

The computer as a tool to acquire and evaluate skills in Math courses

A. Queiruga Dios¹⁺, V. Gayoso Martínez², A. Hernández Encinas¹, and L. Hernández Encinas²

¹ Dpt. of Applied Mathematics, E.T.S.I.I. of Béjar, University of Salamanca, Salamanca, Spain.

² Dpt. of Information Processing and Coding, Information Security Institute (ISI), CSIC, Madrid, Spain

Abstract. A new experience in teaching-learning Mathematics to engineering students is shown. The aim is to give them a better understanding of the contents of the course by using Matlab software, in a graduate-level course. In this paper we discuss how to structure, define, and implement a web-based assessment as a part of the traditional classes. The proposed course facilitates the use of the information and communication technologies. We pay special attention to the possibilities offered by online platforms that allow writing math symbols and formulas.

Keywords: teaching-learning system, online platform, Matlab, questionnaires.

1. Introduction

Linear Algebra, Calculus, Statistics or Numerical Methods are some compulsory subjects for all the students in the first and second years of the engineering graduate studies in our universities. At the end of their courses students will have a greater appreciation of how mathematics can be used in real life situations.

As it is well known, Matlab works very well with matrices, it is one of the most natural for use in Linear Algebra [2], as it was developed specifically for matrix operations, that is why it results a good option to learn Linear Algebra. Computer files for use with Matlab are available from some textbook publishers, and specially designed Matlab teaching codes developed at MIT are available at web.mit.edu/18.06/www/. As a case study we will focus this paper in Linear Algebra, as this is a subject that just ended these days and students are doing their final exams.

Trigueros, Possani [9], and some others lecturers pointed out the difficulties in Algebra teaching, and how to improve the teaching of Linear Algebra, and that an effort is needed to understand why students find this material difficult, and how to help them to learn it effectively. With this in mind we proposed to change the traditional learning way to a new one based on computers.

Linear Algebra is a part of a semester subject and it is separated in 5 modules: 1) Vector spaces, 2) Linear mappings and matrices, 3) Euclidean vector spaces, 4) Systems of linear equations, and 5) Diagonalization.

The case of Calculus, Statistic or Numerical Methods can be considered in a similar way: 5 to 7 modules as semester subjects. The great difference in these issues is that Matlab is not the most adequate program to work with. In the case of Statistics for example, SPSS or Excel are considered better options.

This paper is organized as follows: Changes in traditional classes are explained in Section 2. In Section 3 we present the advantages of the computer as a tool to teach Linear Algebra and other related subjects. Finally, we present some conclusions in Section 4.

2. Changes in Traditional Classes

⁺ Corresponding author. Tel.: + 34 923 408080; fax: +34 923294786.
E-mail address: queirugadios@usal.es

In 2010 the educational system in Spain changed from a traditional model to a more advanced model. During the academic year 2010/2011 most of the Spanish universities started the change to a new qualification system that has been proposed in the Bologna agreement [1]. The Bologna accord has supposed a lot of changes in higher education, more specifically in the contents of the subjects, in the duration of bachelor's degree, and in the higher educational system itself. The educational system was redefined in order to obtain a common European space of university education.

At the colleges of engineering where we teach, we have started the implementation of the new teaching-learning techniques in 2010. During this time, we have moved from a blackboard-based education to a computer-based one, and from a teaching system, where teachers were mere lecturers, to a new teaching-learning system, where students must look for their education and training and be responsible for their learning.

Before the Bologna changes, the Linear Algebra subject was a single semester subject, but now it is a part of the Mathematics I subject. Mathematics I is divided in two parts: Linear Algebra and a small part related to Numerical Methods.

During classes, teachers used to explain the contents and used to solve different types of problems to make the students understand all the contents of the course. As mathematics is not an easy matter, we tried to find different ways to explain the same things, offering some problems so the students could practice how to solve different questions.

The traditional assessment process included a review of the subject contents and represented the global mark of the course. The test used to have 2 parts, with the same percentage for the final mark:

- 1) *Theoretical part*, where students should demonstrate a theorem or solve some theoretical questions.
- 2) *Practical part*, where students should solve some problems related to the contents that were previously explained in classes.

It was not compulsory to assist to classes, and the only requirement was to pass the exam obtaining with at least 5 over 10 points in the final exam.

The Bologna Accord expressed the need for a change, and after 2010 one of the consequences was the use of the information and communication technologies as an important part of the recently established new teaching-learning system.

As a consequence of the use of computers, we propose an ongoing assessment using the math software Matlab and the Learning Management System (LMS), Moodle.

3. The Computer as Useful Tool

We have proposed the students to carry out some exercises developed with Matlab and some questionnaires.

3.1. The use of Matlab

It is important to pay attention to the fact that Matlab is a useful tool for students, though it requires teachers to spend a lot of time evaluating activities.

Regarding Matlab, students must learn how to use such a programming language. Moreover, Matlab offers engineer students a simulation package that represents an important tool to the field of electrical and electronic engineering. The simulation is a computer program running on the computer and students can use the program whenever they want in a relatively lower cost [3]. As a solution to the problems associated with laboratories (the cost of setting-up and maintaining a laboratory), many institutions tend to use virtual laboratories in the form of computer simulation. Due to the fact that engineering is an evolving field, the use of computer software and Matlab in particular, is a necessity to them.

As there are already many documents available on the Internet with information about Matlab and introductory tutorials, we do not need to create new documents for the users. Those documents, including introductions to Matlab and even more complicated problems, are free and available to any student.

Related to the first module of the course, students must solve some problems about vector spaces and bases (Fig. 1). When each activity is finished, students must submit the .m file to the online platform so teachers can evaluate their work by executing all of them and checking the different possibilities. Matlab is the right program for Linear Algebra, as it was designed to work with matrices.

Other tasks that we have proposed to the students are:

- To develop a function that obtains the image of a map.
- To define some figures and make some linear transformations with them.
- To develop a function that obtains the solution of a linear system equation (LSE).
- To develop a function that obtains the solution of a LSE using the LU transformation.

To evaluate these tasks we execute the code submitted by students, we check the validation implemented, and give them a mark.

3.2. Virtual Learning Environment

An educational virtual environment or Virtual Learning Environment (VLE) was defined by [5] as a virtual environment that is based on a certain pedagogical model, incorporates or implies one or more didactic objectives, provides users with experiences they would otherwise not be able to experience in the physical world and produces specific learning outcomes.

The effectiveness of a learning environment is set by its pedagogical approach, the learning theory that follows in order to fulfil the educational goals and reach the desirable learning outcomes.

The use of an online platform provides some advantages to the students:

- They can access the information anytime and anywhere.
- They can interact with teachers and other classmates.
- They can check their marks related to the proposed tasks and questionnaires on the platform.
- They can solve questionnaires and obtain a partial mark by doing so.

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1 EJERCICIO 3.1
2 % Ejemplo de cambio de base
3 % Denotaremos por X, X1, X2 respectivamente a las coordenadas respecto d
4 % las bases canónica, (B1) y (B2) de un vector cualquiera v.
5 % Denotaremos B1, B2, respectivamente, a las matrices cuyas columnas son
6 % de las bases (B1) y (B2) en la base canónica.
7 % Por tanto: X = B1*X1; v = B2*X2; X2 = P*X1.
8 % Luego X = B1*X1 = B2*X2 = B2*P*X1. Es decir B1 = B2*P.
9 B1 = [2 0 0 -1; 0 2 -2 0; 0 2 0 -2; 0 2 2 2];
10 B2 = [1 2 0 2; -1 0 1 1; 0 0 -2 1; -3 0 -3 0];
11 % Matriz P que realiza el cambio de base de B2 a B1.
12 P = inv(B2)*B1;
13 disp('La matriz P buscada es ');
14 disp(P);
15 X2 = [3 -2 -1 2];
16 disp('Las coordenadas pedidas en la base (B1) son: ');
17 X1 = inv(P)*X2;
18 disp(X1);

```

Fig. 1: First practice about vector spaces.

```

BT=B'
repeat
a1=random(1..5); a2=random(1..9); a3=random(1..5);
b1=random(1..5); b2=random(1..5); b3=random(0..8);
c1=random(-5..5); c2=random(1..3); c3=random(1..5);
M = [[a1,b1,c1],[b1,b2,c2],[c1,c2,c3]]
until |M|>0
u = [a2,b2,c2]
v = [a3,b3,c3]
dif = u - v
sol = sqrt(dif.M.dif)
an = arccos( (u.v) / (||u|| * ||v||) ) * 360 / pi

```

Fig. 2: WIRIS question related to Euclidean spaces

The online platform allows the teachers to keep contact with students and follow their work through the course and through the interaction tools like chats or forums, using activities designed specifically for teaching [7]. The management for a Moodle course is a complex and delicate activity, both in educational and computer programming level. The computing science, mathematics, engineering and physics teachers, we have better tasks for follow an eLearning course on the Moodle platform but it take a lot of time to prepare all the activities [6].

The Moodle environment has a graphical user interface intuitive enough for students to use it without prior knowledge of the environment, and it allows students construct their own learning through the use of different activities and collaborative tools as well as the tools of communication between participants (students and teachers) [10]. The use of the LMS allows students to improve their approach to the solution of a given mathematical problem, performing significant signs of autonomy [4] and improving students' performance.

3.3. Online Questionnaires

We started to use WIRIS (<http://www.wiris.com/>) recently because we noticed that Moodle does not offer the option to write symbols and formulas apart from Latex inside Moodle. We use WIRIS as an editor, calculator or to develop some quizzes [8]. We proposed the students 2 or 3 quizzes that must be done in 1 week, according to the outline contents.

One of the great advantages of WIRIS quizzes is that it is possible that every student gets different data for his/her problem. For example, we want them to calculate the distance and angle between 2 vectors; those vectors are different for each student (Fig. 2).

With the command random we can get different values for each execution. WIRIS already has some operations defined by itself, like the distance, module, dot product, etc. The visual WYSIWYG editor facilitates to write all the mathematical formula, including matrices, limits, integral, derivatives, etc.

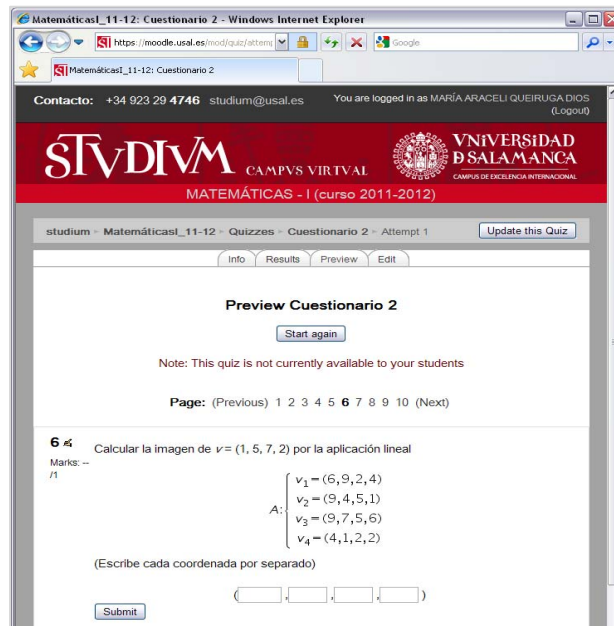


Fig. 3: Question in Moodle platform

Whether working with WIRIS or with any other programming language, one must keep in mind that anyone can make mistakes, so some validations and error controls must be added. Without them, the result is incomplete and may fail. For example, in the case of calculating the distance between 2 vectors, it could be possible that the program returns the same vectors twice, and the student will find without any calculation that the distance equals zero. If we add a question to obtain the inverse of a matrix, that matrix must have the determinant distinct from zero.

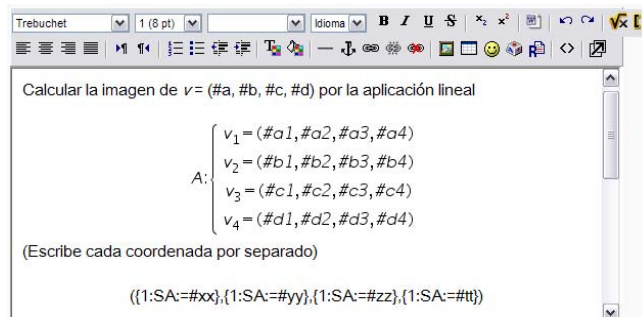


Fig. 4: Wiris formulation for an embedded answers question

The students are unaware that teachers have to develop a short program to create a question and to make each question different. What they see is a question like the one shown in Fig. 3.

In Fig. 4 the corresponding WIRIS formulation is shown. This is the case of an embedded answers question, which means that in a passage of text or math formula there is some empty spaces that must be filled with the right answers.

All the questionnaires of the course can be solved using Matlab or any other similar program. In fact, in some questions we have proposed the codification of a function or the use of some specific Matlab commands. For example, one of the questions about Euclidean spaces was to find an orthonormal base from a given one. The students had to implement a function that calculates such a base.

In the case of module 4 of the outline, students learn how to factorize matrices with LU method to solve linear systems equations. WIRIS instruction to do the LU factorization from a matrix A is:

$$\{L \ U\} = \text{lu_decomposition}(A),$$

which finds an upper triangular matrix (U) and a low triangular matrix (L).

When students try to find the LU decomposition using Matlab, they find that Matlab uses `de lu` command and finds an upper triangular matrix (U) and a “psychologically lower triangular matrix” (L), as can be read from the Matlab product help.

4. Conclusions

In recent years, the proposed educational scheme for graduate students has transformed into a computer-based education. Mathematics is one of the areas in which computer use is very important. In fact, as a result of the implementation of the European Higher Education Area (EHEA) in our universities, we have started the widespread use of computer. Our universities already have online platforms that allow us to work with students and in which we can develop different educational tasks.

We propose the students to work in their own learning process with the use of programs specifically designed for mathematical calculations, as well as new functionalities in the online environment. Particularly it should be mentioned the WIRIS feature, which contributes to Moodle platform with the possibility of having a formula editor, a calculator and a program to develop questionnaires, which can offer different questions to each student.

With the arrival of the changes to get a common European education, the students must change their minds to construct their own learning with the help of computers.

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