2012 4th International Conference on Computer Engineering and Technology (ICCET 2012)
IPCSIT vol.40 (2012) © (2012) IACSIT Press, Singapore

Transient Multiexponential Data Analysis Using A Combination of ARMA and ECD Methods

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Abstract. Another attempt at estimating the time constants and number of components of multiple exponentials in white Gaussian noise is presented. Based on classical Gardner transform, the approach consists of two techniques. First, exponential compensation deconvolution method is used to deconvolved the discrete convolution model arising from the application of Gardner transform. The deconvolved data is then truncated and further processed using autoregressive moving average (ARMA) model whose AR parameters are determined by using high-order Yule-Walker equations via the singular value decomposition (SVD) algorithm. Simulations carried out using a number of synthetic signals demonstrate the effectiveness of the proposed technique. Simulation results shows that this combination is more effective than many existing techniques. It is clearly demonstrated that the proposed approach supersedes a number of popular techniques. Its limitations are also highlighted.

Keywords: ARMA, ECD, deconvolution, Gardner transform, SVD

1. Introduction

Measurement and analysis of transient multiexponential data is an activity that has attracted the attention of many research workers in applied science and engineering. Multiexponential transient data are encountered in semiconductor physics (deep-level transient spectroscopy), biophysics (fluorescence decay analysis), nuclear physics and chemistry (radioactive decays, nuclear magnetic resonance), etc. In these and similar applications, the data signal is of the form

$$S(\tau) = \sum_{i=1}^{M} A_i e^{-\lambda_i \tau} + n(\tau) \tag{1}$$

where $n(\tau)$ is an additive white noise and A_i, λ_i, and M are unknown parameters to be determined. Unlike the usual spectrum analysis where the data are of a multiple damped sinusoidal nature, the main concern here is the estimation of the decay rates from the sampled data sequence.

In this paper, a method of estimating the decay rates and number of components of an exponential signal is presented. The method uses the Gardner transform [1] as modified by Nichols et al [2] to convert the multiexponential signal into a convolution model which is deconvolved using a exponential compensation deconvolution (ECD) [3]. The resulting nonstationary deconvolved data is stationarized and further processed using SVD-ARMA as proposed by Salami and Sidek [4]. Analysis of transient multiexponential data using Gardner transform has been done with different modifications [5]. The novelty in the work presented here is the combination of ECD with SVD-ARMA method which is shown to outperform several existing methods of multicomponent transient data analysis.

2. ECD/ARMA Combination

Application of modified Gardner transform on the signal in Eq. (1) was shown [6] to yield the following input distribution function which contains parameters of interest

$$x(t) = \sum_{i=1}^{M} B_i \delta(t + \ln \lambda_i) \tag{2}$$

where $B_i = A_i(\lambda_i)^{-\alpha}$. The DFT of $x(t)$ yields

$$\hat{X}(k) = \sum_{i=1}^{M} B_i e^{j\frac{2\pi k}{N}\ln \lambda_i} + \varepsilon(k) \tag{3}$$

where $\varepsilon(k)$ is nonstationary noise and $k = 1,2,.....N_d$. The nonstationarity problem was addressed in [7] using a rectangular window and the resulting sequence was:

$$\hat{x}(k) = \sum_{i=1}^{M} B_i e^{j\frac{2\pi k}{N}\ln \lambda_i} + e(k) \tag{4}$$

where $e(k)$ is the new stationary noise and $k = 1,2,.....N_d$. $N_d = 2N_0 + 1$.

The deconvolved and truncated data can now be can be considered to be the output of an ARMA model whose input is a complex white noise sequence $e(k)$ so that

$$\sum_{n=0}^{p} a_n x(k-n) = \sum_{n=0}^{q} b_n e(k-n), \qquad a_0 = 1 \tag{5}$$

where a_n and b_n represent respectively the AR and moving average (MA) model coefficients and p and q are AR and MA model orders respectively.

The remaining procedure is as detailed in [4]. The guess values of the AR and MA model order are respectively p_e and q_e and the desired power distribution of $x(t)$ denoted as $P_x(t)$ is obtained as follows:

$$P_x(t) = S_f(z)\Big|_{z=\exp\left(\frac{j2\pi t}{N\Delta t}\right)} = \sum_{k=1}^{M} B_k^2 \delta(t - \ln \lambda_k) \tag{6}$$

3. Simulation Results

In order to investigate the efficacy of the proposed combination, simulations were carried out and presented in this section. First, the truncation point N_0 was established using the Cramer Rao Lower Bound [7]. Two synthetic signals are then used to establish the effectiveness of the proposed technique. Specifically, what is investigated here is the ability of the combination to analyze a simple two-component signal and its resolving power when high resolution signal is involved.

3.1. Determination of the Truncation Point

The truncation point, N_0 is critical to the performance of any technique to be used to process the deconvolved data, $\hat{x}$. CRLB being a good measure of the efficiency of parameter estimates has been used to establish the quality of the parameter estimates for different values of N_0. By varying the data length and comparing of the resulting estimates with the CRLB we can know which data length would produce the best estimator. Derivation of the CRLB was done as presented in [8]. The signal used for this simulation is

$$S(\tau) = 50e^{-0.025\tau} + 100e^{-0.1\tau} + 200e^{-0.2\tau} + 350e^{-0.7\tau} + n(\tau) \tag{7}$$

The reasons for the choice of this signal were given in [3]

Simulations using the proposed ARMA algorithm with conventional inverse filtering showed that the spectrum is good only for $27 \le N_0 \le 33$ with the best performance at $N_0 = 27$.

3.2. Resolvability of the Components

To investigate the capability of the proposed combination to analyze basic signals, the following two-component signal was used:

$$S_1(\tau) = 0.1e^{-0.1\tau} + 0.2e^{-0.2\tau} \tag{8}$$

The distribution function is:

$$x_1(t) = 0.1^{(1-\alpha)}\delta(t - \ln 10) + 0.2^{(1-\alpha)}\delta(t - \ln 5) \tag{9}$$

The signal was synthesized in MATLAB with noise added using the function *awgn* which is an embedded MATLAB function. Table 2 shows the result of applying the proposed combination. over low ($SNR \le 40dB$), medium ($40dB < SNR < 100dB$) and high ($SNR \ge 100dB$) SNRs. A plot of the distribution function for high SNR is shown in Figure 1. The distribution function was plotted against negative time in order to arrange the exponents in ascending order on the horizontal axis. For the purpose of comparison, DFT plot is shown (broken lines) along with the actual plot (solid). The DFT graphs are obtained by windowing the deconvolved data to remove high frequency noise and then inverse transforming, instead of using SVD-ARMA or any other parametric modelling technique. The results are based on $p_e = 20$, $q_e = 5$ and the choice of deconvolution parameters is according to Table 1.

Table 1: Recommended deconvolution parameters

Parameter	SNR		
	Low	Medium	High
a	0.6	0.6	0.05
b	4	3	1

Table 2: Estimated log of decay rates ($\ln \lambda_i$) for $S_1(\tau)$

Expected Value	LOW SNR	MEDIUM SNR	HIGH SNR
-2.3025	-2.963	-2.45	-2.35
-1.6094	-1.721	-1.75	-1.561

3.3. Response to a high resolution signal

The following signal was used to test the effectiveness of the proposed combination in resolving a high resolution signal.

$$S_2(\tau) = 0.5e^{-0.5\tau} + e^{-\tau} + 2e^{-2\tau} + 5e^{-5\tau} + 10e^{-10\tau} + n(\tau) \tag{10}$$

It is noteworthy that for this signal $\frac{\lambda_1}{\lambda_2} = \frac{\lambda_2}{\lambda_3} = \frac{\lambda_4}{\lambda_5} = \frac{1}{2}$. This signal would prove difficult when applied to most of the popular methods (Prony, original or modified, nonlinear least squares, METS, etc.). This signal is therefore suitable to test the effectiveness of our proposed technique.

The distribution function for the signal $S_2(\tau)$ is as follows:

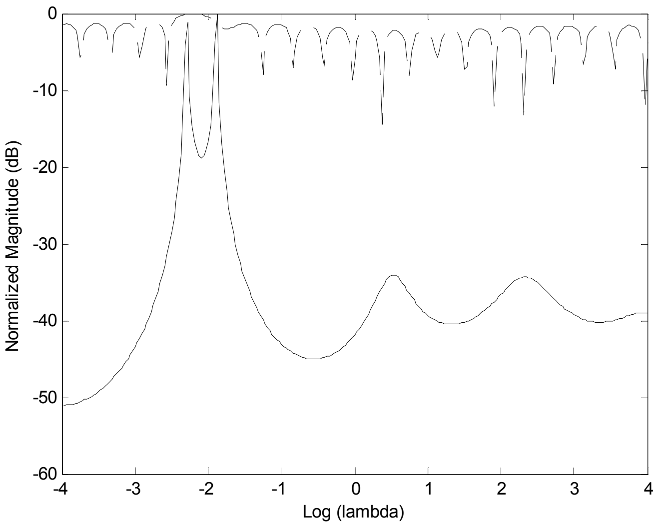

Fig. 1: Power distribution for $x_1(t)$ at high SNR using the proposed combination.

$$x_2(t) = 0.5^{(1-\alpha)}\delta(t-\ln 2) + \delta(t) + 2^{(1-\alpha)}\delta(t+\ln 2) + 5^{(1-\alpha)}\delta(t+\ln 5) + 10^{(1-\alpha)}\delta(t+\ln 10) \quad (11)$$

The result of applying the proposed combination on $S(\tau)$ is shown in Table 3. Figure 2 shows power distribution for $x_2(t)$ at a medium SNR. It is observed that while the combination gives good results over medium and low SNR, it yields poor estimates at low SNRs. Its detection threshold is thus lower than that of MUSIC (Multiple signal classification) and minimum norm methods used in [3].

4. Discussion and Conclusion

In this paper, a new combination of a deconvolution technique and a parametric method has been introduced. The method has been used to analyze two signals, one with two components and another with five components and higher resolution. Although the combination results in a lower detection threshold than that of ECD/MUSIC and ECD/minimum norm, it nevertheless supersedes a few earlier methods like Fast Model-Free Deconvolution [9], Isernberg methods of moments (IMOM) [10] and Multiexponential transient spectroscopy (METS) [11] in terms of the number of components it can be used to analyze.

Table 3: Estimated log of decay rates ($\ln \lambda_i$) for $S_2(\tau)$

Expected Value	LOW SNR	MEDIUM SNR	HIGH SNR
-0.6931	POOR RESULTS	-0.875	-0.7813
0		-0.0721	-0.0625
0.6931		0.921	0.7813
1.6094		1.6812	1.625
2.3025		3.375	2.469

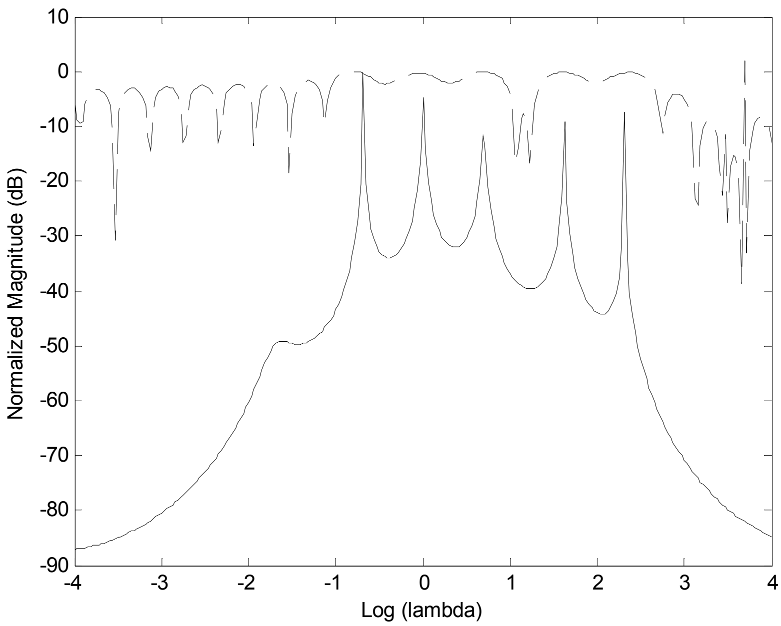

Fig. 2: Power distribution for $x_2(t)$ at medium SNR.

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