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Low-energy Structure of Isotopes $^{152,\,154}$Sm

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Abstract. The $^{152,154}Sm$ isotopes are classified as deformed nuclei. The phenomenological model is presented to describe the complete low-energy structure of $^{152,154}Sm$ isotopes by taking into account the Coriolis mixing between states. The parameters fitted to the model are calculated. The energy spectra of positive-parity states which are in good agreement with the experimental data are presented. It is found that the non-adiabaticity of rotational energy bands occurred at high spin due to the Coriolis effect. Few new states are predicted.

Keywords: Nuclei; energy spectra; band; low-lying state; Coriolis effect.

1. Introduction

In this paper, we describe the complete low-energy structure of deformed nuclei by considering the rotational bands Coriolis states mixing[1]. Since $^{152,154}Sm$ nuclei are classified as deformed nuclei, these nuclei are the best candidates for the study of collective properties of low-lying states. They are quite well studied experimentally[2-4]. By (*t, p*) reaction on even *Sm* isotopes, the excitation spectrum were established below 2-3 MeV[5].

2. The Model

The basic states of the Hamiltonian include ground (gr), $\beta_n - (0^+_{\beta 1}, 0^+_{\beta 2})$, $\gamma -$ vibrational and $K^\pi = 1^+_\nu$ rotational bands. As n is the number of included $\beta -$vibrational states, so ν is the number of 1^+ collective states.

The nuclear Hamiltonian is written in the two-partition form

$$H = H_{rot}(I^2) + H^\sigma_{K,K'}(I) \tag{1}$$

$H_{rot}(I^2)$ is the rotational part and

$$H^\sigma_{K,K'}(I) = -\omega_K \delta_{K,K'} - \omega_{rot}(I)(j_x)_{K,K'}\chi(I,K)\delta_{K,K'\pm 1} \tag{2}$$

$(j_x)_{K,K'}$ is the matrix element describing the Coriolis coupling of rotational bands and $\omega_{rot}(I)$ is the angular frequency of core rotation yielded from

$$\omega_{rot}(I) = \frac{dE_{cor}(I)}{dI}$$

ω_K is the head energy of respective K^+ bands which is the lowest energy level with $I = 0$ and

$$\chi(I,0) = 1\,,\quad \chi(I,1) = \left[1 - \frac{2}{I(I+1)}\right]^{\frac{1}{2}}$$

wave function of the nuclear Hamiltonian

$$\Psi^I_\nu = \sum_K \phi^I_{\nu,K} \left| IMK \right\rangle \tag{3}$$

where $\phi^I_{\nu,K}$ represents the Coriolis mixing coefficient of basis states and

$$\left| IMK \right\rangle = \sqrt{\frac{2I+1}{16\pi^2}} \left\{ \sqrt{2}\psi^I_{gr,K} D^I_{M,0} + \sum \frac{\psi^I_{K',K}}{\sqrt{1+\delta_{K',0}}} \left[D^I_{M,K'}(\theta) b^+_{K'} + (-1)^{I+K'} D^I_{M,-K'}(\theta) b^+_{-K'} \right] \right\} \tag{4}$$

$\psi^{I}_{K',K}$ are the amplitudes of basis states mixing from the $(4+\nu)$ bands includes the ground $|0\rangle$ states bands and the single-phonon $b^{+}_{\lambda=2}|0\rangle = b^{+}_{K}|0\rangle$ with all the mentioned rotational bands before.
By solving the Schrodinger equation

$$H^{\sigma}_{K,n}\psi^{I}_{K',n} = \varepsilon^{\sigma}_{n}\psi^{I}_{K,n} \tag{5}$$

We obtained wave function and energy of states with positive parity.

Total energy of states is determined by following

$$E^{\sigma}_{n}(I) = E_{rot}(I) + \varepsilon^{\sigma}_{n}(I) \tag{6}$$

Energy of rotational core $E_{rot}(I)$ can be determined by different methods. . In this research, we used Harris parameterization of the angular momentum and energy[6].

$$E_{rot}(I) = \frac{1}{2}\Im_0\omega^2_{rot}(I) + \frac{3}{4}\Im_1\omega^4_{rot}(I) \tag{7}$$

$$\sqrt{I(I+1)} = \Im_0\omega_{rot}(I) + \Im_1\omega^3_{rot}(I) \tag{8}$$

where $\Im_0$ and $\Im_1$ are the adjustable inertial parameters of rotational core. A method of defining the even-even deformed nuclei inertial parameters using the experimental data up to $I \le 8\hbar$ for ground band is suggested in recent paper[7] and quoted in Table 1. The linear dependencies of moment of inertia for states $J_{eff}(I)$ on the square of angular frequency of rotation $\omega_{eff}(I)$ are plotted in Figure 1.

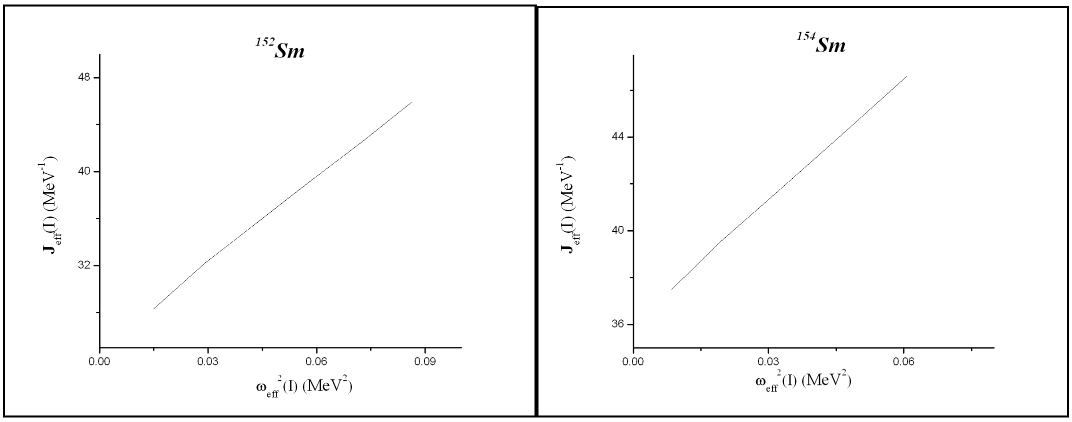

Fig. 1. The linear dependencies of $J_{eff}(I)$ on $\omega_{eff}(I)$.

Table 1. Inertial parameters of rotational core used in the calculations.

Nucleus	$\Im_0(MeV^{-1})$	$\Im_1(MeV^{-3})$
^{152}Sm	24.74	256.57
^{154}Sm	36.07	178.88

The rotational frequency of the core, $\omega_{rot}(I)$ is calculated by solving the cubic equation, which the real root is as follows:

$$\omega_{rot}(I) = \left\{\frac{\tilde{I}}{2\Im_1} + \left(\left(\frac{\tilde{I}}{2\Im_1}\right)^2 + \left(\frac{\Im_0}{3\Im_1}\right)^3\right)^{\frac{1}{2}}\right\}^{\frac{1}{3}} + \left\{\frac{\tilde{I}}{2\Im_1} - \left(\left(\frac{\tilde{I}}{2\Im_1}\right)^2 + \left(\frac{\Im_0}{3\Im_1}\right)^3\right)^{\frac{1}{2}}\right\}^{\frac{1}{3}}. \tag{9}$$

where $\tilde{I} = \sqrt{I(I+1)}$. Equation (9) gives value of $\omega_{rot}(I)$ at the given spin *I*.

3. Results And Discussions

The parameters fitted with the model are presented in Table 2. The lowest energy for ground-state and β_n bands were taken from experimental energies, since they are not affected by the Coriolis forces at spin $I = 0$: $\omega_{gr} = E^{\exp t}_{gr}(0)$ and $\omega_{\beta_n} = E^{\exp t}_{\beta_n}(0)$. The headband energies for the collective 1^{+} states in $^{152,154}Sm$ nuclei are assumed to be $\omega_1 = 3MeV$ because the $K^{\pi} = 1^{+}$ bands have not been observed experimentally for these nuclei respectively[8]. Coriolis rotational states mixing matrix elements $(j_x)_{K,K'}$ and γ – band head energies ω_γ were determined by using the least square method fitting.

In Table 2, the value of interaction matrix elements, $(j_x)_{K,K'}$ and proximity of headband energy, ω_K for certain band determined the strength of states mixing of that band with other bands[9]. Larger value of matrix elements leads to strong states mixing. For ^{152}Sm nucleus, the matrix element $(j_x)_{\beta 2,1}$ of the β_2 - and 1^+ bands is larger than other matrix elements. Unlike ^{152}Sm nucleus, the matrix elements $(j_x)_{\gamma,1}$ of the γ – and 1^+ bands is larger for ^{154}Sm .

Table 3 and 4 give the calculated Coriolis mixing coefficients, $\phi^I_{\nu,K}$ which represents mixture components of other bands in certain band. Structure of $^{152-154}Sm$ can be understood by these calculated values.

The theoretical energy spectra of positive-parity states in $^{152-154}Sm$ are presented in Figure 2 in comparison with the experimental energies[2-4]. From the figures, we see that energy difference $\Delta E(I) = E^{theor}(I) - E^{\exp}(I)$ of the β_1 - band increases with the increase in the angular momentum I . At high spin, I the nonadiabaticity of energy rotational bands occurs. Two states with same spin, I and parity, π from different bands cross in that region causes Coriolis mixing. We predict the existence of s-band states to perturb the pure β_1 - band states. Other than this mentioned obvious deviation, the theoretical positive-parity states energy spectra are in best agreement with the experimental data. Few new states and collective 1^+ bands are predicted.

Table 2. Parameters used in the calculations.

Nucleus	$\omega_{\beta 1}$	$\omega_{\beta 2}$	ω_1	ω_γ	$(j_x)_{gr,1}$	$(j_x)_{\beta 1,1}$	$(j_x)_{\beta 2,1}$	$(j_x)_{\gamma,1}$
^{152}Sm	0.685	1.083	3.0	1.0	0.742	0.821	0.864	0.855
^{154}Sm	1.099	1.203	3.0	1.380	0.345	0.403	0.408	0.417

Table 3. Structure of ^{152}Sm states.

I	gr	$0_{\beta 1}$	$0_{\beta 2}$	1^+	γ	gr	$0_{\beta 1}$	$0_{\beta 2}$	1^+	γ
	Ground-state band					β_1				
2	-0.9997	-0.0025	-0.0016	-0.0227	-0.0014	0.0032	-0.9994	-0.0064	-0.0326	-0.0065
4	-0.9993	-0.0065	-0.0043	-0.037	-0.0044	0.0086	-0.9982	-0.0169	-0.0536	-0.0199
6	-0.9987	-0.0109	-0.0073	-0.0483	-0.0076	0.0148	-0.9964	-0.0283	-0.0704	-0.0343
8	-0.9981	-0.0153	-0.0103	-0.0576	-0.0108	0.0211	-0.9942	-0.0397	-0.0845	-0.0485
10	-0.9975	-0.0197	-0.0132	-0.0657	-0.014	0.0275	-0.9917	-0.0508	-0.0968	-0.0621
12	-0.9968	-0.0239	-0.016	-0.0729	-0.017	0.0338	-0.9888	-0.0615	-0.1078	-0.0752
	γ					β_2				
2	0.0022	0.0078	-0.0302	-0.0326	-0.9990	-0.0025	-0.0075	0.9987	0.0395	-0.0316
3	-	-	-	0.0473	0.9989	-	-	-	-	-
4	-0.0069	-0.0248	0.0888	0.0630	0.9937	-0.006	-0.0182	0.9938	0.0585	-0.093
5	-	-	-	0.0698	0.9976	-	-	-	-	-
6	-0.0122	-0.0442	0.1452	0.0853	0.9846	-0.009	-0.0277	0.9855	0.0686	-0.1526
7	-	-	-	0.0866	0.9962	-	-	-	-	-
8	0.0175	0.0643	-0.1942	-0.1028	-0.9733	0.0115	0.0355	-0.9753	-0.0736	0.2049
9	-	-	-	0.1000	0.9950	-	-	-	-	-
10	-0.0226	-0.0843	0.2352	0.1168	0.9610	-0.0134	-0.0416	0.9645	0.0759	-0.2492
11	-	-	-	0.1112	0.9938	-	-	-	-	-
12	-0.0274	-0.1038	0.2691	0.1282	0.9485	-0.0149	-0.0464	0.9538	0.0766	-0.2865

Table 4. Structure of ^{154}Sm states.

I	gr	$0_{\beta 1}$	$0_{\beta 2}$	1^+	γ	gr	$0_{\beta 1}$	$0_{\beta 2}$	1^+	γ
	Ground-state band					β_1				
2	1.0	0.0002	0.0002	0.0076	0.0001	0.0003	-0.9999	-0.0037	-0.0142	-0.0011
4	-0.9999	-0.0006	-0.0005	-0.0134	-0.0004	0.0009	-0.9996	-0.0114	-0.0250	-0.0041
6	0.9998	0.0011	0.0010	0.0184	0.0009	0.0017	-0.9991	-0.0215	-0.0347	-0.0080
8	0.9997	0.0016	0.0015	0.0228	0.0013	-0.0027	0.9984	0.0329	0.0435	0.0124
10	0.9996	0.0022	0.0021	0.0266	0.0018	-0.0037	0.9975	0.0449	0.0515	0.0172
12	0.9995	0.0029	0.0026	0.0301	0.0023	-0.0048	0.9964	0.0571	0.0589	0.0221

	β_2					γ				
2	0.0003	0.0039	-0.9999	-0.0151	-0.0019	-0.0002	-0.0013	-0.0021	0.0139	0.9999
3	-	-	-	-	-	-	-	-	0.0216	0.9998
4	-0.0009	-0.0121	0.9996	0.0262	0.0067	-0.0008	-0.0047	-0.0075	0.0280	0.9996
5	-	-	-	-	-	-	-	-	0.0344	0.9994
6	-0.0016	-0.0228	0.9990	0.0358	0.0129	-0.0016	-0.009	-0.0145	0.0390	0.9991
7	-	-	-	-	-	-	-	-	0.0451	0.9990
8	0.0025	0.035	-0.9982	-0.0439	-0.0197	-0.0024	-0.0138	-0.0223	0.0480	0.9985
9	-	-	-	-	-	-	-	-	0.0543	0.9985
10	0.0034	0.048	-0.9972	-0.0508	-0.0265	-0.0032	-0.0187	-0.0303	0.0554	0.9978
11	-	-	-	-	-	-	-	-	0.0623	0.9981
12	0.0042	0.0612	-0.9959	-0.0568	-0.0333	-0.004	-0.0236	-0.0383	0.0617	0.9971

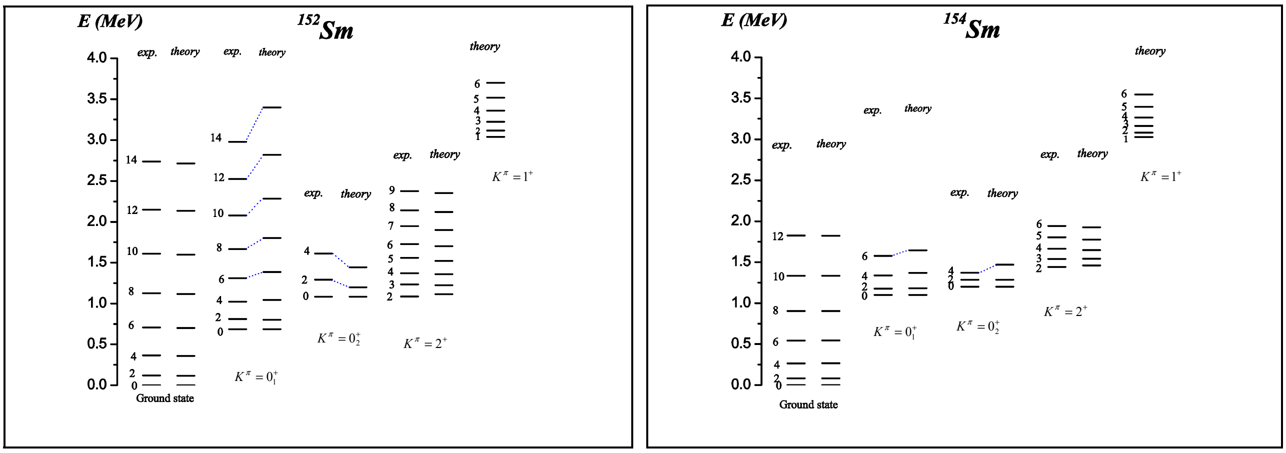

Fig 2. Energy spectrum of positive-parity states in $^{152,154}Sm$.

4. Conclusions

This work is based on the phenomenological model[1], which shows the deviation of the energy spectrum of positive parity states in even-even deformed nuclei from the adiabatic theory. Energy spectra for the isotopes $^{152-154}Sm$ were calculated and the results showed are in good agreement with the experimental data. At high spin I ,the law of $E(I) \sim I(I+1)$[10] is violated. The calculations are done by taking into account the Coriolis mixing of positive parity states. The mixing components of the states is represented by the calculated values of Coriolis mixing coefficients, $\phi^{I}_{\nu,K}$. With the agreement between the theoretical and experimental data, few states that never been observed experimentally are predicted.

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6. References

[1] Usmanov Ph.N. & Mikhailov I.N., “Effects of Non-adiabatics of collective motion in even-even deformed nuclei”, Physics of Elementary Particles and Atomic Nuclei (Fiz.Elem.Chastits At. Yadra) 28, 887-950 (1997).

[2] Begzhanov, R.B., Belinkiy V.M., Zalyubovskiy, I.I. & Kuznichenko, A.B., Handbook on Nuclear Physics Vol. 1 and 2. Tashkent: FAN. (1989).

[3] Cohen, A. A., Nuclear Data Sheets 79(1), 24-76 (1996).

[4] Reich, C.W., Nuclear Data Sheets 110, 2278-2313 (2009).

[5] Bjerregaard J.H, Hansen O., & Nathan O., “The (t,p) Reaction with Even Isotopes of Sm.”, Nucl. Phys., 86(1), 145-166 (1966).

[6] Harris, S.M., “Higher Order Corrections to the Cranking Model.”, Phys. Rev. 138, B509 - B513 (1965).

[7] Okhunov A.A., Kassim H.A., & Usmanov Ph.N., “Determination of Moment of Inertia for 162-168Hf and 164-

176Yb Deformed Nuclei.", Sains Malaysiana 40(1), 1-3 (2011).

[8] Richter A., Proceedings of the Niels Bohr Centennial Conference. Nuclear Structure.North-Holland, Amsterdam. p. 469. (1985).

[9] Okhunov A.A., "Electric Properties of Levels of 156Dy Rotational Bands.", Yadernaya Fizika 69 (4), 617-621(Physics of Atomic Nuclei 69(4), 593-597) (2006).

[10] Bohr, A. & Mottelson, B.R. Nuclear Structure. Singapore: World Scientific,(1998).