the clockwise, the symbol changes into " \vee ", We can intuitively imagine " \vee " as concave curve " \cup ", That is, simply and quickly determine $f(x)$ concave on the interval $[a, b]$ by $f''(x) > 0$. Similarly, if $f''(x) < 0$ on the interval (a,b) , this time rotate " $<$ " 90° in accordance with the clockwise, the symbol changes into " \wedge ", We can intuitively imagine " \wedge " as convex curve " \cap ", That is, simply and quickly determine $f(x)$ convex on the interval $[a, b]$ by $f''(x) < 0$.

5.2 The Distinguishing Skills of Function Extreme Value

Suppose function $f(x)$ has second derivative at point x_0 , and $f'(x_0) = 0$, $f''(x_0) \neq 0$, thus when $f''(x_0) < 0$, rotate " $<$ " 90° in accordance with the clockwise, the symbol changes into " \wedge ", We can intuitively imagine " \wedge " as convex curve " \cap ", By the $f''(x_0) < 0$ we can directly determine the $f(x_0)$ as the maximum value; Similarly, when $f''(x_0) > 0$, rotate " $>$ " 90° in accordance with the clockwise, the symbol changes into " \vee ", We can intuitively imagine " \vee " as concave curve " \cup ", By the $f''(x_0) > 0$ we can directly determine the $f(x_0)$ as the minimum value;

6. Conclusions

The main object of calculus research is the primary function; function's some basic behaviors such as function's monotonic, curve convexity and function's extreme value, most value often use first

derivative, second derivative to examine. In the teaching process, found students not grasp solid enough on distinguishing some function behaviors, student's memory is not strong, and easily confused. By teaching visually distinguishing skills, students not only memory solid problem-solving fast and accurate, but also simulate their interest in learning. In addition, this paper's analysis method has some reference to similar problems.

7. References

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