

Dynamic response of split feed multi-effect evaporators using Mathematical Modelling and Simulation

Deepak Kumar^{1,2} Vivek Kumar², V. P. Singh³⁺

¹ Department of Basic & Applied Sciences, Galgotias University, Greater Noida, U.P, (India)

^{2,3} Department of Paper Technology, IIT Roorkee, Saharanpur Campus, Saharanpur, U.P, (India)

Abstract. A wide range of mathematical models for multiple effect evaporators in process industry including paper industry are well reported in the literature but not so extensive work on the dynamic behavior of MEE system is available in the literature. The present study attempts to develop an unsteady-state model for the Multi-effect evaporator system of a paper industry to obtain the dynamic response of the system. Each effect in the process is represented by a number of variables which are related by the energy and material balance equations for the feed, product and vapor flow. In this study a generalized mathematical model is given which could be applied to any number of effects and all kinds of feeding arrangements like forward feed, backward feed, mixed feed and split feed in the MEE system with simple modifications. In the present study the split feed is used for a sextuple effect falling film evaporators. For the steady state and dynamic simulation the 'fsolve' and 'ODE45' solvers in MATLAB source code is used respectively.

Keywords: Multiple effect evaporator, split feed, fsolve, ODE45, boiling point rise, dynamic response.

1. Introduction

In later years steady state simulation of MEE system were studied by various other researchers such as [1, 2, 3, 4, 7, 8, 9, 10] by using different solution techniques. Ref [11] and [12] developed a distributed parameter model (dynamic model) of black liquor falling film evaporator, based on first principles knowledge and heat transfer processes. In the present investigation the mathematical model of sextuple effect (tubular falling film with liquor flow inside the tube) falling film evaporator system of a paper industry is developed by using unsteady state energy and material balance equations to study the dynamic behavior of the system by using split feed of the flow. In the split feed sequence the feed is equally given to 5th and 6th effect simultaneously and thick liquor coming out from 5th and 6th goes into 4th and from 4th to 3rd and so on up to the 1st effect. The effect of BPR is also considered for the present investigation. The parametric equations were solved for steady state conditions by vanishing the accumulation terms in model equations in our earlier work [5]. To study the transient behavior of the MEE system the solution of simultaneous ordinary differential equations for sextuple mixed feed evaporator is obtained by using 'ode45' solver in MATLAB source code and the steady state solution is used as the initial value.

2. Mathematical Modelling

In the present investigation model is developed for sextuple falling film evaporator system with split feed based on the work proposed by [6]. Model is extended by considering boiling point rise (BPR) with increasing concentration of solution. The Mathematical modelling is carried out for sextuple falling film evaporator system with split feed. In the split feed the feed is equally given to 5th and 6th effects and the thick liquor from both 5th and 6th effect is move into 4th effect and from 4th to 3rd effect and so on up to the 1st effect as shown in the Fig. 1.

⁺ Corresponding author. Tel.: + 91-9412619735; fax: +91-132-2714032
E-mail address: vivekfpt@iitr.ernet.in

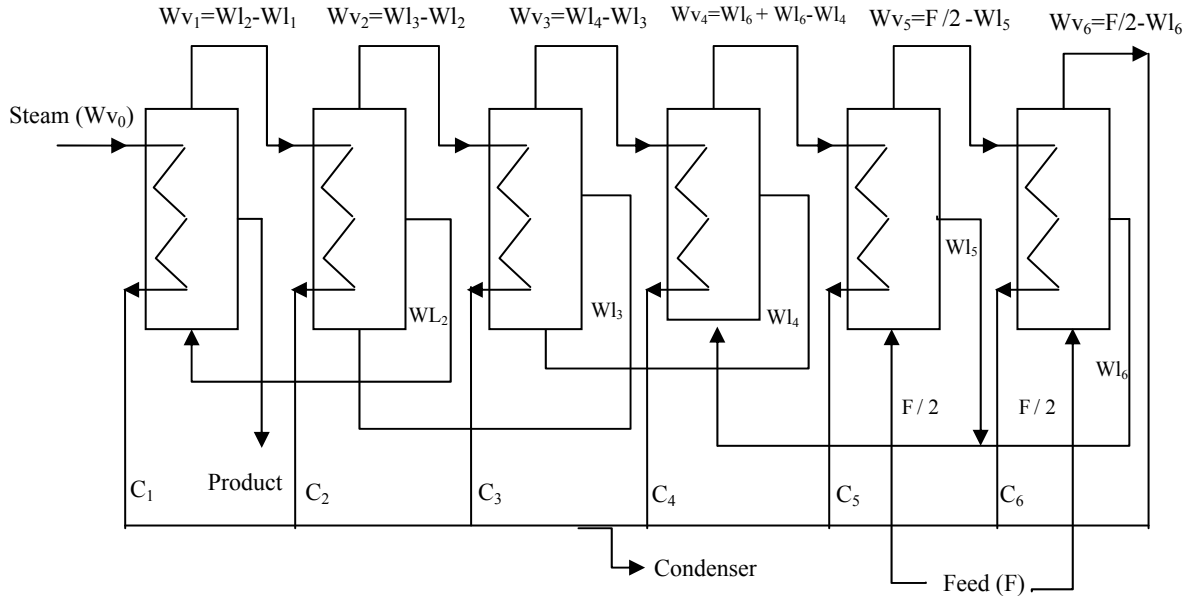


Fig. 1: Flow diagram of Sextuple Split feed evaporator

Model equations are developed for i^{th} effect using material and energy balance equations. The equations stating the physical properties of the black liquor are used and presented in our earlier work [5]. It is assumed that the vapor generated by the process of concentration of black liquor is saturated. It is also assumed that the energy and mass accumulation in the vapor lump is very small as compared to the enthalpy of the steam and can be neglected.

Material balance for liquor in the i^{th} effect for $i = 1, 2$ and 3 :

$$\frac{d}{dt} Ml_i(t) = Wl_{i+1} - Wl_i - Wv_i \quad (1)$$

For $i = 4, 5$ and 6 the equation is slightly different and are given by. For $i = 4$ the material balance equation is

$$\frac{d}{dt} Ml_4(t) = Wl_5 + Wl_6 - Wl_4 - Wv_4 \quad (1a)$$

Similarly for $i = 5$ the material balance equation is

$$\frac{d}{dt} Ml_5(t) = \frac{L_F}{2} - Wl_5 - Wv_5 \quad (1b)$$

In this way for $i = 6$ the material balance equation

$$\frac{d}{dt} Ml_6(t) = \frac{L_F}{2} - Wl_6 - Wv_6 \quad (1c)$$

Similar changes are effective for $i = 4, 5$ and 6 in energy and solid balance and in other equations which are obtained by using these balances.

Energy balance for liquor in the i^{th} effect:

$$\frac{d}{dt} (Ml_i(t) hl_i(t)) = Wl_{i+1} hl_{i+1} - Wl_i hl_i - Wv_i hv_i + Wv_{i-1} hv_{i-1} - Wv_{i-1} hc_{i-1} \quad (2)$$

where $Wv_{i-1} = \frac{Q}{hv_{i-1} - hc_{i-1}}$ and also $Q = U_i A_i (Tv_{i+1} - Tv_i - BPR_i)$

Material balance for solids in i^{th} effect:

$$\frac{d}{dt} (Ml_i(t) X_i(t)) = Wl_{i+1} X_{i+1} - Wl_i X_i \quad (3)$$

2.1. Formulation of the system equations

The vapor and liquor in i^{th} effect are in equilibrium and the relation for the liquor and vapor temperature defined in terms of BPR is as follows:

$$Tl_i = Tv_i + BPR_i \quad (4)$$

and the BPR is defined in terms of temperature and solid concentration, presented in [5].

Differentiating equation (4) with respect to time we get

$$\frac{d}{dt} Tl_i(t) = \left(\frac{d}{dt} Tv_i(t) \right) \left\{ 1 + \left(\frac{\partial}{\partial Tv} BPR_i \right) \right\} + \left(\frac{\partial}{\partial X} BPR_i \right) \left(\frac{d}{dt} X_i(t) \right) \quad (5)$$

Ml_i can be written as:

$$Ml_i = AL_i Pl_i \quad (6)$$

Differentiating equation (6) with respect to time we get

$$\frac{d}{dt} Ml_i(t) = AL_i(t) \left(\frac{d}{dt} Pl_i \right) + APl_i \left(\frac{d}{dt} L_i(t) \right)$$

Since Pl_i is a function of temperature Tl and concentration X , this equation is reduces into equation (7) by

using the value of $\frac{d}{dt} Tl_i(t)$ from equation (5) with rearranging the terms.

$$\begin{aligned} \frac{d}{dt} Ml_i(t) &= APl_i \left(\frac{d}{dt} L_i(t) \right) + AL_i(t) \left(\frac{\partial}{\partial Tl} Pl_i \right) \left\{ 1 + \frac{\partial}{\partial Tv} BPR_i \right\} \left(\frac{d}{dt} Tv_i(t) \right) \\ &+ AL_i(t) \left\{ \left(\frac{\partial}{\partial Tl} Pl_i \right) \left(\frac{\partial}{\partial X} BPR_i \right) + \left(\frac{\partial}{\partial X} Pl_i \right) \right\} \left(\frac{d}{dt} X_i(t) \right) \end{aligned} \quad (7)$$

Comparing the resultant differential equation (7) with equation (1) we get

$$C1 = C2 \left(\frac{d}{dt} L_i(t) \right) + C3 \left(\frac{d}{dt} Tv_i(t) \right) + C4 \left(\frac{d}{dt} X_i(t) \right) \quad (8)$$

where

$$C1 = Wl_{i+1} - Wl_i - Wv_i$$

$$C2 = APl_i$$

$$C3 = AL_i(t) \left(\frac{\partial}{\partial Tl} Pl_i \right) \left\{ 1 + \left(\frac{\partial}{\partial Tv} BPR_i \right) \right\}$$

$$C4 = AL_i(t) \left\{ \left(\frac{\partial}{\partial Tl} Pl_i \right) \left(\frac{\partial}{\partial X} BPR_i \right) + \left(\frac{\partial}{\partial X} Pl_i \right) \right\}$$

Differentiating $Ml_i(t) hl_i(t)$ with respect to time and using the value of $\frac{d}{dt} Tl_i(t)$ from equation (5) and the value of $\frac{d}{dt} Ml_i(t)$ from the equation (7) we get

$$\begin{aligned} \left(\frac{d}{dt} Ml_i(t) \right) hl_i + Ml_i(t) \left(\frac{d}{dt} hl_i \right) &= APl_i hl_i \left(\frac{d}{dt} L_i(t) \right) + AL_i(t) \left(1 + \frac{\partial}{\partial Tv} BPR_i \right) \\ &\left\{ Pl_i \left(\frac{\partial}{\partial Tl} hl_i \right) + \left(\frac{\partial}{\partial Tl} Pl_i \right) hl_i \right\} \left(\frac{d}{dt} Tv_i(t) \right) + AL_i(t) \left(\frac{d}{dt} X_i(t) \right) \\ &\left\{ Pl_i \left(\frac{\partial}{\partial Tl} hl_i \right) \left(\frac{\partial}{\partial X} BPR_i \right) + Pl_i \left(\frac{\partial}{\partial X} hl_i \right) + hl_i \left(\frac{\partial}{\partial Tl} Pl_i \right) \left(\frac{\partial}{\partial X} BPR_i \right) + \left(\frac{\partial}{\partial X} Pl_i \right) hl_i \right\} \end{aligned}$$

Comparing the resultant differential equation with equation (2) we get

$$C5 = C6 \left(\frac{d}{dt} L_i(t) \right) + C7 \left(\frac{d}{dt} Tv_i(t) \right) + C8 \left(\frac{d}{dt} X_i(t) \right) \quad (9)$$

Where

$$C5 = Wl_{i+1} hl_{i+1} - Wl_i hl_i - Wv_i hv_i + Wv_{i-1} hv_{i-1} - Wv_{i-1} hc_{i-1}$$

$$C6 = APl_i hl_i$$

$$C7 = AL_i(t) \left(1 + \frac{\partial}{\partial Tv} BPR_i \right) \left\{ Pl_i \left(\frac{\partial}{\partial Tl} hl_i \right) + hl_i \left(\frac{\partial}{\partial Tl} Pl_i \right) \right\}$$

$$C8 = AL_i(t) Pl_i \left\{ \left(\frac{\partial}{\partial Tl} hl_i \right) \left(\frac{\partial}{\partial X} BPR_i \right) + \left(\frac{\partial}{\partial X} hl_i \right) \right\} + AL_i(t) hl_i \left\{ \left(\frac{\partial}{\partial Tl} Pl_i \right) \left(\frac{\partial}{\partial X} BPR_i \right) + \left(\frac{\partial}{\partial X} Pl_i \right) \right\}$$

Differentiating $Ml_i(t) X_i(t)$ with respect to time and using the value of $\frac{d}{dt} Ml_i(t)$ from equation (7) and rearranging the equation and representing it in the form of coefficients

$$C9 = C10 \left(\frac{d}{dt} L_i(t) \right) + C11 \left(\frac{d}{dt} Tv_i(t) \right) + C12 \left(\frac{d}{dt} X_i(t) \right) \quad (10)$$

where

$$C9 = Wl_{i+1} X_{i+1} - Wl_i X_i$$

$$C10 = APl_i X_i$$

$$C11 = AL_i X_i \left(\frac{\partial}{\partial Tl} Pl_i \right) \left\{ 1 + \left(\frac{\partial}{\partial Tv} BPR_i \right) \right\}$$

$$C12 = AL_i \left[PI_i + X_i \left\{ \left(\frac{\partial}{\partial T1} PI_i \right) \left(\frac{\partial}{\partial X} BPR_i \right) + \left(\frac{\partial}{\partial X} PI_i \right) \right\} \right]$$

3. Simulation Studies

3.1 Steady state simulation and model validation

To study the dynamic response of any chemical process initial values of the process variables are needed. To fulfill this steady state solution of the system of nonlinear equations is obtained by ‘fsolve’ solver in MATLAB source code and is presented in our earlier work [5]. The range of operational (input) parameters is similar as in [5]. For the validation of the model, steady state solution is compared with the data of a literature. The steady state results are in good agreement with the results of [2].

3.2. Dynamic simulation and model application

For the dynamic simulation first order nonlinear differential equations (8), (9) and (10) are solved simultaneously in the same order for all the six effects of the split feed sextuple effect evaporator system. Steady-state solution of the model provide the, the initial values of the system variables at time, $t = 0$. Solution of such types of simultaneous nonlinear ordinary differential equations is extremely intricate in nature even by using sophisticated numerical techniques. In the present investigation an attempt has been made for dynamic simulation for tubular type falling film MEE system with split feed sequence by using ‘ode45’ solver in MATLAB source code respectively. The dynamic behavior of MEE is studied by creating four types of disturbances namely in feed flow rate, feed concentration, in live steam temperature and in feed temperature on the system variables namely (i) temperature of each effect and (ii) output concentration of each effect. Levels of evaporators are controlled by using single loop-feedback proportional-integral controllers. Responses in the face of disturbances are shown by graphical representation in the Fig. 3-10. These graphs are drawn with respect to time and by the graphical study it could easily be estimated that how the variables approach to the new steady state conditions. The responses obtained by the disturbances i.e. 10% increase and 10% decrease were similar in nature for all the six effects. Thus in this paper responses due $\pm 10\%$ impulse input disturbances are shown for first (1st) and last (6th) effects only for each case.

4. Results and Discussion

4.1 Effect of varying feed flow rate

For the dynamic response of the feed parameters the disturbance is applied in the feed of 5th and 6th effects simultaneously. The effect of $\pm 10\%$ step change in feed flow rate on the temperature and concentration on last (connected with condenser i.e. 6th) and first effect (connected with live steam i.e. 1st) are shown from Figure 3 to Figure 4 respectively. The temperature and concentration of both the effects show an increase or decrease with decrease or increase in the feed flow rate. It is obvious as fresh steam supply rate is constant and water to be evaporates and decreases per unit time.

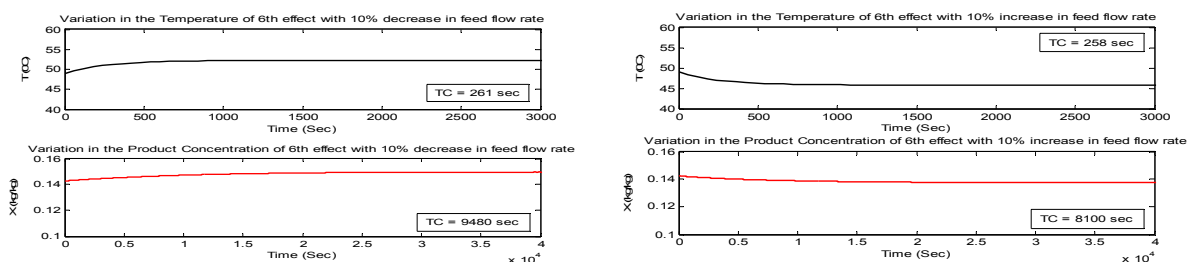


Figure 3 Response of 6th effect by disturbing $\pm 10\%$ in the feed flow rate

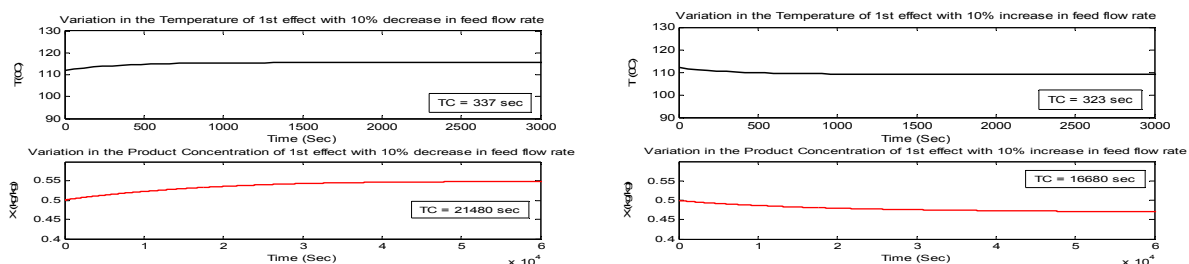


Figure 4. Response of 1st effect by disturbing $\pm 10\%$ in the feed flow rate

4.2 Effect of varying feed concentration

For the dynamic response of the feed concentration the disturbance is applied in the feed of 5th as well as in the 6th effect. The effect of $\pm 10\%$ step input in feed concentration on the temperature and concentration of last and first effect are shown from Figure 5 to Figure 6 respectively. The dynamic behavior of effect's temperature with respect to disturbances in feed concentration shows slight but insignificant change in temperature. However the change, it observed is unidirectional i.e. the temperature increases irrespective of increase or decrease in feed concentration. The changes in product concentration of both the effect show increase or decrease according as the feed concentration is increase or decrease. This may be due to the fact that ΔT across the evaporator system remains constant and vapor-liquid equilibrium of each effect remains almost unchanged for the optimum performance.

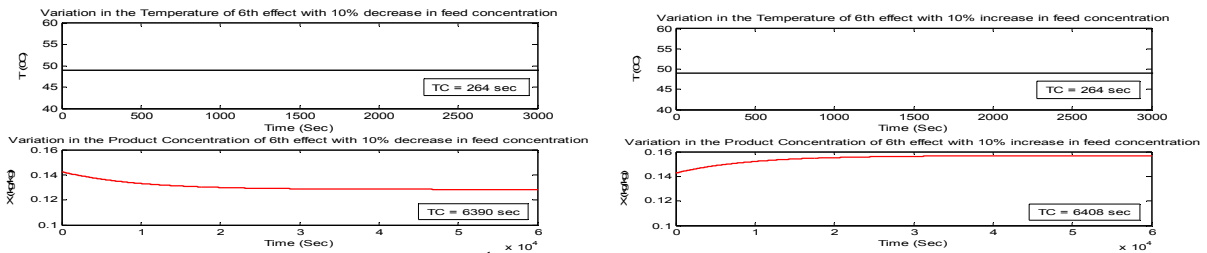


Figure 5. Response of 6th effect by disturbing $\pm 10\%$ in the feed concentration

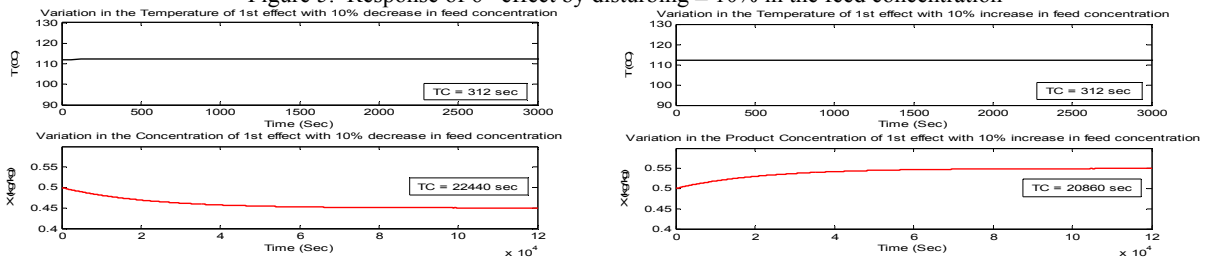


Figure 6. Response of 1st effect by disturbing $\pm 10\%$ in the feed concentration

4.3 Effect of varying steam temperature

In MEE system with split feed live steam enters in the first effect. Thus for the dynamic response of the steam temperature, the disturbance is applied in the first effect. The effect of $\pm 10\%$ step input in steam temperature on the temperature and concentration of last and first effect are shown by the Figure 7 to Figure 8 respectively. The 10% change in steam temperature results in increase or decrease in the temperature of both the effects before obtaining the steady state for 10% increase or decrease respectively. 10% disturbance in steam temperature does not result in any noticeable change in the product concentration of both the effects. However after scale down Y-axis value it was observed that product concentration increase and then decrease or decrease and then increase for 10% increase or decrease in the steam temperature respectively.

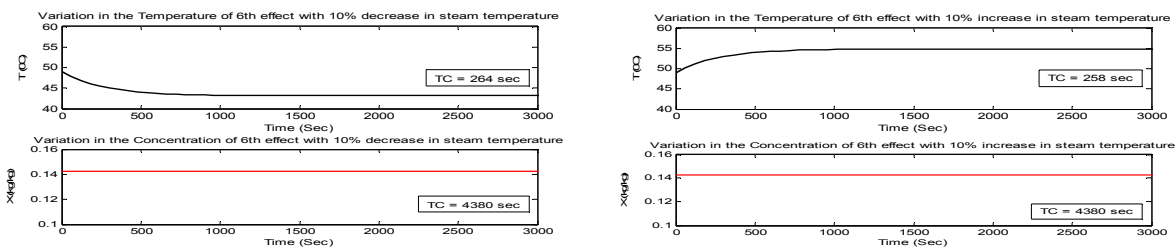
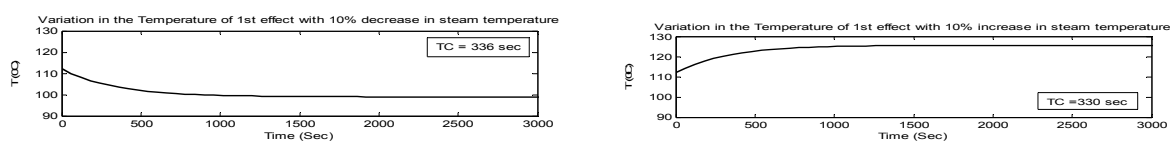


Figure 7. Response of 6th effect by disturbing $\pm 10\%$ in the steam temperature



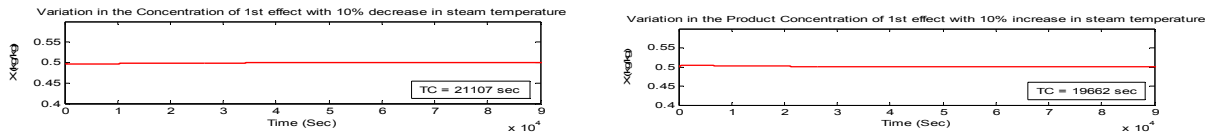


Figure 8. Response of 1st effect by disturbing $\pm 10\%$ in the steam temperature

4.4 Effect of varying feed temperature

For the dynamic response of the feed temperature the disturbance is applied in the 5th and 6th effects simultaneously. The effect of $\pm 10\%$ variation in feed temperature on the temperature and concentration of both the effect are shown from Figure 9 to Figure 10 respectively. It is evident from the figures that 10% disturbance in feed temperature does not bring noticeable change in the temperature and product concentration each effect. However after scale down Y-axis, it is observed that temperature of the both the effect increases and decreases to obtain the steady state with an increase and decrease in feed temperature and the product concentration of each effect first decreases and then increases to obtain the steady state with a very small fluctuations about the steady state up to four to five decimal places in the value of concentrations of both the effect and conversely for 10% increase in feed temperature.

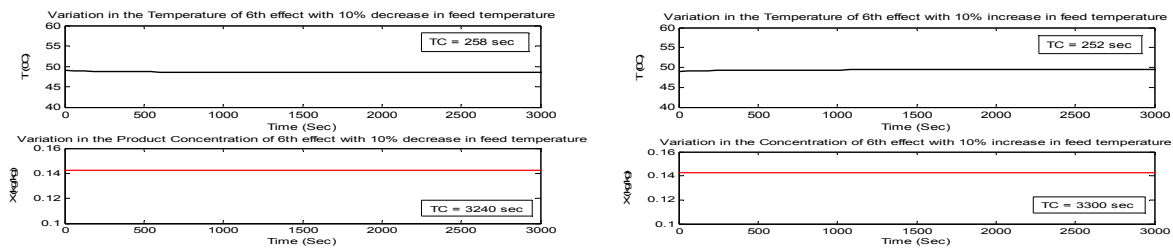


Figure 9. Response of 6th effect by disturbing $\pm 10\%$ in the feed temperature

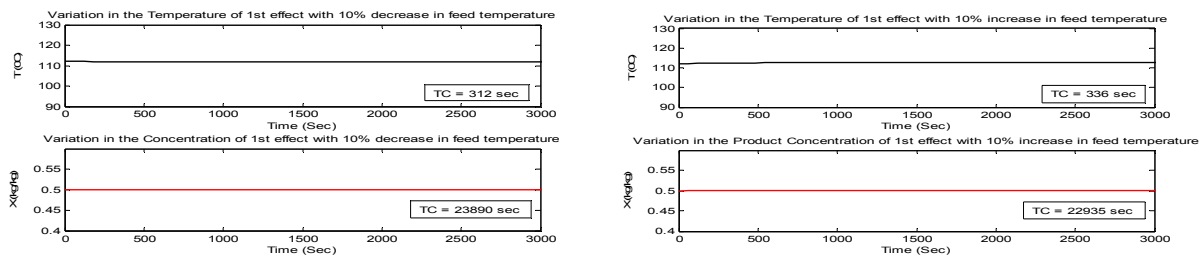


Figure 10. Response of 1st effect by disturbing $\pm 10\%$ in the feed temperature

5. Conclusion

An unsteady-state mathematical model was developed for a sextuple effect split feed evaporator for concentrating the black liquor by using material, energy balance equations and parametric correlations. For the dynamic simulation 'ODE45' solvers in MATLAB source code is used fruitfully. The dynamic behavior of each effect's temperature and product concentration was studied by disturbing the liquor flow rate, feed concentration, steam and feed temperatures by $\pm 10\%$. The transient study shows that the steady state is reached more quickly for temperature in comparison of the solid concentration and all of the responses converge in a smooth fashion.

Nomenclature:

A	Shell area, m ²	U	Overall Heat Transfer Coefficient (OHTC), kJ/sec.m ² .°C
BPR	Boiling point rise, °C	W	Mass flow rate, kg/s
C	Constant	X	Solid content, %
Cp	Specific heat of water at constant pressure, kJ/kg	λ	Latent heat of vaporization
h	Enthalpy, kJ/kg °C		Subscripts
L	Liquor level, m	c	Condensate
M	Mass, kg	l	Liquor
ρ_l	Liquor density	i	Effect number
t	Time, sec	v	Vapor
T	Temperature, °C		

6. References:

- [1] A.K. Ray, N.J. Rao, M.C. Bansal, B. Mohanty. Design data and correlations of waste liquor/black liquor from pulp mills. *IPPTA Journal*, (1992), 4: 1-21.
- [2] A.K. Ray, N.K. Sharma, P. Singh. Estimation of Energy Gains through Modelling and Simulation of Multiple Effect Evaporator System in a Paper Mill. *IPPTA*, (2004), 16(2): 35-45.
- [3] A.K. Ray, N.K. Sharma. Simulation of Multi-Effect Evaporator for Paper Mill-Effect of Flash and Product Utilization for Mixed Feeds Sequences. *IPPTA*, (2004), 16(4), 55-64.
- [4] A.K. Ray, P. Singh. Simulation of Multiple Effect Evaporator for black Liquor Concentration. *IPPTA*, (2000), 12(3): 35-45.
- [5] Deepak Kumar, Vivek Kumar, V.P.Singh, To study the parametric effect on optimality of various feeding sequences of a multi-effect evaporators in Paper industry using Mathematical modelling and simulation with MATLAB, *International Journal of Chemical and Biological Engineering*, 3 (3), 2010, 129-136.
- [6] H.A. Narmine, M.A. Marwan. Dynamic response of Multi-effect evaporators. *Desalination*, (1997), 114: 189-196.
- [7] N. J. Rao, R. Kumar. Energy conservation approaches in a paper mill with special reference to the evaporator plant. In Proceedings of the IPPTA international seminar conservation in pulp and paper industry (1985), 58–70.
- [8] S. Khanam, B. Mohanty. Energy reduction schemes for multiple effect evaporator systems. *Applied Energy*, (2010), 87: 1102–1111.
- [9] V. Miranda, R. Simpson. Modelling and simulation of an industrial multiple effect evaporator: tomato concentrate. *Journal of Food Engineering*, (2005), 66: 203–210.
- [10] V.K. Agarwal, M.S. Alam, S.C. Gupta. Mathematical model for existing multiple effect evaporator systems. *Chemical Engineering World*, (2004), 39:76-78.
- [11] Z.I. Stefanov, K.A. Hoo. Distributed Parameter Model of Black Liquor Falling-Film Evaporators. Part 1. Modelling of a Single plate. *Ind. Eng. Chem. Res.* (2003), 42: 1925-1937.
- [12] Z.I. Stefanov, K.A. Hoo. Distributed Parameter Model of Black Liquor Falling-Film Evaporators. 2. Modelling of a Multiple-Effect Evaporator Plant. *Ind. Eng. Chem. Res.*, (2004), 43: 8117-8132.