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Scaling and Translation Resistant Tchebichef Moments in Image Watermarking

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Abstract. The use of moments in Image Watermarking has gained wide attention recently primarily owing to its ability to convey image shapes and contours. These moments are however susceptible to common geometric attacks like Scaling and Translation. Our aim in this paper is to develop Image Moments (Tchebichef Moments) that are resistant to Scaling and Translation. Previous researchers have introduced the continuous orthogonal Legendre moments which enable easier image reconstruction compared to the geometric moments. However, it needs approximation from continuous to Digital domain and also the variable used in the Legendre polynomial need Scaling between [0 1]. The orthogonal Tchebichef polynomials are more accurate and need no Scaling of the variable. We propose here a novel method to generate Invariant Tchebichef Moments by placing constraints on the Tchebichef polynomial. Some attempts have been made earlier by other researchers to develop Invariant Tchebichef moments primarily for pattern recognition but they have not emphasized on image reconstruction. Thus these methods can't be employed in image watermarking directly. Our method proposes a constraint on the Tchebichef polynomial variable based on the image centroid approach for Translation al invariance. We use one of the terms in Hu's Invariant s approach to develop a scaling invariance too. To the best of our knowledge this approach to generate Scaling and Translation resistant Tchebichef moments for image watermarking has not been attempted before. The results on both binary and grayscale images are encouraging wherein we have been able to achieve a NCC of above 0.9 in all the cases validating our novel approach.

Keywords: tchebichef moments, invariant moments, digital watermarking, scaling and translation resistant, image processing.

1. Introduction

Image moments have recently gained wide usage in Image Processing. Since Hu [1] introduced the seven moment Invariants, the usage of image moments has been widely reported in pattern recognition, face recognition and contour matching. Teague [2] further introduced the concept of image moments using different polynomials. The geometric moments were not orthogonal and hence image reconstruction from geometric moments was a complex procedure involving large redundant data. Further the dynamic range of the Geometric Moments increases for large image sizes inducing instabilities. Using an orthogonal set of kernel function to generate moments has obvious advantages. The orthogonal kernel function ensures that minimum information redundancy occurs as also a one to one correspondence between the kernel set and the moments. This greatly reduces complexity in image reconstruction.

Legendre moments have been introduced earlier by Teague and also Mukundan [3] using a set of orthogonal Legendre polynomials. However the Legendre polynomials are orthogonal in the continuous domain and hence the procedure involves approximation from continuous domain to discrete domain. Further, since the orthogonality of the polynomials is defined over the range [0 1], the variable in the Legendre polynomial needs Scaling. This essentially implies that the image of size N x N needs to be scaled

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between [0 1] to achieve orthogonality of the Legendre polynomials. These two factors introduce errors in the Legendre Moment calculation. Liao and Pawlak [4] propose more accurate approximation formula for computing the 2D Legendre moments of a Digital image when an analog original image is digitized. They use an alternative extended Simpson's rule (AESR) to numerically calculate a double integral function for a higher order of Legendre moments in each pixel. Khalid Hosny [5] proposed a new method to improve the approximation of Legendre polynomials while converting from continuous domain to discrete domain to get more accurate Legendre Moments. However in spite of all techniques, error is unavoidable primarily because of the fact that Legendre polynomials are orthogonal in the continuous domain. Mukundan [6] proposed the concept of a Discrete Tchebichef polynomial orthogonal in the Digital domain itself.

In this paper using the Tchebichef polynomials we propose a new method to generate Scaling and Translation resistant moments and demonstrate embedding/extraction of a watermark. In section II we have an overview of the Tchebichef moments, in section III and IV we introduce a methodology to achieve Translation invariance and Scaling invariance. In section V we explain the watermarking process followed by the conclusion in section VI.

2. Introduction to Tchebichef Moments

2.1. Orthogonal polynomials.

Consider a discrete orthogonal system $\{f_n(i)\}$, where 'i' lies between 'a' and 'b' including both limits. The orthogonality property, as shown in [6], in the above domain can then be written as :

$$\sum_{i=a}^{i=b} [w(i) f_m(i) f_n(i) = \rho(n,a,b)\delta_{mn} \tag{1}$$

Here in this case 'w(i)' is the weighting function (also called the jump function) and $\rho(.)$ is the squared norm. Given an *N*x*N* image, we first seek discrete orthogonal polynomials $\{t_n(x)\}$ that satisfy the condition:

$$\sum_{x=0}^{N-1} \sum_{y=0}^{N-1} t_m(x) \quad t_n(y) = \rho(n,N)\delta_{mn}$$
$$where\ m,n = 0,1 \ldots N-1 \tag{2}$$

Where $\rho(n,N)$ is the squared norm of the polynomial set t_n.

2.2. Tchebichef polynomials

The classical discrete Tchebichef polynomials satisfy the property of orthogonality with

$$\rho(n,N) = \frac{N(N^2-1)(N^2-2^2)\ldots\ldots(N^2-n^2)}{2n+1}$$
$$n = 0,1,2 \ldots. N-1 \tag{3}$$

It also follows a recursive formula:

$$(n+1)t_{n+1}(x) - (2n+1)(2x-N+1)t_n(x) + n(N^2-n^2)t_{n-1}(x) = 0\,,$$

Where n = 1…..N-1 (4)

The first two Tchebichef polynomials are $t_0(x)=0$ and $t_1(x)= (2x+1-N)/N$. The Tchebichef polynomials as defined above together with their norms, become numerically unstable for large values of N. It can be easily verified that the magnitudes of t_n grow at the rate of N^n. Mukundan[6] proposed to scale the Tchebichef polynomials $t_n(x)$ by a factor N^{-n} to make them suitable for image analysis. We then define the Tchebichef moments as follows:

$$T_{pq} = \frac{1}{\rho(p,N)\,\rho(q,N)} \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} t_p(x) t_q(y) f(x,y)$$

Where p, q=0, 1….*N - 1* (5)
and

$$\rho(p,N) = \frac{N\left(1-\frac{1^2}{N^2}\right)\dots\left(1-\frac{p^2}{N^2}\right)}{2p+1}$$
$$p = 0,1,2 \dots . N-1$$

(6)

We have used the following recursive formula for calculating the Tchebichef polynomials:

$$t_p(x) = \frac{(2p-1)t_1(x)t_{p-1}(x) - (p-1)\left(1-\frac{(p-1)^2}{N^2}\right)t_{p-2}(x)}{p}$$

where p > 1 (7)

The classical Tchebichef polynomials modified as above do not lead to numerical overflows for large images. Both the polynomials and the associated moments do not show large variation in the dynamic range of values, unlike the case of geometric moments. Since the Tchebichef polynomials are exactly orthogonal in the discrete coordinate space of the image, we can reconstruct the image from the following theorem:

$$f(x,y) = \sum_{p=0}^{N-1}\sum_{q=0}^{N-1} T_{pq}\, t_p(x) t_q(y)$$

(8)

3. Proposed Translation al Invariance

3.1. Translation al invariance in geometric moments

Several authors have proposed techniques to generate Translation al invariance [7],[8],[9] but the image reconstruction has not been propounded successfully. We will also explain the reason for inaccuracies in image reconstruction if we use standard image reconstruction formula (8). We first find the geometric moments of the given image:

$$m_{pq} = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} x^p\ y^q\ f(x,y)$$

(9)

We find the image centroid, $x_0 = (m_{10}/m_{00})$ and $y_0 = (m_{01}/m_{00})$. Translation al Invariant Geometric moments are achieved by the following:

$$\mu_{nm} = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} (x-x_0)^n\ (y-y_0)^m f(x,y)$$

(10)

3.2. Proposed translation al invariance in tchebichef moments

We can now generate the Translation Invariant Tchebichef moments using:

$$T_{pq} = \frac{1}{\rho(p,N)\,\rho(q,N)} \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} t_p\,(x-x_0) t_q(y-y_0) f(x,y)$$

where p, q=0,1….*N - 1*

(11)

Several authors state that the Translation al Invariant moments so generated can be used to reconstruct the image using (8). This however is not feasible because for positive values of x_0 and y_0 (the image translated to the right and/or down) the range of the value of the variable in the Tchebichef polynomial changes and also becomes negative. When the range changes from [0 N-1] to some other range (in this case it becomes negative as well) the kernel function loses its orthogonality and image reconstruction is not possible.

To maintain the orthogonality of the kernel function we need to achieve two important aims,

- The range of the translated polynomial variable should be between [0, N-1] and cyclic.
- All the values of the translated polynomial variable should be integers.

To simply put, the term $(x\text{-}x_0)$ should lie between [0 N-1] and should be integer.

We propose to achieve the same in two simple steps:

The centroid is rounded off to its nearest integer value such that x_{abs}= integer(x_0) and y_{abs}= integer(y_0). Now our next aim is to ensure that the kernel function is orthogonal such that $(x\text{-}x_0)$ and $(y\text{-}y_0)$ are always between [0 N-1].

For all values of ‘x’ where ‘$x\text{-}x_{abs}<0$’ we propose to translate the variable by a range of ‘N’. The new Tchebichef polynomial can then be expressed as:

$$t'_p(x - x_{abs}) = x - x_{abs} \quad , \quad if\ \ x - x_{abs}\ is\ non\ negative$$
$$= x - x_{abs} + N, x - x_{abs}\ is\ negative \tag{12}$$

These two modifications ensure that the kernel function remains orthogonal by achieving the two aims. The Translation Invariant Tchebichef moments can now be expressed as:

$$T'_{pq} = \frac{1}{\rho(p,N)\,\rho(q,N)} \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} t'_p\,(x - x_{abs})t'_q(y - y_{abs})f(x,y)$$
$$where\ p,q=0,1....N - 1 \tag{13}$$

While the image can be reconstructed by

$$f(x,y) = \sum_{p=0}^{N-1}\sum_{q=0}^{N-1} T'_{pq}\, t'_p(x - x_{abs})\, t'_q(y - y_{abs}) \tag{14}$$

We tested this approach on a 32x32 size binary images of different letters. In this case image containing letter ‘E’ using Translation parameters of ‘x_t=2’ and ‘y_t=2’ is shown in Fig 1. The first few Translation Invariant Tchebichef moments are mentioned in Table I.

Fig. 1: a) Original Image b) Translated Image

Table. 1: Example

Order of Moment	T_{oo}	T_{10}	T_{01}	T_{21}
x_t=0, y_t=0	0.29757785467	0.1425515660809	-0.0158899923	0.3007756132756
x_t=2, y_t=2	0.29757785467	0.1425515660809	-0.0158899923	0.3007756132756

We tested this approach on standard 64x64 size grayscale images. In this case of image of a rose shown in Fig 2, we show Translation parameters ‘x_t=3’ and ‘y_t=2’. The first few Translation Invariant Tchebichef moments are mentioned in Table 2.

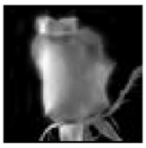
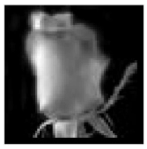
 

Fig. 2: a) Original Image b) Translated Image

Table. 2: Example

Order of Moment	T_{oo}	T_{10}	T_{01}	T_{21}
x_t=0, y_t=0	0.2217351137986	0.0564922743290	0.0114684045661	0.3713483453015
x_t=3, y_t=2	0.2217351137986	0.0564922743290	0.0114684045661	0.3713483453015

As can be seen the proposed method is suitable for binary as well as grayscale images. Further the image reconstruction using (14) is also accurate as the modified Tchebichef polynomial is an orthogonal kernel set. The reconstructed image of the Rose with first 26 moments is shown in Fig 3.

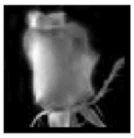


Fig. 3: Reconstructed Image 'Rose'

4. Proposed Scaling Invariance

4.1. Scaling invariance in geometric moments

The Scaling Invariant s for geometric moments have been proposed by some authors based on the Hu's Invariant s. Mukundan [10] proposed a simple way to generate fairly accurate geometric moments resistant to Scaling using the following formula.

$$\mu_{nm} = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1}(x-x_0)^n\,(y-y_0)^m f(x,y) \tag{15}$$

where μ_{nm} is the Translation al Invariant geometric moment then we can obtain the scale as well as Translation al Invariant geometric moment by :

$$\mathrm{GM_{st}} = \mu_{nm}/\ \mu_{00}^{\frac{n+m+2}{2}} \tag{16}$$

4.2. Proposed scaling invariance in tchebichef moments

We apply the same analogy to the Tchebichef moments calculated in (13) to generate fairly accurate scale and Translation Invariant Tchebichef moments.

$$T''_{pq} = \frac{T'_{pq}}{\mu_{00}^{\frac{p+q+2}{2}}}$$

Note that $T_{00}=\mu_{00}/(N^2)$

(17)

The inverse transformation formula now gets modified to

$$f(x,y) = \sum_{p=0}^{N-1}\sum_{q=0}^{N-1}(T''_{pq} * \mu_{00}{}^{(p+q+2)/2}) * t'_p(x-x_{abs}) *\ t'_q(y-y_{abs}) \tag{18}$$

Eqn. (17) gives us Tchebichef moments that are resistant to Scaling and Translation while (18) gives us the image reconstruction. We tested the approach on the same binary image containing letter 'E' as in Fig 1. In our case, the images have been scaled by integral values only. To ensure that the scaled and translated image have the same size we have done zero padding suitably. After Scaling the original image by a factor of 2, we translated the image by values of x_t=2 and y_t=2 as shown in the Fig 4.

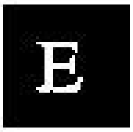
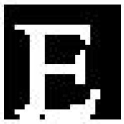



Fig. 4: a) Original Image b) Scaled and Translated Image

The scale and Translation Invariant moment values of the original image and the 'translated+scaled' image for all moment values are almost identical. The scaled and translated image which was reconstructed using the first 36 moments is shown in Fig 5.

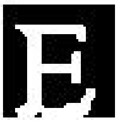


Fig. 5: Reconstructed: Scaled and Translated Image 'E'

We tested the same on the previous grayscale image of size 64x64 with Translation of x_t=4 and y_t=4 and Scaling ratio of 1:2.

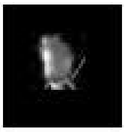



Fig. 6: a) Original Image b) Scaled and Translated Image

The scale and Translation Invariant moment values of the original image and the 'translated+scaled' image for all moment orders are almost identical.

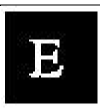
5. Watermarking Using The Scaling and Translation Tchebichef Invariant s

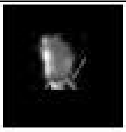
We have established invariance in Scaling and Translation of Tchebichef moments using our proposed method. We embed a 10 bit unique secret code indicating watermark information. The watermarked images are shown below. To ensure imperceptibility of the embedded watermark, we ensured that the watermarked image have PSNR of a minimum 20 dB. To assess the robustness of the watermark we use the NCC in extraction of watermark. For the embedded watermark "w" and extracted watermark " w' " The NCC is given by,

$$NCC(w(i), w(i)') = \frac{\sum_i w(i)w\prime(i)}{(\sum_i (w(i)^\wedge 2))^\wedge 0.5 * (\sum_i (w\prime(i)^\wedge 2))^\wedge 0.5} \quad (19)$$

We tested the NCC for images of varied sizes. The NCC for a watermarked 32x32 binary image and a 64x64 grayscale image for various cases are shown in Table 3.

Table. 3: Example

Watermarked Image	NCC without any attacks	NCC after Translation attacks	NCC after Scaling and Translation attacks
 32x32 Binary	0.99	0.989	0.96

 64x64 Grayscale	0.98	0.976	0.93

6. Conclusion

We have now demonstrated a new approach to generate Scaling and Translation resistant Tchebichef Moments. We have also emphasized on Image Reconstruction using Tchebichef Moments and in order to achieve the same we have added a new constraint on the variable in the Tchebichef polynomial to maintain orthogonality of the kernel function. We then employ embedding and extraction of watermark to check our approach. The results are, as mentioned, fairly accurate. Our approach has been tested on both grayscale and binary images of varied sizes upto 128x128. Although Image Watermarking based on any Moment based technique is computationally intensive this disadvantage is offset by its ability to offer robustness against geometric attacks. Further research will focus on improving the SNR value more than 35dB and achieving more robustness towards other intentional attacks.

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