

## Transient Analysis of Dam-Reservoir Interaction

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**Abstract.** In this study the linear dynamic behavior of the Pine Flat concrete dam is applied in the time domain to analyze reservoir – dam interaction. Also the effect of fluid compressibility and the Sommerfeld boundary are used to determine the hydrodynamic pressure. Hence, a finite element program is written in the FORTRAN language, and eight-noded serendipity element is implemented in the modeling. The fluid is assumed as compressible and inviscous, and the foundation is considered to be the rigid. Comparison between present study based on compressibility assumption of water and previous researches based on incompressibility assumption show increasing of maximum hydrodynamic pressure about 113% for Taft earthquake. Therefore, compressibility assumption of water is played important role to evaluate Pine Flat dam.

**Keywords:** fluid-structure interaction, finite element, dynamic analysis of dam, hydrodynamic pressure.

### 1. Introduction

In fluid-structure interaction one of the main Problems is the identification of the hydrodynamic pressure applied on the dam body during the time of earthquake. The analysis of dam-reservoir system is much more complicated than that of the structure alone and that is because of the difference between the characteristics of fluid and dam's concrete on one side and the interaction between the reservoir and dam on the other side. One of the early methods is based upon fluid incompressibility assumption. In this method Westergard solved the equation governing the hydrodynamic pressure in the dam reservoir domain (Helmholtz's equation)[1]. In this method for the analysis of concrete dams, fluid is treated as an added mass to the body of the dam. Then the studies of Chopra [2-3] showed that fluid incompressibility assumption does not predict correctly the applied hydrodynamic pressure on the dam body. The early studies on 2-dimensional gravity dam roots back to late 1970 s, in which interaction effects were considered through the exact and non-numerical solutions of the governing equations [4-5].

Zienkiewicz and Taylor [6] explained the governing fluid-structure equations through using the finite element method. For modeling the upstream boundary of the reservoir, they used radiative boundaries. Aznarez and et al. [7] studied the effects of reservoir bottom absorbent materials, on the dynamic analysis of fluid-structure interaction problem through the use of the boundary element method in the frequency domain. In this study the effects of absorbent materials degree of consolidation, compressibility and permeability were considered on the dynamic response of the system. Akkose and et al. [8] have studied the effects sloshing on the nonlinear dynamic response of the arch dam.

In 2007 Xiuli Du and et al. [9] studied the nonlinear seismic response analysis of a foundation-arch dam system, then they compared the results of their proposed method which is a combination of the implicit finite element method, the transmitting boundaries and the "Relaxation" dynamic method, with the common method which disregards the mass of the foundation. They found that the maximum dynamic stress obtained frame their method is lower than that of the common method. Bonnet and et al. [10] used the combination of

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the finite element and boundary element methods to simulate the dam-reservoir interaction in the frequency domain. They considered an elastic material behavior and a rigid reservoir bottom. The results of the analysis showed good agreement between theoretical and numerical methods. Then, Mitra and et al. [11] used the finite element method to simulate the 2D fluid-structure interaction, followed by considering the sloshing effect on a water storage tank. They considered the a wall of the reservoir as being either rigid or flexible, and the results showed that the applied hydrodynamic pressure on flexible walls is more than that on rigid walls. Seghir and et al.[12] used the coupled finite element and symmetric boundary element to model the interaction problem. In this study fluid compressibility is not considered, and the reservoir intersection points are considered at distances  $L_F=3 H_B$  and distance  $L_F=0.25 H_B$  ( $H_B$ = the height of dam). In this study it has been shown that for the reservoir to be at a distance close to the dam, the hydrodynamic pressure calculated by the F.E.M would not provide good results compared to the boundary element method formulation.

Endrani and et al. [13], studied the fluid-structure interaction in the frequency domain. They considered the sloshing effect in their studies. They assumed the material behavior to be linear-elastic and water as being compressible also the Sommerfeld boundary is used at a distance 3 times the height of the dam from the dam body.

According to these studies, in this paper we are going to consider the fluid-structure interaction phenomenon considering the water compressibility effect and using the Sommerfeld boundary. The obtained results would be compared with the results of Seghir [12] which are based on the incompressibility assumption. For this purpose a program was written in the FORTRAN Language, and in order to model the dam and reservoir, eighth noded serendipity elements are used.

The formulation is based on the finite element method in the plane strain form. The Euler-Lagrange method which the hydrodynamic pressure is the variable in the fluid would be used.

## 2. Governing Equation of Wave Propagation Through Fluid

In both the Eulerian and Lagrangian methods, The governing fluid-structure system equation is solved using wave propagation through the fluid by assuming linear compressibility and inviscosity. The wave propagation equation through fluid is as follow [6]:

$$\nabla_p^2 = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = \frac{\rho}{k} \frac{\partial^2 p}{\partial t^2} \quad (1)$$

In which  $p$  is the pressure function and  $c$  is the acoustic wave speed. If the fluid would be in compressible, equation (1) would take the following form [6]:

$$\nabla_p^2 = 0 \quad (2)$$

This equation has been solved and studied by many researchers through different analytic and analytical-numerical methods. But some of simplifying assumptions such as fluid incompressibility and not considering the nonlinear behavior in extreme earthquakes, the obtained solutions are not that practical.

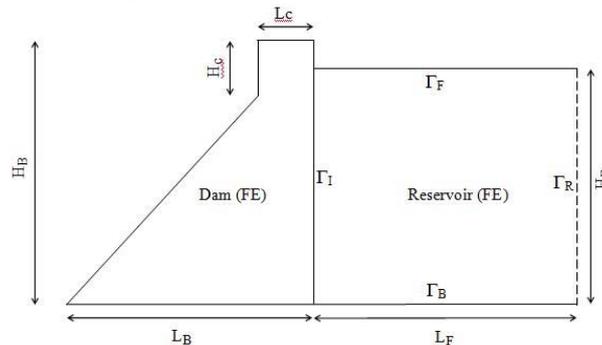


Fig. 1: Dam-reservoir coupled model

Therefore, Eq. (1) is used to solve the dam-reservoir interaction problem. Hence, the boundary conditions of the governing equation are stated as below.

## 2.1. Reservoir upstream boundary ( $\Gamma_R$ ):

With the vibration of the dam, volumetric hydrodynamic pressure waves are created in the reservoir and propagate toward the upstream, if the length of the dam is assumed to be infinity, then these waves would approach to vanish. It should be noted that the length of reservoir is assumed as a finite length,  $L_F$ , in numerical modeling. Hence, an artificial boundary is applied to simulate effect of infinite reservoir. This boundary is modeled based on the Sommerfeld boundary as

$$\frac{\partial p}{\partial n} = -\frac{1}{c} \frac{\partial p}{\partial t} \quad (3)$$

## 2.2. Reservoir bottom ( $\Gamma_B$ ):

According to the rigidity of the reservoir bottom, by assuming the horizontal movement of the earth, the pressure gradient is neglected.

$$\frac{\partial p}{\partial n} = 0 \quad (4)$$

## 2.3. Reservoir free surface ( $\Gamma_F$ ):

By neglecting the effects of surface waves, the governing boundary condition is as follow:

$$p = 0 \quad (5)$$

## 2.4. Fluid-structure interface ( $\Gamma_I$ ):

In the common boundary between the reservoir and the dam body, an interaction between these two occurs which is the result of an inertia force caused by the movement of the reservoir wall. Hence, the applied pressure on the reservoir wall caused by the inertial force is as follow.

$$\frac{\partial p}{\partial n} = -\rho \cdot \ddot{u}_n \quad (6)$$

In which,  $\rho$  is the density of fluid and  $\ddot{u}_n$  is the structure's acceleration vector in the direction normal to the common boundary of the fluid and structure.

By using the Galerkin method, the weak form of equation (1) is as below:

$$\int N_p^T \left[ \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) \right] dv = 0 \quad (7)$$

In which,  $N_p$  is the hydrodynamic nodal interpolation function in the finite element method. So by using discretization in finite element method, and placement the boundary conditions in relation (7), the equation governing the fluid can be summarized as follows:

$$S\ddot{p} + \tilde{C}\dot{p} + Hp + Q^T \ddot{u} + \rho Q^T \cdot I \cdot \ddot{u}_g(t) = 0 \quad (8)$$

In which in Eq. (8),  $[S]$ ,  $[\tilde{C}]$  and  $[H]$  are pseudo mass matrix, pseudo damping matrix and pseudo fluid stiffness matrix, respectively. The finite element form of these matrices is as follow:

$$\begin{aligned} [S] &= \int_{\Omega} N_p^T \cdot \frac{1}{c^2} \cdot N_p \cdot d\Omega \\ [\tilde{C}] &= \int_{\Gamma_R} N_p^T \cdot \frac{1}{c} \cdot N_p \cdot d\Gamma \\ [H] &= \int_{\Omega} (\nabla N_p)^T \cdot (\nabla N_p) \cdot d\Omega \end{aligned} \quad (9)$$

## 3. Structure Governing Equations

Based on the theory of the finite element method [11], the governing equation of the structure dynamic response to the support excitation in the time domain, can be shown as below:

$$M \ddot{u} + C \dot{u} + Ku - Qp + M \cdot I \cdot \ddot{u}_g(t) = 0 \quad (10)$$

In which  $[M]$ ,  $[C]$  and  $[K]$  are mass, damping and stiffness matrices of the structure respectively.  $u$ ,  $\dot{u}$  and  $\ddot{u}$  are displacement, velocity and acceleration vectors respectively. In Eq. (10),  $[I]$  is a unit matrix which

transmits the support acceleration vector  $\{\ddot{u}_g(t)\}$  to the structure degrees of freedom. Matrix  $[Q]$  is shown in eq.(8). Its duty is to transform the accelerations of the structure to pressure flux, and also transforming the hydrodynamic pressure into applied loads on the structure. In fact the  $[Q]$  matrix causes interaction in fluid and structure. In the same relation vector  $\underline{n}$  is normal to the common surface of the structure and fluid.

Hence, the coupled equation of the fluid-structure system based on relations (8) and(10) in the Euler- Lagrange formulation are presented as follow :

$$\begin{bmatrix} M & O \\ \rho Q^T & S \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \dot{p} \end{Bmatrix} + \begin{bmatrix} C & 0 \\ 0 & \tilde{C} \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix} + \begin{bmatrix} K-Q \\ 0 & H \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{bmatrix} -M.I.\ddot{u}_g(t) \\ -\rho Q^T.I.\ddot{u}_g(t) \end{bmatrix} \quad (11)$$

#### 4. Analysis of Pine Flat Dam under Taft Earthquake

In order to study the effect of fluid compressibility, the dynamic analysis of the Pine Flat was performed. The geometrical and dimensional properties are presented in Fig.1 and Table 1. Recorded horizontal components of the ground acceleration during the Taft earthquake (S69E) shown in Fig.2 are used in this analysis. Table 2 summarizes results obtained for periods of the first five free vibration modes. The stiffness proportional damping equivalent to  $\xi = 5\%$  damping for all the modes is used. A time step of 0.005 sec is chosen for the analysis. The density and velocity of pressure wave in fluid are taken as  $1000 \frac{kg}{m^3}$  and  $1438.7 \frac{m}{s}$ , respectively. The dam and reservoir is discretized with 8-noded quadratic element and is analyzed using plain strain behavior for serendipity element.

Table. 1: Geometry and material properties of the Pine Flat dam

Dimensions (m)						Material properties		
$H_B$	$H_C$	$L_B$	$L_C$	$L_F$	$H_F$	$E(N/m^2)$	$\nu$	$\rho(Kg/m^3)$
122.0	18.5	96.0	9.75	366.0	116.0	$34.47 \times 10^9$	0.2	2440

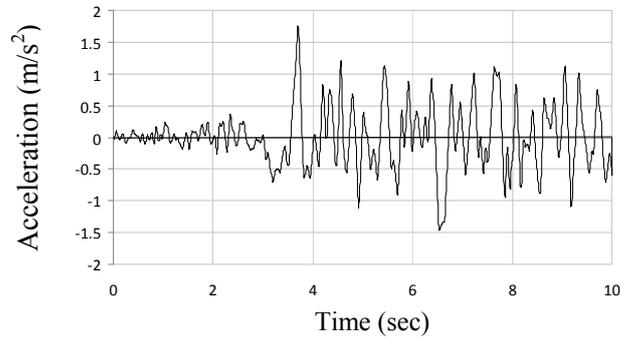


Fig. 2: Horizontal ground acceleration record of the Taft earthquake

Table. 2: Periods (sec) for only structure of the dam

Mode number	1	2	3	4	5
Only structure of Dam	0.2558	0.1241	0.0921	0.0705	0.0466

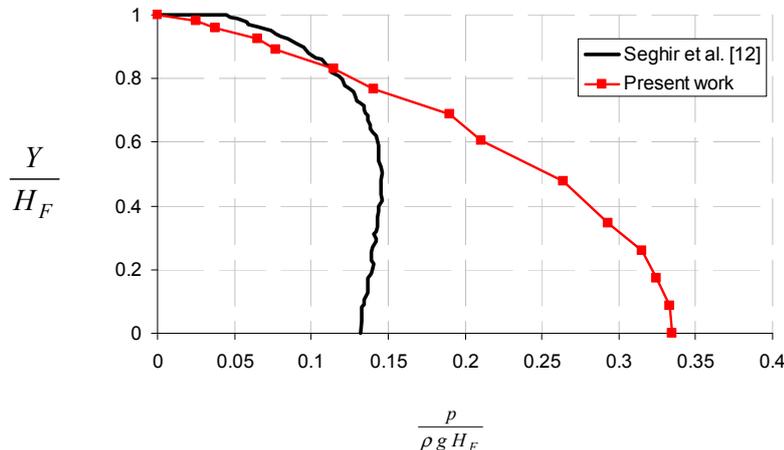


Fig. 3: Hydrodynamic Pressure envelope acting on upstream face of the dam

Fig.3 shows that the fluid compressibility assumption causes the hydrodynamic pressure to increase. Therefore if we assume fluid to be incompressible, there would be a large error in the evaluation of the applied hydrodynamic pressure on the dam body, in which the maximum hydrodynamic pressure obtained from the compressibility and incompressibility assumptions are 380.7 kPa and 150 kPa respectively.

## 5. Conclusion

In this study the fluid compressibility effect was examined, followed by the dynamic analysis of the Pine Flat dam under Taft earthquake. For this purpose a program was written in the FORTRAN language, in which eight-noded serendipity element with plane strain behavior were used to model the dam-reservoir. In this study was assumed that the foundation is the rigid and fluid is inviscous and compressible. Also in this study the fluid-foundation interaction was neglected. The performed analysis shows that the maximum applied hydrodynamic pressure on the dam body by assuming fluid compressibility is 2.5 times more than that with the fluid incompressibility assumption.

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