

Particle Swarm Optimization with Adaptive Mutation in Local Best of Particles

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Abstract. Particle Swarm Optimization (PSO) has shown its good search ability in many optimization problems. But PSO easily gets trapped into local optima while dealing with complex problems due to lacks in diversity. In this work, we proposed an improved PSO, namely PSO-APMLB, in which adaptive polynomial mutation strategy is employed in local best of particles to introduce diversity in the swarm space. In the first version of this method (PSO-APMLB1), each local best is perturbed in the current search space instead of entire search space. In the second version of this method (PSO-APMLB2), each local best is perturbed in terms of the entire search space. We also proposed another local best mutation method, namely, PSO-AMLB, in which mutation size is controlled dynamically in terms of current search space. In this work, we carried out our experiments on 8 well-known benchmark functions. Finally the results are compared with PSO. From the experimental results, it is found that the proposed algorithms performed better than PSO.

Keywords: Particle swarm optimization; adaptive polynomial mutation; adaptive mutation

1. Introduction

Particle swarm optimization [1] is a population based global search technique having a stochastic nature. It has shown its good search ability in many optimization problems with faster convergence speed. However, due to lack of diversity in population, PSO easily trapped into local optima while dealing with complex problems. Different mutation strategies like Cauchy Mutation [3], Gaussian Mutation [2], Power Mutation [7], Adaptive Mutation [6] is introduced into PSO for solving local optima problem. Changhe Li et al. [4] introduced fast particle swarm optimization with cauchy mutation and natural selection strategy. Andrew Stacey et al. [5] used mutation in PSO with probability $1/d$, where d is the dimension of particles. JIAOWei et al. [9] proposed elite particle swarm optimization with mutation. Xiaoling Wu et al.[7] introduced power mutation into PSO (PMPSO). Coello [13] presented a hybrid PSO algorithm that incorporates a non-uniform mutation operator similar to the one used in evolutionary algorithms. Pant [12] used an adaptive mutation operator in PSO. Higashi [2] proposed a PSO algorithm with Gaussian mutation (PSO-GM). A new adaptive mutation by dynamically adjusting the mutation size in terms of current search space is proposed in [6]. A.J. Nebro et al. [10] applied a polynomial mutation to the 15 percentage of the particles. Tapas Si et al. [17] introduced adaptive polynomial mutation in global best particles in PSO (PSO-APM).

In this work, our objective is to use adaptive polynomial mutation and adaptive mutation in local best solution in PSO to solve local optima problem and to analysis the performance and effectiveness of adaptive polynomial mutation. A comparative study is also made with PSO having linearly decreasing inertia weight.

2. Particle Swarm Optimization(PSO)

PSO is an optimization algorithm by simulating the behaviour of fish schooling and bird's flocking. PSO algorithms use a population of individual called particles. Each particle has its own position and velocity to move around the search space. Particles have memory and each particle keep track of previous best position and corresponding fitness. The previous best value is called as pbest. Thus pbest is related only to a

particular particle. It also has another value called gbest, which is the best value of all the particles pbest in the swarm. The basic concept of PSO technique lies in accelerating each particle towards its pbest and the locations at each time step.

$$S_{ij}(t+1) = w * v_{ij}(t) + c_1 r_1 (x_{ij}^{pbest}(t) - x_{ij}(t)) + (x_j^{gbest}(t) - x_{ij}(t)) \quad (1)$$

$$x_{ij}(t+1) = v_{ij}(t) + x_{ij}(t) \quad (2)$$

c_1 and c_2 are personal and social cognizance of a particles respectively and r_1 and r_2 are two uniformly distributed random numbers in the interval (0,1).The inertia weight w in (7) was introduced by Shi and Eberhart [14]. They proposed a w linearly decreasing with the iterative generation as

$$w = w_{\max} - (w_{\max} - w_{\min}) \frac{g}{G} \quad (3)$$

where g is the generation index representing the current number of evolutionary generation and G is the predefined maximum number of generations.

3. Proposed Algorithms

Polynomial mutation is based on polynomial probability distribution.

$$X_i = (t+1) = x_j(t) + (x_j^u - x_j^l) \times \delta \quad (4)$$

Where x_j^u is the upper bound and x_j^l is the lower bound of x_j . The parameter $\delta \in [-1, 1]$ is calculated from the polynomial probability distribution.

$$p(\delta) = 0.5 \times (\eta_m + 1) (1 - |\delta|)^{\eta_m} \quad (5)$$

η_m is the polynomial distribution index.

$$\delta = \begin{cases} (2r)^{1/(\eta_m+1)} - 1, r < 0.5 \\ 1 - 2[(1-r)]^{1/(\eta_m+1)}, r \geq 0.5 \end{cases} \quad (6)$$

$$\eta_m = 100 + t \quad (7)$$

In the proposed algorithm, adaptive polynomial mutation is done in local best of the particles with the mutation probability:

$$p_m = \frac{1}{d} + \frac{t}{t_{\max}} \left(1 - \frac{1}{d} \right) \quad (8)$$

where d is the dimension of the problem, t is the current iteration number and t_{\max} is the maximum iteration number.

$$mx_j^{pbest} = x_j^{pbest} + (b_j(t) - a_j(t)) * \delta \quad (9)$$

Where δ is calculated using Eq.(6) and η_m is calculated using Eq.(7)

$$a_j(t) = \min(x_{ij}(t)) \quad b(t) = \max(x_{ij}(t)) \quad (10)$$

In this work, mutation is employed in PSO with decreasing inertia weight. The main feature of this method is that each local best of particles is muted with the increasing probability P_m by dynamic mutation size in terms of current search space and decreasing polynomial index with increasing iteration.

3.1. PSO-APMLB1 Algorithm

1. Initialize the particle with uniform distributed random numbers.
2. Update velocity and position vectors using Eq.(1) and Eq.(2).
3. Calculate fitness value, $f_{fitness}(i)$
4. if ($fitness(i) \leq Pbest(i)$) then set $Pbest(i) = fitness(i)$. if ($Pbest(i) \leq gbest$) then set $gbest = Pbest(i)$.
5. if ($U(0,1) \leq P_m$) then Apply mutation to $Pbest$ using Eq.(8)
6. Evaluate the fitness value, $P_{fitness}(i)$

7. if (fitness(i) <= Pbest(i)) then set Pbest(i) = fitness(i).
8. if (Pbest (i) <= gbest) then set gbest = Pbest(i).
9. Repeat the loop until maximum iteration or maximum number of function evaluation is reached.

In PSO-APMLB2 algorithm, mutation is done by using the following equation

$$mx_j^{pbest} = x_j^{pbest} + (x_j^u - x_j^i) * \delta \quad (11)$$

In PSO-AMLB algorithm, mutation is done by using the following equation

$$mx_j^{pbest} = x_j^{pbest} + (b_j(t) - a_j(t)) * rand0 \quad (12)$$

4. Experimental Studies

There are 8 different global optimization problems, including 4 uni-modal functions (f1- f4) and 4 multi-modals functions (f5- f8), are chosen in our experimental studies. These functions were used in an early study by X. Yao et al. [18]. All functions are used in this work to be minimized. The description of these benchmark functions and their global optima are given in Table 1.

In this experiment, the parameters are set as following: population size is set to 20, D=30, 10^5 number of function evaluations are allowed for each problem. Initial distribution index for polynomial mutation = 100. $c_1 = c_2 = 1.49445$, $w_{max}=0.9$ and $w_{min}=0.4$

The obtained results are presented in Tables 2 & 3. ‘‘Mean Best’’ is the mean of best solutions and standard deviation of the best solution of 30 runs for each test problem are reported in Table 2 & 3. In Fig. 1, convergence graph of PSO-APLB1 for function f5 is given. From the Table 2& 3, it can be easily said that our proposed algorithms PSO-APMLB and PSO-AMLB performed better than Adaptive PSO. PSO-AMLB performed better for functions f_1, f_3, f_5 and f_7 . PSO-APMLB2 performed better for function f_2 . PSO-APMLB1 performed better for functions f_4, f_6 and f_8 . For a comparison of overall performance, PSO-APMLB1 perform better for all functions except the highly multi-modal function f_5 comparative to PSO-AMLB.

Table 1: The 8 benchmark functions used in our experiments, where D is the dimension of the functions, f_{min} is the minimum values of the functions, and $S \subseteq R^D$ in the search space

Test Function	S	f_{min}
$f_1(x) = \sum_{i=1}^D x_i^2$	[-100,100]	0
$f_2(x) = \sum_{i=1}^D \left(\sum_{j=1}^i x_j \right)^2$	[-100,100]	0
$f_3(x) = \sum_{i=1}^D \left(10^6 \right)^{\frac{i-1}{D-1}} x_i^2$	[-100,100]	0
$f_4(x) = \sum_{i=1}^{D-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	[-100,100]	0
$f_5(x) = \sum_{i=1}^D -x_i * \sin(\sqrt{ x_i })$	[-500,500]	-12569.5
$f_6(x) = \sum_{i=1}^D \frac{x_i}{4000} - \prod_i \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	[-600,600]	0
$f_7(x) = -20 * \exp(-0.2 * \sqrt{\frac{1}{D} \sum_{i=1}^D x_i^2}) - \exp\left(\frac{1}{D} \sum_{i=1}^D \cos(2\pi x_i)\right) + 20 + e$	[-32,32]	0
$f_8(x) = \sum_{i=1}^D [x_i - 10 \cos(2\pi x_i)] + 10$	[-5.12,5.12]	0

5. Conclusion

In this work, we proposed adaptive polynomial mutation and adaptive mutation in local best of particles in particle swarm op-best of particles to introduce diversity in the population and to solve local optima problem. PSO with adaptive polynomial mutation and adaptive mutation in local best of particles produces the better results than PSO. But PSO-APMLB produces poor performance for multi-modal function f_5 and f_8

with respect to the global optimums of the aforementioned functions. Our future works will be directed towards solving local minima problem in complex multi-modal function optimization.

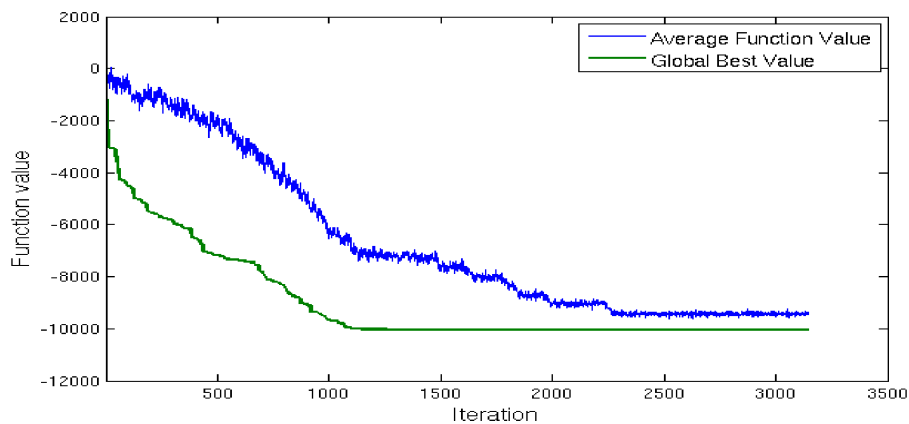


Fig. 1: Convergence graph of PSO for f5

Table 2: Function values achieved by PSO and PSO-APMLB1

Problem	PSO with decreasing ω		PSO-APMLB1	
	Mean	Std. Dev.	Mean	Std. Dev.
f_1	2.58e-11	1.37e-10	1.52e-13	5.34e-013
f_2	8.54	29.5	3.84	5.21
f_3	1.31e-10	2.45e-10	5.41e-10	2.16e-9
f_4	51.7	32.3	21.9	4.31
f_5	-9221.46	530.14	-10249.32	759.474
f_6	1.81e-2	2.41e-2	1.54e-2	1.50e-2
f_7	0.566	0.722	0.464	0.753
f_8	51.2	15.4	49.35	14

Table 3: Function values achieved by PSO-APMLB2 and PSO-APML

Problem	PSO-APMLB2		PSO-APML	
	Mean	Std. Dev.	Mean	Std. Dev.
f_1	2.64E-013	1.31E-012	3.27e-15	1.68e-14
f_2	0.169	0.154	6.68	12.2
f_3	3.36e-9	1.56e-8	1.86e-14	5.93e-014
f_4	26.5	10.6	51.2	39.2
f_5	-9008.2570	622.96	-12301.68	250.9641
f_6	2.54e-2	2.05e-2	1.76e-002	1.77e-002
f_7	0.746	0.835	0.0621	0.236
f_8	51.4	14.5	91.0	24.1

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