2012 International Conference on Computer Networks and Communication Systems (CNCS 2012)
IPCSIT vol.35(2012) © (2012) IACSIT Press, Singapore

An Approximation Algorithm for Vertex Cover Problem

Yaser Khamayseh, Wail Mardini, Anas Shatnawi
{yaser, mardini}@just.edu.jo, anasshatnawi@gmail.com
Computer Science Department, Faculty of Computer and Information Technology
Jordan University of Science and Technology, Irbid 22110 Jordan
+962-27101000

Abstract. An emerging problem in wireless sensors network is the need to increase the network life time. In a network with two power modes, this goal can be achieved by choosing the minimum number of sensors that have high power mode (long range radio) in the service area. The vertex cover assist in organizing clusters and determine the cluster head to save power and ensure that all sensors in the network are accessible by the cluster heads, The minimum (optimal) number of vertices (sensors) problem is NP-Complete. Therefore, a near-optimal solution to the problem can be obtained by an approximation algorithm that solves the problem in polynomial time. In this paper, we propose a new algorithm for minimizing the number of node that represents the wireless sensor with high transition power, thus we minimize the power consumption of the wireless sensor network and prolong the network life time.

Keywords: Sensor Networks, Vertex Cover Problem, Approximation Algorithms

1. Introduction

Wireless Sensor Networks (WSN) are an emerging communication technology that offers a rich interaction model with the environment. Sensors are equipped with data processing and communication capabilities. Sensors gather and send data to a base-station either directly or through another sensor node [15]. WSN supports nodes mobility and sensors are have limited capabilities. Such limitations enforce the need for power-efficient resource management protocols to extend the network lifetime. [15]. WSNs offer a wide range of possible applications both military and civil [13].

Maximizing the network life time is an important issue in sensor networks due to its scarce resources. Several schemes were proposed to prolong the life time, one such schemes is using minimal cover set algorithm. A vertex cover is a set of vertices V'; such that V' is a subset of V, where V is a set of vertices in an undirected graph $G = (V, E)$, such that for each edges with two vertices (u, v), either u or v or both must be a member of V'. The number of the vertices in V' represent the vertex cover size. For example if we have a graph G with vertices $V = \{A,B,C,D,E,F\}$ and edges $E = \{(A,B), (A,C), (A,E), (A,D), (C,D), (C,E), (D,F), (E,F)\}$, the graph G has a vertex cover $V' = \{A,D,E\}$ of size 3 that covers all the edges of the graph, as shown in Figure 1.

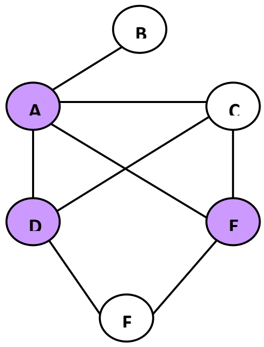

Figure 1: An undirected graph G, the shadowed nodes represent the vertex cover.

A vertex cover problem is a problem to determine the minimum (optimal) number of vertices that cover all the edges in the graph G; in other words, we want to get the minimum vertex cover size [9]. For example, the minimum cover is 2, as shown in Figure 1. The problem of determining the minimum number of vertices is classified as NP-complete problem [1, 2, 5, 7]. Therefore, we can't find an optimal vertex cover size in polynomial algorithm. For this reason an approximation algorithm is used to find an approximate solution for the vertex cover.

2. Related Works

This section presents some proposed algorithms to find vertex cover for graph G, with polynomial time complexity. The algorithm in [7] finds the vertex cover for graph with n elements and maximum degree Δ, so that the vertex cover size is no more than (n - ceiling (n/ Δ+1)), which is the best possible solution for n and Δ. The algorithm firstly defines vertex cover *Ci* for vertex *i* as all vertices except *vi*, then it is search for removable vertices in vertex cover *Ci* to decrease vertex cover size. In [4], the authors focus on the communication issues by assuming the wireless sensor network consist of two types of sensor devices: coverage sensors and communicating sensors. Coverage sensors are responsible for monitoring (sensing) the surrounding area, periodically collecting the data from surrounding area and combine it in form of data packets, then forward this data packet along with data packets from other sensors to the communication sensors. In order to reach the optimal or near-optimal solution, the genetic algorithm uses the population of chromosomes to find a better candidate solution for the next generation (step) by applying genetic functions such as mutation and crossover.

In [3], sensors have been designed in two power modes, which are low power mode (short range radio) and high power mode (long range radio), sensors can adjust its power mode, the goal is to minimize total number of nodes that have high power mode by a heuristic algorithm that can find the near optimal solution. This solution identifies the cluster head to assign it's mode to high power mode and other nodes in the same cluster to low power. The author in [15] define the dual power assignment problem as the problem of assigning the high power level to the minimum number of nodes, same as the author in [3], but in this model the author provide a more efficient approximation algorithm.

3. Proposed Scheme

In this section we present the proposed algorithm to obtain near optimal solution by finding minimum number of sensor nodes with high power transmission. The proposed model solves the same problem as in [3, 15], by finding the minimum number of nodes that assign the high power mode, the algorithm produce the set of nodes, which has high power mode, based on the nodes that have maximum degree (node connect max number of component), then the algorithm find the removable edges, where the graph is still connected, and assigning the low power mode to it's nodes if the node did not connect it's component with any other component lonely, the complexity of proposed model is *O(n^2),* the algorithm steps are shown in the following procedure.

1. Divide the network area into several components by assigning each node mode to low power transmission.
2. Build virtual graph *G* based on the resulting components without edges.
3. Let *M=0*;
4. While *G* is not Connected

- get node *N*, which has maximum degree;
- add edge between *N1* and node *N2*, where *N2* is belong to other component and *N1*, *N2* can reach each other in the high power mode and add *N1*, *N2* to *M*;

5. For each edge *e* in *E* (check if there is removable edges):

- *E=E-e*;
- If *G* is still connected
- If there is a node *N* correspond to *e* has zero degree, set *N* to low power mode and remove N from *M*.
- If G is Not connected *E=E+ e*.

6. Return size of *M*;

4. Experimental Results

Simulation experiments were conducted to evaluate the performance of the proposed scheme. The performance was compared against two other techniques: Optimal and DPMP [3]. The network area is a two dimensional space with x and y coordination. The node of the wireless sensor network is distributed randomly in the area. Various scenarios with different number of node (30, 50, and 100) were examined. Network area size was fixed at (1000*1000), and with different ratios (20%, 50%, and 80%). Figure 2 shows the results of 30 nodes, Figure 3 shows the results of 50 nodes, and Figure 4 shows the results of 100 nodes.

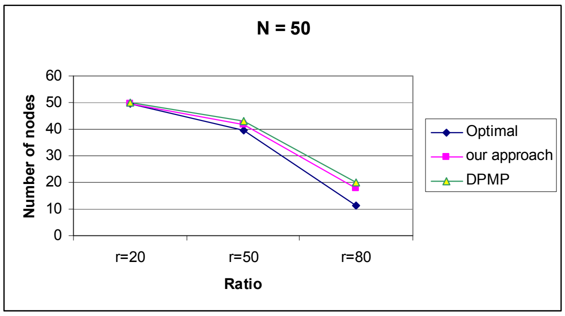

Figure 2: Various ratio percentages with 30 nodes.

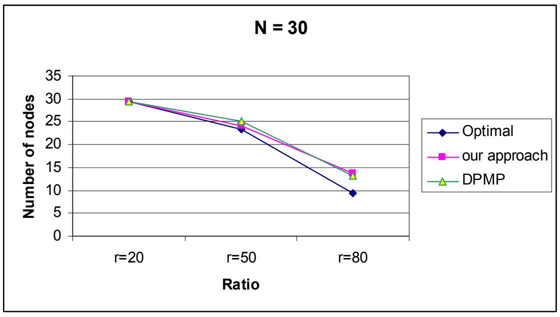

Figure 3: Various ratio percentages with 50 nodes.

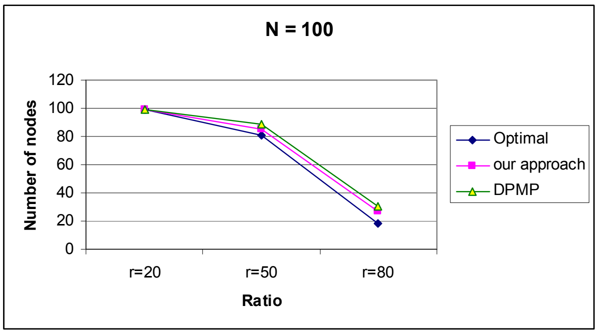

Figure 4: Various ratio percentages with 100 nodes.

The obtained results show that the proposed scheme achieves slightly better results than the DPMP scheme. As expected, the optimal solution achieves the best results.

5. Conclusion

Network life time and energy consumption are crucial issues in wireless sensor networks. In this paper we presented the vertex cover problem as well as an approximation algorithm that find near optimal solution for choosing near optimal number of cluster heads in wireless sensor networks to maximize the network life time. This paper introduced an enhanced algorithm for the vertex cover problem that can find more optimal solution in term of the set of the number that represent the vertex cover. The proposed algorithms adopt the vertex cover technique to enhance the live-time of the wireless sensor network by selecting minimum number of nodes that reduce power consumption. Finally, simulation experiments were conducted to evaluate the performance of the proposed scheme.

6. References

[1] Thomas J. Cormen, Charles E. Leiserson, Ronald L. Rivert, and Clifford Stein, “Introduction to algorithms”, third

edition, 2009.

[2] Stephen Cook, “The Complexity of Theorem Providing Procedures”, ACM, 1971.

[3] Yanxia Rong, Hongsik Choi, and Hyeong-Ah Choi, “Dual Power Management for Network Connectivity in Wireless Sensor Networks”, Proceedings of the 18th International Parallel and Distributed Processing Symposium (IPDPS’04), IEEE, 2004.

[4] Maytham Safar, Mohammad Taha, and Sami Habib, “Modeling the Communication Problem in Wireless Sensor Networks as a Vertex Cover”, IEEE, 2007.

[5] D. S. Hochbaum, “Approximation Algorithms for NP Hard Problems”, PWS Publishing Company, Boston, MA, 1997.

[6] V. Vazirani, “Approximation algorithms”, Springer-Verlag, 2001.

[7] Matti Astrand, Patrik Floreen, Valentin Polishchuk, Joel Rybicki, Jukka Suomela, and Jara Uitto, “A Local 2-approximation Algorithm for the Vertex Cover Problem”, 23rd International Symposium on Distributed Computing (DISC), Elche, Spain, September 2009.

[8] Wendi R. Heinzelman, A. Chanrakasan, and H. Balakrishnan, “Energy-Efficient Communication Protocol for Wireless Microsensor Networks”, IEEE Proceedings of the Hawaii International Conference on System Sciences, 4 January 2000.

[9] Ashay Dharwadker, “The Vertex Cover Algorithm”, Proceedings of the Institute of Mathematics, 2006.

[10] Wendi B. Heinzelman, Anantha P. Chandrakasan, and Hari Balakrishnan, “An Application-Specific Protocol Architecture for Wireless Microsensor Networks”, IEEE Transactions on Wireless Communications, V.1, No.4, October 2002.

[11] P. Agarwal and C. Procopiuc, “Exact and Approximation Algorithms for Clustering”, Proceedings 9th Annual ACM-SIAM Symposium Discrete Algorithms, P.658-667, Jan 1999.

[12] T. Murata, and H. Ishibuchi, “Performance Evaluation of Genetic algorithms for Flowshop Scheduling Problems”, IEEE Conference Evolutionary Computation, V.2, P.812-817, June 1994.

[13] Kemal A., Mohamed Y., “A survey on Routing Protocols for Wireless Sensor Networks”, Ad Hoc Networks, volume 3, 2005.

[14] Ossama Y., and S. Fahmy, HEED: “A Hybrid, Energy-Efficient, Distributed Clustering Approach for Ad-hoc Sensor Networks”, IEEE Transactions on Mobile Computing, V.3, No.4, P.366-379, October - December 2004.

[15] Jian-Jia Chen, Tei-Wei Kuo, Hsueh-I Lu, Chuan-Yue Yang, and Ai-Chun Pang, "Dual Power Assignment for Network Connectivity in Wireless Sensor Networks", in IEEE Global Telecommunications Conference (GlobeCom), in St. Louis, Missouri, USA, Nov. 28 - Dec. 2, 2005.