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Simulation of Fluid Flow Inside a Back-ward-facing Step by MRT-LBM

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Abstract. In this paper, the lattice Boltzmann method (LBM) is applied to simulate the two-dimensional in compressible steady low Reynolds number backward-facing step flow in a channel. The selected Reynolds number is limited to a maximum value of Re=105.in this Letter, we propose a numerical scheme to solve the velocity field using the multi-relaxation-time (MRT-D2Q9).The main objective of this study is to analyze the flow field in the back-ward-facing step which is including a blockage and also to study the shape of vortexes that are happened after the blockage. This model is validated by the numerical simulation of a classical benchmark problem. The results demonstrate the accuracy and effectiveness of the proposed methodology.

Keywords: Lattice Boltzmann method; Multi-relaxation-time; Back-ward-facing step

1. Introduction

Lattice Boltzmann method with Bhatnagar-Gross_Krook collision model(LBGK)[3,6,7,10] also called single-relaxation-time(SRT)LB method has gotten noticeable success in simulation of hydrodynamic issues and the main important advantages of the LBM can be classified as its explicitly, simplicity performance, and natural to parallelize. there are many other factors that are addressed for improving the lattice Boltzman method such as capability for complex geometry[2,9,10,13] applying of the boundary conditions[5,12,14,15] and also simulation of high Reynolds number. However, despite the presented advantages, some weakness points of the LBGK model are clear. For example this method may cause to numerical instability when the dimensionless relaxation time τ is close to 0.5.one way for solving these weakness points of the LBGK model is to use a multi-relaxation-time(MRT),which has been shown to make stable solution for high Reynolds number flows[4,8,11] and has the simplicity and computational of the LBGK method. One the main advantages of MRT-LB is that it has better numerical stability and also has more degrees of freedom rather than SRT-LB model.

Following the method of MRT-LB model, an incompressible MRT-LB(IMRT-LB) model has been presented in this Letter. The simulating results agree well with the analytical solutions or other benchmark data [1]. In addition, the numerical results demonstrate that the presented (IMRT-LB) model has better stability in comparison to the LBGK model.

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2. Numerical Method

2.1 Multi Relaxation time Lattice Boltzmann model

In multi-Relaxation method the collision operator is clearly classified as

$$f_i(x+c\Delta t, t+\Delta t) - f_i(x,t) = -\Omega[f_i(x,t) - f_i^{eq}(x,t)] \quad (1)$$

In this equation Ω is the collision step and this term is changed to momentum space and illustrated as

$$f_i(x+c\Delta t, t+\Delta t) - f_i(x,t) = -M^{-1}S[m(x,t) - m^{eq}(x,t)] \quad (2)$$

M presents Matrix transform and S is defined as a diagonal matrix. m(x,t) and m^{eq} are moments vectors. in this modelling, Matrix transform for D2Q9 is going to be used like following;

$$M = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\ 4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & -2 & 0 & 2 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ 0 & 0 & -2 & 0 & 2 & 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \end{pmatrix} \quad (3)$$

The momentum vector here is $m = (\rho, e, \varepsilon, j_x, q_x, j_y, q_y, p_{xx}, p_{xy})^T$

And equilibrium of the moment is

$$m_0^{eq} = \rho$$
$$m_1^{eq} = -2\rho + (j_x^{\ 2} + j_y^{\ 2})$$
$$m_2^{eq} = \rho - 3(j_x^{\ 2} + j_y^{\ 2})$$
$$m_3^{eq} = j_x$$
$$m_4^{eq} = j_{-x}$$
$$m_5^{eq} = j_y$$
$$m_6^{eq} = j_{-y}$$
$$m_7^{eq} = j_x^{\ 2} - j_y^2$$
$$m_8^{eq} = j_x j_y \quad (4)$$

$j_x j_y$ Also is defined by following equations:

$$j_x = \rho u_x = \sum_i f_i^{eq} c_{ix}$$
$$j_y = \rho u_y = \sum_i f_i^{eq} c_{iy} \qquad (5)$$

$$s = diag(1,1.4,1.4,s_3,1.2,s_5,1.2,s_7,s_8) \qquad (6)$$

The macroscopic properties such as density and velocity are categorized by

$$\sum_i f_i = \rho, \sum_i f_i e_i = \rho \vec{u} \qquad (7)$$

On the other hand equilibrium distribution function is calculated as following

$$f_i^{eq} = w_i \rho [1 + \frac{3\vec{e_i}.\vec{u}}{c^2} + \frac{9(\vec{e_i}.\vec{u})^2}{2c^4} - \frac{3\vec{u}.\vec{u}}{2c^2}] \qquad (8)$$

dx, dt and c are defined as lattice width, time step for lattice and lattice speed respectively

In backward-spacing flow problem c is assumed 1 and $dt = dx$

Particle speed $\vec{e_i}$ are determined as

$$\begin{aligned} &\vec{e_0} = 0 \\ &\vec{e_i} = (\cos[\pi(i-1)/2], \sin[\pi(i-1)/2])c \\ &i = 1,2,3,4 \\ &\vec{e_i} = (\cos[\pi(i-4-1/2)/2], \sin[\pi(i-4-1/2)/2])\sqrt{2c} \\ &i = 5,6,7,8 \end{aligned} \qquad (9)$$

w_i has been presented weighting factors that is classified in three terms

$$w_0 = 4/9, w_i = 1,2,,3,4 = 1/9, w_i = 5,6,7,8 = 1/36 \qquad (10)$$

The geometric parameter of the inserted square blockage is $w = \frac{1}{3}H$ and the location is as plotted in Fig (1).

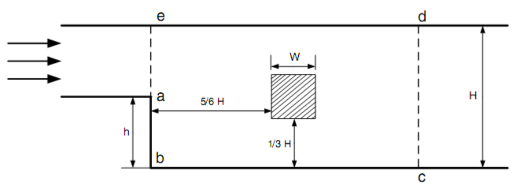

Fig. 1.The geometry of backward-facing step with inserted square blockage

3. Results and Discussion

3.1. Overview of the velocity field

In this letter a uniform rectangular mesh (200*40) for simulating of the flow was conducted. After and before the recirculation zone, Uniform square lattice mesh was applied. The Reynolds number of the flow is presented as $\frac{4U(H-h)}{3\upsilon}$,where U is defined as the maximum velocity in the inlet.

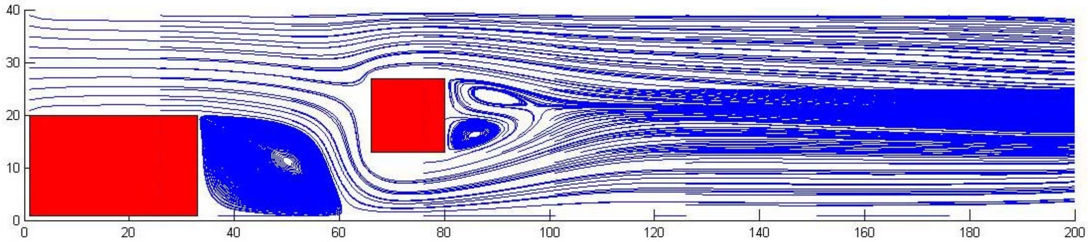

Fig. 2. Velocity vector with the inserted square blockage

Figure 2 shows the velocity vector field with the inserted square blockage after the enlargement.

3.2 Influence of Reynolds number

In this letter different Reynolds numbers (50,130,105) are used in order to analyze different vortexes after the blockage and also to figure out the reattachment area and deformation of vortex after this blockage.

According to analyzing different velocity vector fields figures it can be obtained that, by performing high Reynolds numbers shape of vortex will be changed to bigger one as opposed to the small one for low Reynolds numbers.

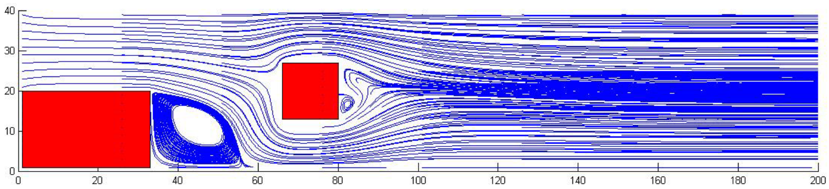

Fig. 3. Velocity vector with the inserted blockage for Reynolds number 50

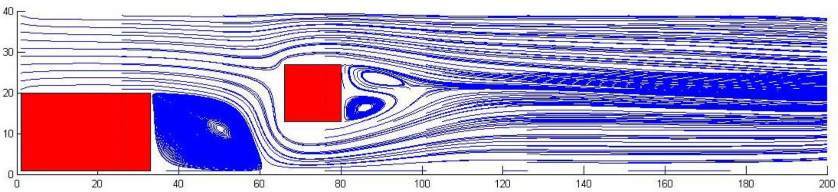

Fig. 4. Velocity vector with the inserted blockage for Reynolds number 130

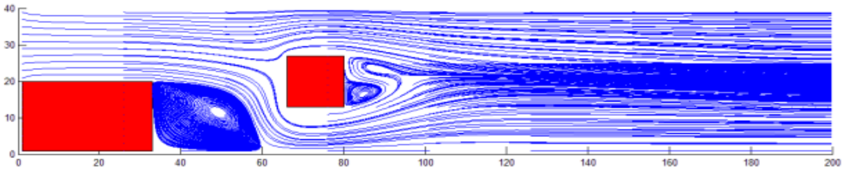

Fig. 5.Velocity vector with the inserted blockage for Reynolds number 105

By making a comparison between the illustrated figures for three Reynolds numbers which are 50,130 and 105 respectively, it is clear to note that by increasing the Reynolds number (Re=130,105) the shape of the vortex after the blockage changes and its size turns to be bigger whereas its small size for low Reynolds number(Re=50).

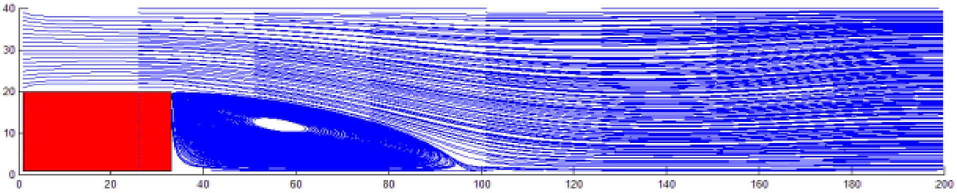

Fig. 6. Velocity vector without blockage for Reynolds number 105

Figure 6 clearly illustrates the velocity vector without the blockage. As can be seen when there is no inserted blockage in the channel the shape, size and length of the vortex will be changed to a bigger one as opposed to the channel including the inserted blockage that were shown in Figure 2,3,4and 5.

4.Conclusion

In this study, different Reynolds numbers backward-facing step flows are simulated using multi-relaxation-time model based on D2Q9 LBM. The numerical results obtained for the velocity field are in good agreement with the published numerical results. When we applied high Reynolds numbers in the channel including inserted blockage the shape, size and length of the vortex will have significant deformation in comparison to the situation with low Reynolds number. Finally the channel without blockage is discussed in order to figure out the effect of inserted blockage on the vortex in different aspects such as shape, size and its length.

5.Acknowledgments

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6.References

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