*2012 IACSIT Coimbatore Conferences*
***IPCSIT vol. 28 (2012) © (2012) IACSIT Press, Singapore***

# Formulation of Seismic Active Earth Pressure of Inclined Retaining Wall Supporting c-Φ Backfill

Sima Ghosh [1] and Satarupa Sengupta [2+]

[1] Assistant Professor, Civil Engg. Department, National Institute of Technology
[2] M.Tech Student, Civil Engg. Department, National Institute of Technology, Agartala, PIN – 799055

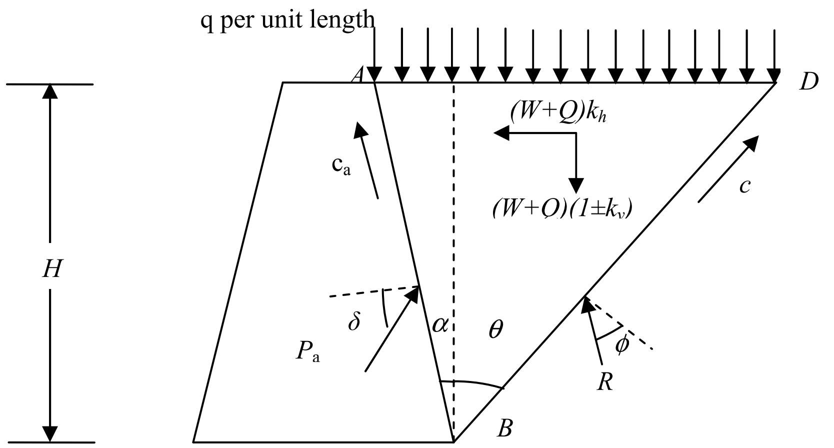
**Abstract.** An effort is made to evaluate the formulation of seismic active earth pressure behind a non-vertical retaining wall supporting c-Φ backfill. The formulation is done to get a single critical wedge surface for simultaneous action of weight, surcharge, cohesion and adhesion. The effect of various parameters viz. internal friction ($\Phi$), angle of wall friction ($\delta$), wall inclination angle($\alpha$), cohesion ($c$), adhesion ($c_a$), seismic accelerations ($k_h$, $k_v$), surcharge loading ($q$), unit weight ($\gamma$), height ($H$) are also taken into account to provide the variation of seismic active earth pressure coefficient.

**Keywords:** seismic active earth pressure, pseudo-static, retaining wall, *c-Φ* backfill, single wedge, wall inclination.

## 1. Introduction

The pioneering work in determining the static earth pressure was done by Coulomb (1773). Mononobe-Okabe extended the theory including earthquake loads introducing horizontal and vertical inertia forces for retaining wall having cohesionless backfill. The Coulomb's theory is further extended to evaluate seismic active earth pressure considering c-ϕ backfill by Prakash and Saran (1966), Saran and Prakash (1968), Saran and Gupta (2003). In all these methods three separate wedge surfaces generated for the indivisual action of unit weight, surcharge and cohesion considering unit adhesion is equal to unit cohesion. This assumption is not at all true. Thereafter, Shukla et al. (2009) was the first person who give the idea to extend this Mononobe-Okabe concept for c-Φ backfill in such a way to get single critical wedge surface. Therefore this paper aimed to give a satisfactory formulation to evaluate seismic active earth pressure including the influence of both adhesion and cohesion for a non-vertical retaining wall.

## 2. Method of Analysis




+ Corresponding author. Tel.: + 919774596933.
*E-mail address*: satarupas@yahoo.com.

Fig. 1: Forces acting on retaining wall – soil wedge system during active state of equilibrium

A schematic diagram of seismic active earth pressure is shown in the fig.1. Here a rigid retaining wall of height H supporting *c-Φ* backfill of unit weight *γ*, unit cohesion *c*, unit adhesion $c_a$, angle of wall friction *δ*, angle of soil friction *Φ,* retaining wall inclination angle α is shown. On the top of the backfill a surcharge load of intensity *q* per unit length is acting. At any stage of earthquake (having seismic acceleration coefficients $k_h$ and $k_v$) during active state of equilibrium, if the planer wedge surface *BD* generates an angle *θ* with the vertical, then the forces acting on the wedge system as shown in Fig.1, $P_a$ and *R* being the force on the retaining wall and reaction offered by the retained earth on the sliding wedge *ABD* at the face *BD* respectively.

The other forces are total cohesion $C = cH \sec\theta$, total adhesion $C_a = c_a H \, Sec\alpha$, weight of wedge, $W = \{\gamma H^2 (\tan\theta + tan\alpha)\}/2$, surcharge load, $Q = qH (\tan\theta + tan\alpha)$, horizontal inertia force = $(W+Q)K_h$ and vertical inertia force = $(W+Q)K_v$.

Applying the equilibrium conditions, $\sum H = 0$ and $\sum V = 0$ we get respectively,

$$P_a \cos(\alpha+\delta) - R\cos(\phi+\theta) + cH\tan\theta - c_a H\tan\alpha = (W+Q)k_h \quad (1)$$

$$P_a \sin(\alpha+\delta) + R\sin(\phi+\theta) + cH + c_a H = (W+Q)(1 \pm k_v) \quad (2)$$

On simplification of Eqn 1 and 2 substituting the values of *C*, *Q*, *W* etc. we get,

$$P_a \sin(\alpha+\phi+\delta+\theta) = \left(\gamma + \frac{2q}{H}\right)\frac{H^2}{2}(1 \pm k_v)\frac{(\tan\theta+\tan\alpha)}{\cos\psi}\cos(\phi+\theta-\psi) - cH\sec\theta\cos\phi - c_a H\cos(\alpha+\phi+\theta)\sec\alpha \quad (3)$$

Replacing ($\gamma + 2q/H$) by $\gamma_e$, Eqn 3 can be written as,

$$P_a = \gamma_e \frac{H^2}{2}(1 \pm k_v)\frac{\left[\frac{(\tan\theta+\tan\alpha)}{\cos\psi}\cos(\phi+\theta-\psi) - \frac{2c}{\gamma_e H(1\pm k_v)}\sec\theta\cos\phi - \frac{2c_a}{\gamma_e H(1\pm k_v)}\cos(\alpha+\phi+\theta)\sec\alpha\right]}{\sin(\alpha+\phi+\delta+\theta)} \quad (4)$$

Substituting $\frac{2c}{\gamma_e H(1\pm k_v)} = n_c$ and $\frac{2c_a}{\gamma_e H(1\pm k_v)} = m_c$

$$P_a = \gamma_e \frac{H^2}{2}(1 \pm k_v)\frac{[\sin(\alpha+\theta)\cos(\phi+\theta-\psi) - n_c\cos\alpha\cos\phi\cos\psi - m_c\cos\theta\cos\psi\cos(\alpha+\phi+\theta)]}{\sin(\alpha+\phi+\delta+\theta)\cos\theta\cos\psi\cos\alpha} \quad (5)$$

which can also be written as

$$P_a = \gamma_e \frac{H^2}{2}(1 \pm k_v)k_a \quad (6)$$

where,

$$k_a = \frac{[\sin(\alpha+\theta)\cos(\phi+\theta-\psi) - n_c\cos\alpha\cos\phi\cos\psi - m_c\cos\theta\cos\psi\cos(\alpha+\phi+\theta)]}{\sin(\alpha+\phi+\delta+\theta)\cos\theta\cos\psi\cos\alpha} \quad (7)$$

In Eqn 7, all the terms are constant except *θ*. On optimizing this coefficient for seismic active earth pressure we get the value of *θ* which is represented here as $\theta_c$ and given by

$$\theta_c = \cos^{-1}\sqrt{\frac{(r+s)s + t^2 + t\sqrt{s^2+t^2-r^2}}{2(s^2+t^2)}} \quad (8)$$

Where

$$r = -\sin(\psi+\delta) - m_c\cos\psi\cos\delta \quad (9)$$

$$s = 2n_c \cos\phi \cos\alpha \cos\psi \cos(\alpha + \phi + \delta) + m_c \cos\psi \cos\delta + \sin(\phi - \psi)\cos(2\alpha + \phi + \delta) + \sin(\phi + \delta)\cos(\phi - \psi) \quad (10)$$

$$t = 2\cos\alpha \sin(\alpha + \phi + \delta)[\sin(\phi - \psi) + n_c \cos\phi \cos\psi] \quad (11)$$

## 3. Parametric Study

Parametric study is done to clarify the effects of different soil and wall parameters on the variation of seismic active earth pressure coefficient $K_a$.Fig.2. shows the variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for different value of $\Phi$. From the plot it is clear that increase in $\Phi$ decreases the magnitude of seismic active earth pressure coefficient $K_a$ but increase in seismic acceleration coefficient $K_h$ increases the magnitude of seismic active earth pressure coefficient $K_a$. Fig.3. shows the variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for different value of $\delta$. From the plot, it is seen that $K_a$ decreases with increase in $\delta$ from $-\Phi/2$ to 0. But for increase in $\delta$ from $\phi/2$ to $\phi$, $K_a$ remains constant upto a certain level of $K_h = 0.1$ then increases with increase in $K_h$. Fig.4. shows the variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for different ratio of $K_v/K_h$. From the plot, it is clear that $K_a$ is constant upto a certain level of $K_h$=0.15 approximately after that increase in $K_v/K_h$ ratio increases the magnitude of seismic active earth pressure coefficient $K_a$. Fig.5. shows the variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for different values of unit cohesion. From the plot, it is seen that cohesion decreases the magnitude of seismic active earth pressure coefficient. For example, for $K_h$ = 0.4, due to increase in cohesion from 0 to 10 KN/m$^2$ and 0 to 20 kN/m$^2$, seismic active earth pressure coefficient $K_a$ decreases by 27.78% and 44.25% on $c$ = 0 value. Fig.6. shows the variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for different ratio of $c_a/c$. There is very nominal decrease in seismic active earth pressure coefficient due to the

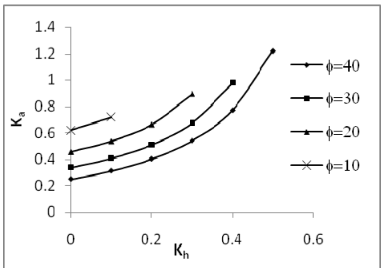



Fig.2.Variation of seismic active earth pressure coefficient ($k_a$) with $k_h$ for $K_v$=$K_h$/2, δ=Φ/2, γ = 18 kN/m$^3$, c=10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, q = 15 kN/m, H=10 m, α=20°

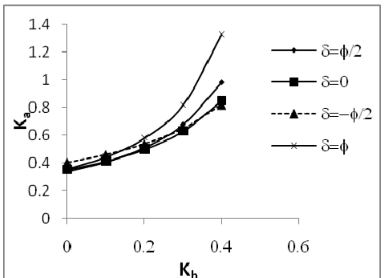



Fig.3.Variation of seismic active earth pressure coefficient ($k_a$) with $k_h$ for $K_v$=$K_h$/2, ϕ=30°, γ = 18 kN/m$^3$, c=10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, q = 15 kN/m, H=10 m, α=20°

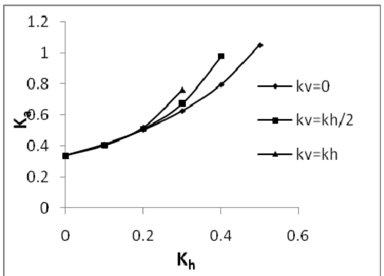



Fig.4.Variation of seismic active earth pressure coefficient ($k_a$) with $K_h$ for ϕ=30°, δ=Φ/2, γ = 18 kN/m$^3$, c=10 kN/m$^2$, $c_a$ = 8 kN/m$^2$, q = 15 KN/m, H=10 m, α=20°

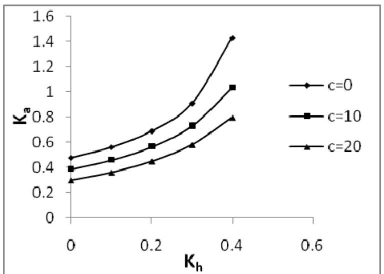



Fig.5.Variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for $k_v$=$k_h$/2, ϕ=30°, δ=Φ/2, γ = 18 kN/m$^3$, $c_a$ = 0 kN/m$^2$, q = 15 kN/m, H=10 m, α=20°

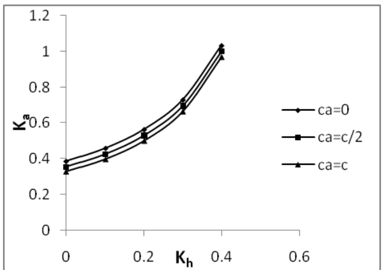



Fig.6.Variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for $k_v$=$k_h$/2, ϕ= 30°, δ=Φ/2, γ = 18 kN/m$^3$, c=10 kN/m$^2$, q = 15 KN/m, H=10 m, α=20°

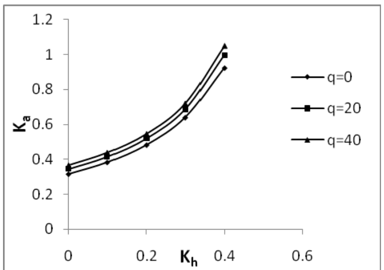



Fig.7.Variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for $K_v$=$K_h$/2, ϕ=30°, δ=Φ/2, γ = 18 KN/m$^3$, c=10KN/m$^2$, ca = 8 KN/m$^2$, H=10 m, α=20°

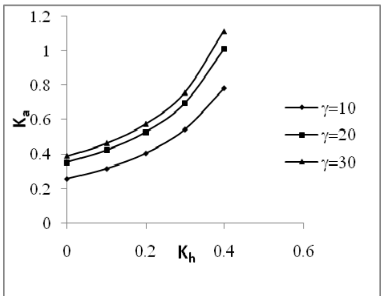



Fig.8.Variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for $K_v=K_h/2$, $\phi=30°$, $\delta=\Phi/2$, c=10 KN/m², ca = 8 KN/m², q=15 KN/m, H=10 m, α=20°

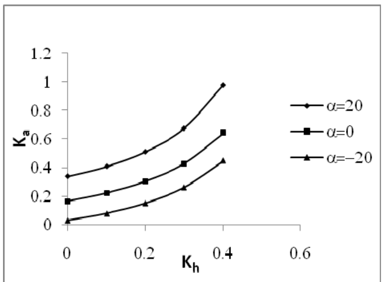



Fig.10.Variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for $K_v=k_h/2$, $\phi=30°$, $\delta=\Phi/2$, γ = 18 KN/m³, c=10 KN/m², ca = 8 KN/m², q=15 KN/m, H=10 m

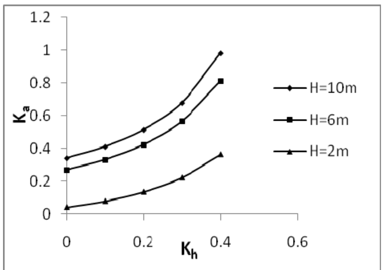



Fig.9.Variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for $K_v=k_h/2$, $\phi=30°$, $\delta=\Phi/2$, γ = 18 KN/m³, c=10 KN/m², ca = 8 kN/m², q=15 KN/m, α=20°

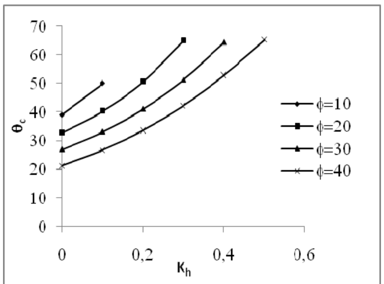



Fig.11.Variation of critical wedge angle ($\theta_c$) with $k_h$ for $K_v=K_h/2$, $\delta=\Phi/2$, γ = 18 kN/m³, c=10 kN/m², $c_a$ = 8 kN/m², q = 15 kN/m, H=10 m, α=20°

increase in $c_a/c$ ratio. The rate of decrease again becomes lesser for higher value of $K_h$. Fig.7. shows the variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for different value of loading $q$ affects significantly the magnitude of $k_a$. Fig.8 shows the variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for different value of unit weight. From the plot, it is seen affects significantly the magnitude of $k_a$. $K_a$ increases with the increase in the value of γ. For example at $K_h$ = 0.2, due to change in $\gamma$ from 10 to 20 KN/m³, $K_a$ increases by 30.20% on $\gamma$ = 10 KN/m³ value. Fig.9 shows the variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for different height of retaining wall. From the plot, it is seen that height of the retaining wall affects significantly the magnitude of $K_a$. For smaller height of retaining wall effect of $K_h$ is small. Fig.10 shows the variation of seismic active earth pressure coefficient ($K_a$) with $K_h$ for different wall inclination angle (α). From the plot, it is seen that the effect of wall inclination angle is a very prominent factor for the determination of seismic active earth pressure coefficient ($K_a$). In the case of evaluating seismic active earth pressure the critical wedge angle is the maximum angle made by the failure surface. The wedge angle is presented graphically in Fig.11. for different value of ϕ. $\theta_c$ decreases with the increases in the value of ϕ which means lesser participation of soil mass in vibration due to increase in $\Phi$.

## 4. Comparison of Results

Table 1 shows the comparison of results as obtained from present study with Sharma and Ghosh'2010.

Table. 1: Comparison of the results obtained from present study with Sharma and Ghosh'2010 [Φ = 20°, δ = 2Φ/3, γ = 18kN/m3, q = 15 KN/m, H = 10 m, i= 0, α=30°]

| Value of cohesion and adhesion | $K_h$=0, $K_v$=0 | | $K_h$=0.1, $K_v$=0.05 | | $K_h$=0.2, $K_v$=0.1 | |
|---|---|---|---|---|---|---|
| | Sharma and Ghosh(2010) | Present study | Sharma and Ghosh(2010) | Present study | Sharma and Ghosh(2010) | Present study |
| $c = c_a$ = 10 KN/m² | 0.622 | 0.569 | 0.698 | 0.663 | 0.828 | 0.821 |
| $c = c_a$ = 20 KN/m² | 0.502 | 0.395 | 0.578 | 0.467 | 0.695 | 0.582 |

## 5. Conclusion

The present study describes an analytical formulation for the coefficient of total active force on the back of a retaining wall supporting a *c-Φ* backfill considering simultaneous action weight, surcharge, adhesion and cohesion to get a single critical wedge angle. Depending on the developed formulation, a thorough parametric study is done for the variation of different soil and wall parameters. From the basis of parametric study, it is seen seismic earth pressure coefficient shows an inverse relation with the angle of soil internal friction, cohesion, $c_a/c$ ratio. On the other hand, it increases with the increase in the value of $K_v/K_h$ ratio, surcharge loading, unit weight of backfill material. The effect of $K_a$ is greater for high retaining wall and it increases due to change of wall inclination from negative to positive.

## 6. References


[1] Coulomb, C. A. (1773), "Essai sur une application des regles des maximis et minimis a quelque problems de statique relatifs 1' architecture, *Memoires d'Academie Roy. Pres. Diverssavants.7*.

[2] Mononobe, N. and Matsuo, H. (1929), "On the Determination of Earth Pressure during Earthquakes", *Proceedings, World Engg. Conference*, Vol. 9, 176 p.

[3] Okabe, S. (1926), "General Theory of Earth Pressure", *J. of the Japanese Society of Civil Engineers, Tokyo, Japan*, 12(1).

[4] Saran, S. and Gupta, R. P. (2003), "Seismic Earth Pressure Behind Retaining Walls", *Ind. Geotech.*, 33(3), 195 - 213.

[5] Prakash, S. and Saran, S. (1966), "Static and dynamic earth pressures behind retaining walls", *Proc., 3rd Symposium on Earthquake Engineering, University of Roorkee, Roorkee*, Vol. 1, pp. 277-288.

[6] Saran, S. and Prakash, S. (1968), "Dimensionless Parameters for Static and Dynamic Earth Pressure Behind Retaining Walls", *Indian Geotechnical J.*, 7(3), 295 – 310.

[7] Shukla, S. K., Gupta, S. K. and Sivakugan, N. (2009), "Active earth pressure on retaining wall for *c – Φ* soil backfill under seismic loading condition", *J of Geotechnical and Geoenvironmental Engineering*", Vol. 135, No.5.