2012 International Conference on Innovation and Information Management (ICIIM 2012)
IPCSIT vol. 36 (2012) © (2012) IACSIT Press, Singapore

The Cost Problem of Technological Research - A path Integral Approach

Yue Fang [1 +]

[1] Georgia Institute of Technology

Abstract. This paper explores the cost problem of technological research, within a path integral approach which shows three characteristics of technology research. Firstly, most technological progress has the characteristics of path dependence. Second, the technological research and development has the characteristics of "constructive destruction", that is, some technological characteristic improves at the expense of temporarily decreasing other technological characteristics. Thirdly, although path dependence is the general case in technological research, there are some exceptions that some technologies have the characteristics of the path independence, i.e. the research cost has nothing to do with the path adopted.The paper suggested an agreeable model that could embrace three of the characteristics above. Under the assumptions of the model, this approach can deal with the optimal path of technological research, which minimizes the total cost. This is meaningful for providing a new method to guide the research activity.

Keywords: Cost of technological research, Path integral, Optimal path of technological research

1. Background of the Model

There are plenty of technological research cases in reality. In the research and development of a technology, the cost may be one of the most important variables because both the costs and the benefits that technology brings about determine the research and development decision of an enterprise. This importance drives us to find a model which describes the rules of the research and development costs. In the previous economic literatures, no models describing the technological research costs have ever existed. In this paper, I will try to identify some general rules of technological research and development activities and suggest a satisfactory mathematical model, so as to properly describe the technological research and development activities in a general framework.

Broadly speaking, the function of cost spent in the technological research should be a function of the initial condition and target technological feature, that is, the cost of technological advancement is related to initial state and final state of technological research. Generally speaking, the more a firm wants to improve technological property, the more cost will it generate. For example, improving the capacity of hard disk to 500G bytes will cost more than just improving to 100G. Therefore, the function has the following mathematical form:

$$C = f[(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)]$$

where (x_i, y_i) indicate n technological characteristics parameter vectors of a technology, such as power consumption, running speed etc. in CPU; (x_1, y_1) indicates the initial technological index before starting the research and development; while (x_n, y_n) indicates the index of the research and development target. The technological characteristics of a research target determine the costs required in researches. The purpose of this study is to find a mathematical model consistent with the general functional form mentioned above. Besides the general rule above, I believe that such a satisfactory model describing the cost of technological research should be able to reflect the following three features of technological research: it should reflect the

+ Corresponding author. Tel.: +(01)404-232-5424
E-mail address: :yfang39@gatech.edu

"path property" of technological research and be compatible with path dependence/independence cases. And it should reflect the characteristic of "constructive destruction", that is, some technological characteristic improves at the expense of temporarily decreasing other technological characteristics.

First of all, it can be observed from daily experience that technological progress has the characteristics of path dependence, that is, different sequence of research often leads to the different costs and difficulties of research and development. Take CPU research and development in IT industry for an example, CPU has several technological properties, like power consumption, main frequency, size, etc... To be simple, let assume there are only two technological properties, power consumption and main frequency. In this case the result obtained by first reducing energy consumption and then improving performance may be different from that obtained by reducing energy consumption and improving performance at the same time. As an extreme example, suppose Intel is intended to enhance the main frequency of the Pentium CPU from 133MHz to 1000MHz, while reducing CPU power consumption from 40W to 15W. What the Intel chooses is a strategy of equally enhancing two technological characteristics, that is, the main frequency is firstly slowly raised to 500MHz while power consumption is decreased to 25W, and then the main frequency and power consumption indices are raised to the target level in similar path. If Intel corporation adopts the research strategy of improving one technological characteristic (for example, the main frequency is increased at one-time to 1000MHz, and then power consumption is decreased to the target level), the costs may be different from those of balanced development strategy of two technological characteristics because it is an extremely difficult process for the main frequency to be increased to a very high level based on the CPU design and process at 133MHz. In other words, in a sense, technological research process usually is path dependent. And the cost of technological research depends on the upgrading of the order of a variety of technological attributes, namely, research and development strategy. This characteristic implies that, there is an "optimal" path in technological research which minimizes the total cost. Thus, an agreeable model should be able to enable us to find such an optimal path mathematically.

Secondly, the technological research and development has the characteristics of "constructive destruction" that is, a certain technological characteristic makes a progress at the expense of temporarily decreasing another technological characteristic. Technology often evolves in a fluctuated way. For example, during the course of CPU research and development, power consumption needs to be temporarily increased in order to increase running speed due to the constraints of processing techniques and electrical properties. However, all technological indices of any technology are not decreased at the same time; otherwise, it runs counter to the purpose of technological research. Therefore, an agreeable model of technological research should tolerant temporary backward of some technological properties in the path of technological advancement. This backward can be expressed mathematically as the partial derivative of path equation with respect to some variables is negative in a value range.

Thirdly, some technologies have the characteristics of the path independence, i.e. the research cost has nothing to do with the path adopted. In reality, an extreme example is the manual razor. Among many technological parameters of the manual razor, some parameters (such as grip comfort level and sharpness) can be studied and developed in an almost completely independent way: they can be viewed as a combination of two independent parts, the researches of their technological properties are therefore unrelated at all. In this case, the research on two techniques has nothing to do with the path, so the costs spent on research make no difference no matter the manufacturer chooses to enhance the sharpness and then the grip comfort level or enhance the grip comfort level and then the sharpness or simultaneously enhance them. Therefore, a satisfactory model describing the cost of technological progress should be "degenerated" so as to show the path independent features, and technological research activities in which cost of research has nothing to do with the path can be better described.

To summarize the above three characteristics, a technological research cost model describing path integral of coordinate is proposed. The model is described in detail in the following paragraphs.

2. The Path Integral Model

In order to describe the issue of cost of technological research using path integral, it is necessary to first make an introduction of some concepts concerned and its economic significance. In terms of the application of path integral in this study, two most important concepts are the field equation and the path equation, as well as the economic significance of their integrals. For convenience, I assume there are only two technological properties in one technology, x and y.

Definition 1. Field equation of technological research

The field equation of technological research reflects an inherent rule of technological research, the mathematic form of which is a vector function, and the economic meaning of which is the track of vector $\vec{v} = [P(x,y), Q(x,y)]$ of the cost consumed if two technological characteristics are increased by Δx and Δy separately on the basis of a specific technological level (x_i, y_i). $P(x,y)\Delta x$ reflects how much cost is needed to increase x by a small margin. $P(x,y)$ can be decomposed as $P(x,y) = \phi(x) + \varphi(y) + \gamma(x,y)$, in this $\phi(x)$ denotes the "pure" effect of x on itself, $\varphi(y)$ denotes the "pure" effect of y on x, and $\gamma(x,y)$ denotes the "cross" effect. In practice, the field equation can be estimated with the already acquired data about technological research by means of fitting.

Assumption 1. Due to the non-negative constraint to the cost of technological research, it is prescribed that each component in $[P(x,y), Q(x,y)]$ should be non-negative for any x$>$0 and y$>$0.

Assumption 2. $P(x,y)$ and $Q(x,y)$ are continuous with partial derivatives existing assumed x$>$0 and y$>$0. This means in the process of technology research, all technological properties cannot move back together.

Definition 2. The path equation of technological research.

The path equation of technological research represents the research track of a specific technology, which is a non-closed curve $y = y(x)$ from the initial technological characteristic point (x_0, y_0) along (x_i, y_i) to the research target point (x_n, y_n), i.e. $y_i = y(x_i)$ for all i.

Assumption 3. The partial derivatives of the path equation of technological research with respect to both x and y are not negative. Its significance lies in that the scientific research in practice may result in one technological characteristic improves while another declines, but will not cause all technological characteristics decline.

Based on the above definition, a path integral model can be built. For one technology, its field equation under investigation is given. Assumed that the path which the research work follows satisfies equation $y(x)$, the starting point of research is A, the terminal is B, and the research path is represented by the curve $\widehat{AB}$. Point A_i is used to segment curve $\widehat{AB}$, so that A_iA_{i+1} can be as small as possible, where the coordinate of A_i is (x_i, y_i). A point M is taken from A_iA_{i+1}, and the value of the field function on M is $P(\xi_i, \eta_i)$ and $Q(\xi_i, \eta_i)$. Thus, the cost for the slight increase from technological characteristic (x_i, y_i) to (x_{i+1}, y_{i+1}) is:

$\Delta C = P(\xi_i, \eta_i)\Delta x_i + Q(\xi_i, \eta_i)\Delta y_i (1)$

where

$\Delta x_i = x_{i+1} - x_i$

$\Delta y_i = y_{i+1} - y_i$

In the equation (1), $P(\xi_i, \eta_i)\Delta x_i$ represents the cost for improving the technological characteristic x, and $Q(\xi_i, \eta_i)\Delta y_i$ represents the cost for improving the technological characteristic y. With the limits of both sides calculated, the cost for promoting one technology from (x_0, y_0) to (x_1, y_1) along the research and development path is:

$$C = \int_{AB} P(x,y)dx + \int_{AB} Q(x,y)dy$$

In this equation, $\int_{AB} P(x,y)dx$ denotes the cost of improving technological property x from x_0 to x_n, $\int_{AB} Q(x,y)dy$ denotes the cost of improving technological property x from y_0 to y_1. The total cost of research is the direct aggregation of the two.

3. Optimal Path of Technological Research

With the definitions above, the model can be used to deal with some problems in technological research. Theoretically, this model can be applied to the case of any n number of technological properties. However, for the convenience of explanation, I assume there is a technology N with only two technological properties x and y, and in this two dimensional case, the field equation of technological research is $[P(x,y), Q(x,y)]$. The initial technological condition is $(1,1)$, the purpose of research is to elevate technological properties to $(2,2)$.

For rational enterprises who conduct technological research development, their objectives are generally to minimize the cost of research. Under the presumption of the model, the problems can be summarized as: to select a curve between (x_0, y_0) and (x_n, y_n) so as to obtain the minimum path integral, i.e., the cost of research. Such problems belong to the problem of dynamic optimization, which can be solved with the variation method by means of establishing Euler equations. For the example above, the solution is as follows:

In this case, from the formula of path integral, the problem could be converted to another problem of solving a y(x), maximizing the function below:

$$C = \int_{y(x)} (e^x + y(x))dx + \int_{y(x)} xdy = \int_0^2 (e^x + y(x))dx + \int_0^2 xy'(x)dx$$

The initial condition of the differential equation is:

$$y(1) = 1, y(2) = 2,$$

Establish the Euler function:

$$F_x[x, y'(x)] = \frac{dF_x[x, y'(x)]}{dx} = e^x + 2y' + xy' = 0$$

Solve the equation we can get

$$y^* = \frac{-e^x}{x} + (e^2 - e + 4) + \frac{2e - e^2 - 4}{x}$$

This is the obtained optimal path of technological research.

In this kind of problems, not all the Euler functions have analytical solutions. For this kind of equation, the way of finding out the optimal path is through the methods of solving numerical solution of partial differential equations, like Euler method or Runge-Kutta method, etc...

4. Path Independence Conditions for Cost of Technological Research

The value of the path integral exhibits the feature of path independence for a specific field equation, i.e., the integrals for fixed starting points and terminals are always fixed values irrelevant to the integral path, without reference to the integral path selected. If a field equation of technological research meets the condition of path independence in research activities, then the cost of technological research, i.e. the integral along the research path, will remain unchanged without reference to the sequence of research adopted by enterprises. For a technology involving only two technological characteristics, the path independence of the cost of research satisfies the condition for the independent path integral of two variables, i.e.:

$$\frac{\partial P(x,y)}{\partial y} = \frac{\partial Q(x,y)}{\partial x}$$

If the field equation of a technology meets the above condition, then the research process of the technology is path independent. No matter what research strategy the firm adopts, the cost of research is a constant value.

There is a special form in the field equation which satisfies the condition, i.e., each component in the field equation is independent to other variables without correspondence. For the two-dimensional field equation, such independence can be expressed in mathematics as: for the field equation of research, the partial derivative of $P(x,y)$ with respect to y equals to the partial derivative of $P(x,y)$ with respect to x, and equals to 0. That is, according to the condition of integral

$$\frac{\partial P(x,y)}{\partial y} = \frac{\partial Q(x,y)}{\partial x} = 0$$

The technologies which satisfy such a mathematic form meets the third characteristic mentioned above, that is, the process of the research and development of each technological property is completely independent, and the instance mentioned above about shavers exactly illustrates such a technology. Actually, this kind of technology can be viewed as the simple combination of two “simple technologies” which are independent to each other; there is no interrelation between the two technologies. That is to say, the research condition of one “simple technology” has no influence to the research process another “simple technology”.

5. Summary

This paper has attempted to innovate by introducing the mathematic tool of path integral to explain the cost of a certain technological research. It illustrates the path dependency in technological research and the relationship between the two technological properties in the research process, and how these would influence the cost of technological research. This paper also provides a practical method to determine the best strategy of technological research, that is, to find an optimal path which minimizes the total cost.

6. References

[1] R. H. Schmidt, G. Spindler, Path Dependence, Corporate Governance and Complementarily, International Finance 2002 5:3: pp.311-333.

[2] P. Milgrom, J. Roberts The Economics of Modern Manufacturing: Technology, Strategy, and Organization, American Economic Review, 1990, 80: 511-28

[3] R.R. Nelson The Simple Economics of Basic Scientific Research, The Journal of Political

[4] Economy, 1959, 67, pp.397-306

[5] W. B. Author, Competing Technologies, Increasing Returns, and Lock in by HistoricalEvents, Economic Journal, 1989, 99, 116-131

[6] S. J. Liebowitz Path Dependence, Lock-In, and History, The Journal of Law, Economics & Organization, 1995, 11:205-226.

[7] D. C.North Institutions and Economic Development, Taiwan Economic Review, 1995,23:1～24

[8] E. Mansfield Industrial Research and Technological Innovation, Norton, W.W. &Company, ch.1-3, 1968

[9] B. Yannis, C.F. Kemerer Recent Applications of Economic Theory in InformationTechnology Research, Decision Support Systems, 1992, 12:365-386

[10] C. Freeman, The economics of technological change, Cambridge Journal of Economics,1994, 18:463-514