

## PERISTALTIC PUMPING OF A CONDUCTING BINGHAM FLUID IN A CHANNEL

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**Abstract.** Peristaltic transport of a conducting Bingham fluid is studied under long wavelength and low Reynolds number assumptions. The expressions for the velocity field and volume flux in the wave frame of reference are established. The pressure rise and mechanical efficiency are obtained in integral form and they are evaluated using Mathematica. The pumping characteristics are discussed for various values of yield stress, Hartmann number and amplitude ratio. It is observed that, yield stress parameter is influencing the pumping characteristics. The results obtained have potential application in the design of peristaltic pumps, corrugated pumps and sanitary flows.

**Keywords:** peristaltic, Bingham fluid, pumps

### 1. Introduction

Peristaltic pumping is a form of fluid transport generally from a region of lower to higher pressure, by means of a progressive wave of area contraction or expansion, which propagates along the length of a tube-like structure. Peristalsis occurs naturally as a means of pumping physiological fluids from one place in the body to another. Some electro-chemical reactions are held responsible for this phenomenon. This mechanism occurs in swallowing of food through oesophagus, in the ureter, the gastro intestinal tract, the bile duct and even in small blood vessels.

Even though peristalsis is a well-known pumping phenomenon observed in biological systems for many decades, Latham [1] was the first to make an experiment to study the fluid mechanics of peristaltic transport. Based on this experimental work, Burns and Parkes [2] investigated the peristaltic motion of Newtonian fluid through a pipe and channel by considering sinusoidal waves along the walls. Shapiro et al. [3] made a study on peristaltic pumping through a tube and a channel under the assumptions of low Reynolds number and long wavelengths. Brasseur et al. [4] discussed the influence of a peripheral layer of different viscosity on peristaltic pumping with Newtonian fluids. Usha and Ramachandra Rao [5] studied the peristaltic transport of two immiscible Newtonian fluids in a circular tube. Most of the investigations reported deal with peristaltic transport of Newtonian fluids. But the rotating viscometer data of Rand et al [6], Bugliarello et al. [7] and Chien et al. [8] made the researchers to study on the non-Newtonian behavior of biofluids such as blood. Lew et al. [9] reported that chyme is a non-Newtonian material having plastic like properties. Bugliarello and Sevilla [10] and Cockett [11] have experimentally shown that blood has a peripheral layer of plasma and a core region of suspension of all the erythrocytes when it flows through small blood vessels. Raju and Devanathan [12] made a theoretical investigation for blood flow by considering blood as a power-law fluid. Shukla and Gupta [13] studied the peristaltic transport of a power-law fluid with variable consistency. Srivastava and Srivastava [14] made a study on the peristaltic transport of blood by considering blood as a two-layered fluid model consisting of a central layer of suspension of erythrocytes, as a casson fluid and peripheral layer of plasma as a Newtonian fluid. Srivastava and Srivastava

[15] investigated peristaltic transport of a power-law fluid and gave its applications to the ductus efferentes of the reproductive tract. Usha and Ramachandra Rao [16] made a theoretical study of peristaltic transport of two-layered power-law fluids. In order to study the effect of yield stress in peristaltic pumping, Vajravelu et al. [17] and [18] investigated the flow of a Herschel-Bulkley fluid in a channel and in an inclined tube. They observed that for a given pressure difference, the flux depends on yield stress. They also obtained the trapping limits for Bingham and power-law fluids. All these investigations on peristaltic are made considering the pumping fluid as a non-Newtonian fluid, omitting the effect of yield stress.

In certain situations such as magneto therapy, the bio-fluids (for e.g. Blood, chyme) in a human body are subjected to magnetic fields. In view of this, the perishable peristaltic flow of a bio-fluid has to be investigated taking the influence of magnetic field and yield stress into consideration.

In this paper peristaltic transport of a conducting Bingham fluid in a channel under long wavelength and low Reynolds number assumptions is studied. This model is probably suitable for the blood flow in the sense that erythrocytes region and the plasma regions may be described as plug flow and non-plug flow regions. Further the blood is experimentally proved by Sud et al [19] to be a conducting fluid in several physiological applications. Motivated by this fact it is proposed to study the peristaltic transport of a Bingham fluid. It is observed that for a Bingham fluid the pressure difference and the mechanical efficiency of pumping depend on the yield stress and the magnetic field.

## 2. Mathematical formulation and solution

Consider the peristaltic pumping of a conducting Bingham fluid in a channel of half-width ( $a$ ). A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity, we restrict our discussion to the half width of the channel as shown in Figure - 1. The region between  $Y = 0$  and  $Y = y_0$  is called plug flow region. In this region,  $|\tau_{yx}| \leq \tau_0$ . In the region between  $Y = Y_0$  and  $Y = H$ ,  $|\tau_{yx}| > \tau_0$ . The wall deformation is given by

$$H(x, t) = a + b \sin \frac{2\pi}{\lambda} (x - ct) \quad (1)$$

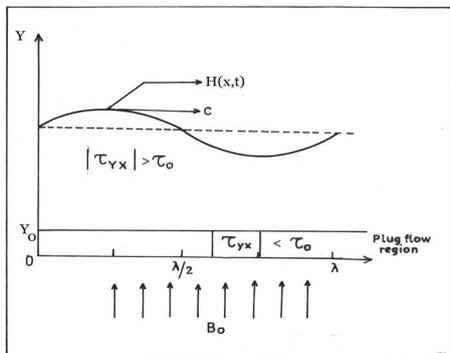


Figure - 1 Physical Model

### 2.1 Equations of Motion

Under the assumptions that the tube length is an integral multiple of the wave length  $\lambda$  and the pressure difference across the ends of the tube is a constant, the flow becomes steady in the wave frame  $(x, y)$  moving with velocity  $c$  away from the fixed (laboratory) frame  $(X, Y)$ . The transformation between these two frames is given by

$$\begin{aligned} x &= X - ct, & y &= Y \\ u(x, y) &= U(x - ct, Y) - c \\ v(x, y) &= V(x - ct, Y) \end{aligned} \quad (2)$$

In many physiological situations it is proved experimentally that the Reynolds number of the flow is very small (for example in ureter,  $R_e \cong 1$ ). So, we assume that the flow is inertia – free. Further, we assume that the wavelength is infinite. So the flow is of Poiseuille type at each local cross-section. Using the non-dimensional quantities,

$$\bar{u} = \frac{u}{c}; \quad \bar{x} = \frac{x}{\lambda}; \quad \bar{y} = \frac{y}{a}; \quad \bar{h} = \frac{h}{a}; \quad \bar{t} = \frac{ct}{\lambda}; \quad \bar{F} = \frac{Fa}{\lambda\mu c}; \quad \bar{\tau}_0 = \frac{a\tau_0}{\mu c}; \quad \bar{\Psi} = \frac{\Psi}{ac}; \quad \bar{p} = \frac{Pa^2}{\lambda\mu c}; \quad \bar{y}_0 = \frac{y_0}{a}; \quad \bar{q} = \frac{q}{ac}; \quad \bar{\phi} = \frac{b}{a}$$

The non-dimensional form of equations governing the motion (dropping the bars) is

$$\frac{d}{dy} (\tau_0 - \Psi yy) + M^2 \Psi y = -\frac{dp}{dx} \quad (3)$$

$$0 = \frac{\partial p}{\partial y} \quad (4)$$

where  $M = \sqrt{\frac{B_0^2 a^2 \sigma_e}{\mu}}$ , the Hartmann number

The non-dimensional boundary conditions are

$$\psi = 0; \quad \psi_{yy} = \tau_0 \text{ at } y = 0; \quad u = \psi_y = -1 \text{ at } y = h \quad (5)$$

$$\text{Where } h = 1 + \varphi \sin 2\pi x \quad (6)$$

Solving equation (4) with the boundary conditions (5), we obtain the velocity as

$$u = A_1 e^{My} + B_1 e^{-My} + \frac{p}{M^2} \quad (7)$$

where  $A_1 = a_1 p + a_2$ ,  $B_1 = a_3 p + a_4$

$$a_1 = \left[ \frac{-\frac{1}{M^2} + \frac{1}{M^2} e^{-Mh}}{2 \sinh M} \right]; \quad a_2 = \frac{-1 - \tau_0 e^{-Mh}}{2 \sinh M}; \quad a_3 = \frac{-\frac{1}{M^2} e^{Mh} + \frac{1}{M^2}}{2 \sinh M}; \quad a_4 = \frac{\tau_0 e^{Mh} + 1}{2 \sinh M}$$

$$p = -\frac{\partial p}{\partial x} \quad (8)$$

Taking  $y = y_0$  in equation we get the velocity in plug flow region as

$$u_p = A_1 e^{My_0} + B_1 e^{-My_0} + \frac{p}{M^2} \quad (0 \leq y \leq y_0) \quad (9)$$

the volume flux  $q$  through each cross-section in the wave frame is given

by

$$q = \int_0^{y_0} u_p dy + \int_{y_0}^y u dy = a_5 P + a_6 \quad (10)$$

where

$$a_5 = a_1 e^{My_0} \left( y_0 - \frac{1}{M} \right) + a_3 e^{-My_0} \left( y_0 + \frac{1}{M} \right) + \frac{a_1}{M} e^{Mh} - \frac{a_3}{M} e^{-Mh} + \frac{h}{M^2}$$

$$a_6 = a_2 e^{My_0} \left( 1 - \frac{1}{M} \right) + a_4 e^{-My_0} \left( 1 + \frac{1}{M} \right) + \frac{a_2}{M} e^{Mh} - \frac{a_4}{M} e^{-Mh}$$

From the above equation

$$\frac{\partial p}{\partial x} = \frac{a_6 - q}{a_5} \quad (11)$$

Averaging equation over one period yields the time mean flow (time-averaged volume flow rates)  $\bar{Q}$  as

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = q + 1 \quad (12)$$

## 2.2 The Pumping characteristics

Integrating the equation (10) with respect to  $x$  over one wavelength, we get the pressure rise over one cycle of the wave as

$$\Delta P = \int_0^1 \left( \frac{a_6 - \bar{Q} + 1}{a_5} \right) dx \quad (13)$$

The pressure rise required to produce zero average flow rate is denoted by  $\Delta P_0$  and is given by

$$\Delta P_0 = \int_0^1 \left( \frac{a_6 + 1}{a_5} \right) dx \quad (14)$$

The dimensionless friction force F at the wall across one wave length is given by

$$F = \int_0^1 h \left( -\frac{dp}{dz} \right) dx = \int_0^1 h \left( \frac{\bar{Q} - 1 - a_6}{a_5} \right) dx \quad (15)$$

### 2.3 Mechanical Efficiency:

The mechanical efficiency of pumping for one wave length is given by

$$E = \frac{\bar{Q}\Delta p}{\int_0^1 p_x (h-1) dx} = \frac{\bar{Q} [I_1 - (\bar{Q}-1)I_2]}{Q [I_3 - (\bar{Q}-1)I_4]} \quad (16)$$

$$\text{Where } I_1 = \int_0^1 \frac{a_6}{a_5} dx ; \quad I_2 = \int_0^1 \frac{1}{a_5} dx ; \quad I_3 = \int_0^1 \frac{a_6}{a_5} \sin 2\pi x dx ; \quad I_4 = \int_0^1 \frac{1}{a_5} \sin 2\pi x dx$$

## 3. Results and conclusion

From the equation (13) we have calculated the pressure difference as a function of  $\bar{Q}$  for different values of yield stress  $\tau_0$  for a fixed  $\phi$  and shown in Figure - 2. It is observed that for a given  $\Delta P$  the flux  $\bar{Q}$  depends on yield stress and it decreases with increase in yield stress. For a given flux  $\bar{Q}$  the pressure rise  $\Delta P$  decreases with increase in yield stress  $\tau_0$ . For free pumping, the flux  $\bar{Q}$  increases with decreasing yield stress.

From the variation of pressure rise with time averaged flow rate is calculated from equation (13) for different amplitude ratios and shown in Figure - 3. For given  $\Delta P$  the flux  $\bar{Q}$  for Bingham fluid depends on  $\phi$  and we find that the larger the amplitude ratio the greater the pressure rise against which the pump works. This observed for small values of  $\Delta P$  ( $\Delta P < 0001$ ) and free pumping cases.

From equation (13) we have calculated the variation of pressure rise with time averaged flow rate  $\bar{Q}$  for different magnetic parameters M and is shown in Figure - 4 we observed that the larger the magnetic parameter the greater the pressure rise against which the pump works. For free pumping the flux  $\bar{Q}$  depends on magnetic field and it increases with increasing magnetic parameter M.

From equation (15), we have calculated the frictional force as a function of  $\bar{Q}$  for fixed amplitude ratio, and is depicted in Figure - 5. It is observed that the frictional force 'F' has the opposite behaviour compared to pressure rise ( $\Delta P$ ). As  $\tau_0$  increases the frictional force also increases with the flux  $\bar{Q}$ . For a given  $\bar{Q}$  the frictional force increases with decrease in yield stress.

From equation (16) we have calculated the mechanical efficiency as a function of  $\bar{Q}$  and is depicted in Figure - 6 and Figure - 7. for different values of magnetic parameters M, yield stress and amplitude ratio  $\phi$ . For the chosen parameter the mechanical efficiency is positive for  $\bar{Q}$  less than or equal to 6 and is negative for  $\bar{Q}$  greater than 6. Further we observed that the pumping efficiency is greater for higher values of magnetic parameters. It is also found that the mechanical efficiency decreases with the increasing yield stress. For lower value flux  $\bar{Q}$ , the efficiency remains constant and is unaffected by variation in M and  $\tau_0$ .

From equation (14), we have calculated the pressure rise required to produce zero average flow rate  $\Delta P_0$  as a function of amplitude ratio  $\phi$  for different values of yield stress  $\tau_0$  and is shown in Figure - 8. It is observed that for a conducting Bingham fluid, the value of  $\Delta P_0$  increases with decrease in yield stress  $\tau_0$ . It is also observed that the increase in the value of  $\Delta P_0$  is more for the amplitude ratio beyond 0.7.

From equation (14), we have calculated the pressure rise required to produce zero average flow rate  $\Delta P_0$  as a function of amplitude ratio  $\phi$  for different values of magnetic parameter m and is shown in Figure - 9. It

is observed that for a conducting Bingham fluid, the value of  $\Delta P_0$  increases with increase in magnetic parameter.

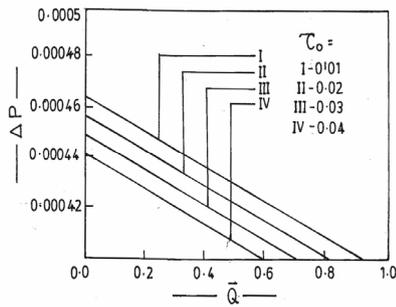


Figure - 2 The variation of  $\Delta P$  with averaged flow rate  $\bar{Q}$  for different  $\tau_0$  with  $\phi = 0.6, m = 0.02, y_0 = 0.2$

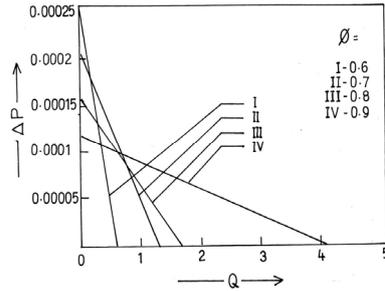


Figure - 3 The variation of  $\Delta P$  with averaged flow rate  $\bar{Q}$  for different  $\phi$ , with  $\tau_0 = 0.01, m = 0.01, y_0 = 0.2$

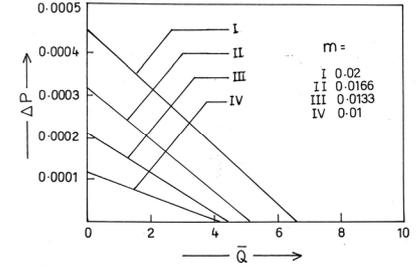


Figure - 4 The variation of  $\Delta P$  with averaged flow rate  $\bar{Q}$  for different  $m$  with  $\tau_0 = 0.01, \phi = 0.6, y_0 = 0.2$

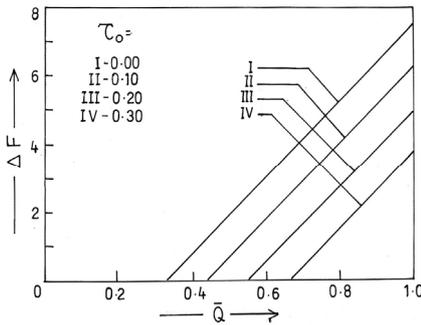


Figure - 5 The variation of  $\Delta F$  with averaged flow rate  $\bar{Q}$  for different  $\tau_0$  with  $\phi = 0.6, m = 0.02, y_0 = 0.2$

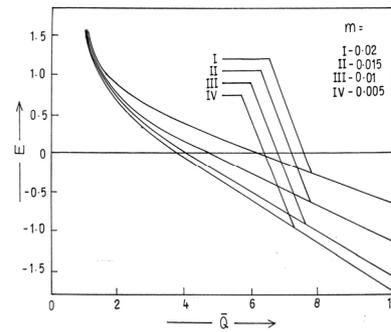


Figure - 6 The variation of Mechanical Efficiency  $E$  with averaged flow rate  $\bar{Q}$  for different  $m$  with  $\tau_0 = 0.01, \phi = 0.6, y_0 = 0.1$

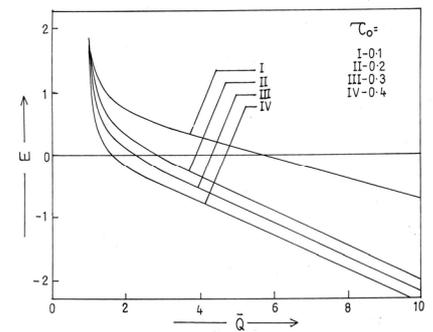


Figure - 7 The variation of Mechanical Efficiency  $E$  with averaged flow rate  $\bar{Q}$  for different  $\tau_0$  with  $\phi = 0.6, m = 0.02, y_0 = 0.1$

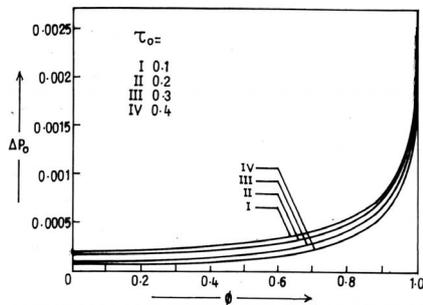


Figure - 8 The variation of  $\Delta P_0$  with Amplitude ratio  $\phi$  for different  $\tau_0$  with  $m = 0.02, y_0 = 0.2$

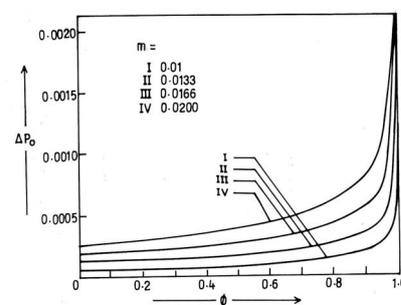


Figure - 9 The variation of  $\Delta P_0$  with Amplitude ratio  $\phi$  for different  $m$  with  $\tau_0 = 0.01, y_0 = 0.2$

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## Nomenclature

a:	Half width of the wave
B <sub>0</sub> :	Applied magnetic field
b:	Amplitude of the wave
c:	Wave speed
E:	Mechanical efficiency
F:	Dimensionless friction force
h:	Height of the wave in the direction of y
m:	Hartmann number
P:	Pressure
p:	Dimensionless pressure
ΔP:	Pressure rise/drop
q:	Volume flux
Q:	Time averaged flow rate
Re:	Reynold's number
t:	Time
U, V:	Axial and transverse velocity components in laboratory frame
u, v:	Axial and transverse velocity components in wave frame
u <sub>p</sub> :	Velocity in plug flow region
x, y:	Axial and transverse co ordinates
X, Y:	Dimensionless axial and transverse co ordinates
y <sub>0</sub> :	Plug flow region
λ:	wave length
σ <sub>e</sub> :	Electrical conductivity
Ψ:	Stream function
φ:	Amplitude ratio (b/a)
τ <sub>0</sub> :	Yield stress
μ:	Viscosity index