2012 International Conference on Information and Computer Networks (ICICN 2012)
IPCSIT vol. 27 (2012) © (2012) IACSIT Press, Singapore

Comparative Study of Radiated Heat by Aluminium & Copper Cable

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Abstract. Underground cables have replaced the use of overhead lines in the main cities around the world, although costly but cables have been preferred because of its higher current carrying capability and very low maintenance. Different properties of the cable itself and also of the environment limit the current carrying capacity, ampacity, of a cable. Ampacity of underground cables mainly depends on the rate at which the cable can dissipate heat, generated from the current flow, to the environment. This is why the thermal properties of the cable material become so crucial. Aluminium and Copper are the most utilized materials that are used for transmission of electrical power. So our aim is to compare the amount of heat dissipated by Aluminium and Copper cables. In the first portion of our paper we have described a mathematical model of underground cables for heat radiation. This heat radiation model is then solved by using analytical solution technique. After that it is used to simulate and to compare the heat radiation profile of different Aluminium and Copper cables. As a result of the simulation we came to know that because of the heat radiation the value of temperature at surface of the cable is always less than the core. It has been observed that the fall in temperature is highest for the lowest number of AWG cable and as we go for higher number, a more flattened temperature profile is obtained.

Keywords: Heat, Aluminium, Copper, simulation and underground cables etc.

1. Introduction

Electric power can be transmitted through overhead lines or underground cables. Though the initial cost is higher, in the urban areas electric power usually transmitted by underground cables because of its higher ampacity (allowable current) and lower maintenance cost. In an underground cable system ampacity is mainly determined by the capacity of the installation to extract heat from the cable and dissipate it in the surrounding soil and atmosphere [1]. The soil thermal resistance, the soil temperature, the environment temperature and the external heat source have predominant influences on this heat transfer from the cable to environment. It is shown that cable ampacity increases as the soil thermal conductivity and the distance from the external heat source increases, while it decreases linearly with the increases of the environment temperature and the soil temperature. [2]. Although several researches have described many ways to calculate [3] or simulate [4, 5] the amount of heat generated by underground cables and to improve ampacity by changing the environment temperature and materials [5, 6], very few researches are there to compare the thermal performances of different suitable materials that can be used to construct better under cables. Aluminium and Copper are the two most widely used materials for transmission purpose because of its conductivity, tensile strength and ductility. In this paper we describe a comparative study between Aluminium and Copper from the point of view of temperature variation when current is passed through it. In this case cable with (theoretically) infinite number of conductors is considered for both type of material and the well-known heat equation is utilized to reach a solution where partial differential equation describes the behavioral change in temperature with respect to time and radial distance. The theoretical results are derived first for a cable made up of an infinite number of conductors distributed uniformly in a cylindrical insulating medium. In order to apply these expressions to actual cables it is necessary to take into account the variations of the thermal constants of the cable with the size and number of wires.

2. Mathematical Model

2.1. Basic Model

The temperature as a function of time in a cable may be obtained by writing the fundamental heat relationship for an element of cylindrical shell of unit length and radii *r* and *r*+Δ*r*:

Heat generated = Heat dissipated + Heat stored.

$$I^2RN\,dt = -k\left[2\pi r\frac{\partial u}{\partial r_r} - 2\pi(r+\Delta r)\frac{\partial u}{\partial r_{r+\Delta r}}\right]dt + 2\pi r\Delta r c\rho\,du$$

where, I is the current through each wire, R is the resistance of each conductor, N is the number of conductors in the shell between radii r and $r+\Delta r = 2\pi m r\Delta r/\pi r^2$, b is the radius of the cable, m is the total number of conductors in the cable, *k* is the thermal conductivity, *ρ* is the density, *c* is the average specific heat, and u is the temperature. In the limit, $\Delta r \longrightarrow 0$, the equation (1.1) becomes,

$$\frac{\partial u}{\partial t} = A + \frac{a^2}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \; 0 \le r < b,\; 0 < t \text{ where } A = I^2Rm/(\pi b^2 c\rho) \text{ , and } a^2 = k/(\rho c)$$

Above equation is one of those standard heat equations in cylindrical coordinate and in non-homogeneous representation (due to the presence of A) [7, 8].

$$u_t = a^2\left(u_{rr} + \frac{1}{r}u_r\right) + A.$$

2.2. Boundary Condition

The boundary condition associated with the equation stands for the boundary (where $r = b$) of the cable (which equals the radius) is: $u_r(b,t) = -hu;$ where h is the surface conductance, the *(-)* sign signifies the radiation of heat out of the cable at the surface. This is one of the homogeneous Robin Boundary Condition type [8].

The other boundary condition stems from the fact that the solutions to the above heat equation must be bounded (say, at the center of the cable $r=0$) (i.e. the temperature of the cable should not rise with the increase of time indefinitely which will cause blowing up of cable.)

2.3. Initial Condition

The initial condition for the aforementioned heat equation is: $u(r,0) = 0$, signifying the fact that initially the temperature of the cable is zero.

3. Analytical Solution

The result must be in the form of transient and steady state solution because of the non-homogeneity of the heat equation itself. So, taking $u(r,t)$, $\omega(r)$ and $v(r,t)$ to be the total, steady state and transient solution for the heat equation we have, $u(r,t) = \omega(r) + v(r,t)$.

3.1. Steady State Solution

The steady state solution $\omega(r)$ satisfies

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = -\frac{A}{a^2} \;or\; w(r) = T_c - \frac{Ar^2}{4a^2}$$

where T_c is the (yet unknown) temperature in the center of the cable.

3.2. Transient Solution

The transient solution $v(r,t)$ is governed by ,

$$\frac{\partial v}{\partial t} = \frac{a^2}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right); \qquad 0 \le r < b,\; 0 < t$$

with the initial condition $v(r,0) = u(r,0) - \omega(r) = 0 - (T_c - \frac{Ar^2}{4a^2})$.

Because the temperature equals the steady-state solution when all the transients die out, $\omega(r)$ must satisfy this radiation boundary condition regardless of the transient solution.

This requires that

$$T_c = \frac{A}{a^2}\left(\frac{b^2}{4} + \frac{b}{2h}\right)$$

Therefore, $v(r,t)$ must satisfy the $v_r(b,t) = -hv(b,t)$ at $r=b$.

We find the transient solution *v(r,t)* by the separation of variables *v(r,t) = R(r)T(t)*. Substituting previous equation,

$$\frac{1}{rR}\frac{d}{dr}\left(r\frac{dR}{dt}\right) = \frac{1}{a^2T}\frac{dt}{dt} = -k^2$$

$\frac{d}{dr}\left(r\frac{dR}{dr}\right) + k^2rR = 0$ and $\frac{dT}{dt} + k^2a^2t = 0,$ with *R'(b) = -hR(b)* as the boundary condition.

The only solution which remains finite at *r = 0* and satisfies the boundary condition is $R(r) = J_o(kr)$, where J_o is the zero-order Bessel function of the first kind. Substituting $J_o(kr)$ into the boundary condition, we have

$$kbJ_1(kb) - hbJ_0(kb) = 0$$

For a given value of h and b, (1.11) yields an infinite number of unique zeros k_n.

The corresponding solution of T_n for the problem is $T_n(t) = A_n e^{(-a^2k_n^2t)}$

So that the sum of the product solutions is

$$v(r,t) = \sum_{n-1}^{\infty} A_n J_0(k_nr)e^{(-a^2k_n^2t)}$$

$v(r,0) = \frac{Ar^2}{4a^2} - T_c = \sum_{n-1}^{\infty} A_n J_0(k_nr)$ which is a Fourier – Bessel series in $J_0(k_nr)$.

The coefficients of a Fourier – Bessel series with the orthogonal function $J_0(k_nr)$ and the boundary condition equals:

$$A_n = \frac{2k_n^2}{(k_n^2b^2 + h^2b^2)J_o^2(k_nr)}\int_0^b r\left(\frac{Ar^2}{4a^2} - T_c\right)J_0(k_nr)dr$$

from $A_k = \frac{1}{C_k}\int_0^L xf(x)J_n(\mu_k x)\,dx$ and $C_k = \int_0^L xJ_n^2(\mu_k x)dx,$

Finally we get

$$A_n = \frac{2}{(k_n^2 + h^2)J_o^2(k_nb)}\left[\left(\frac{Ak_nb}{4a^2} - \frac{A}{k_nba^2} - \frac{T_ck_n}{b}\right)J_1(k_nb) + \frac{A}{2a^2}J_0(k_nb)\right]$$

This value of A_n we get by using the relation:

$$\frac{d}{dx}[x^nJ_n(x)] = x^nJ_{n-1}(x)$$

and using integration by parts.

So, now we can put the value of A_n -

$$v(r,t) = \frac{2}{(k_n^2 + h^2)J_o^2(k_nb)}\left[\left(\frac{Ak_nb}{4a^2} - \frac{A}{k_nba^2} - \frac{T_ck_n}{b}\right)J_1(k_nb) + \frac{A}{2a^2}J_0(k_nb)\right]J_0(k_nr)e^{(-a^2k_n^2t)}$$

Finally the total solution for *u(r,t)* can be summed up and written as $u(r,t) = w(r) + v(r,t)$, where the expressions of *ω(r)* and *v(r,t)* is taken from equations and they represent the steady state and transient solution respectively [7].

4. Case Study

The case study for the change of temperature through a cable is done for two different materials, i.e. Copper and Aluminium. Three different classes of cables (i.e. AWG #1,#6 and #10) are selected for the purpose of study and the values used for Copper and Aluminium for different parameters are listed in Table 1 shows the list for typical data of Copper AWG #6 and maximum safe current for different Copper cables.

Material Name	AWG #	I (amperes)	m	b (cm.)	R (Ω)	A (ºC/sec)	a^2 (cm²/sec)	hb	T_c (ºC)
Copper	1	1.216	37	5.02	9.66	6.08	1.16	1.75	24.94
	6	.594	37	4.4	29.61	2.2747	1.16	1	23.94
	10	.3513	37	2.85	40.98	2.12	1.16	.85	22.54
Aluminium	1	.7816	37	11.2	6.21	.1462	1.026	.4249	24.94
	6	.3824	37	6.4	19.05	.3289	1.026	.2428	23.94

	10	.2259	37	5.44	26.37	.2199	1.026	.2063	22.54

Table 1: Values of different parameters used for simulation [7].

5. Simulation and Results

All equations are converted into MATLAB codes and output of MATLAB simulation can be given below.

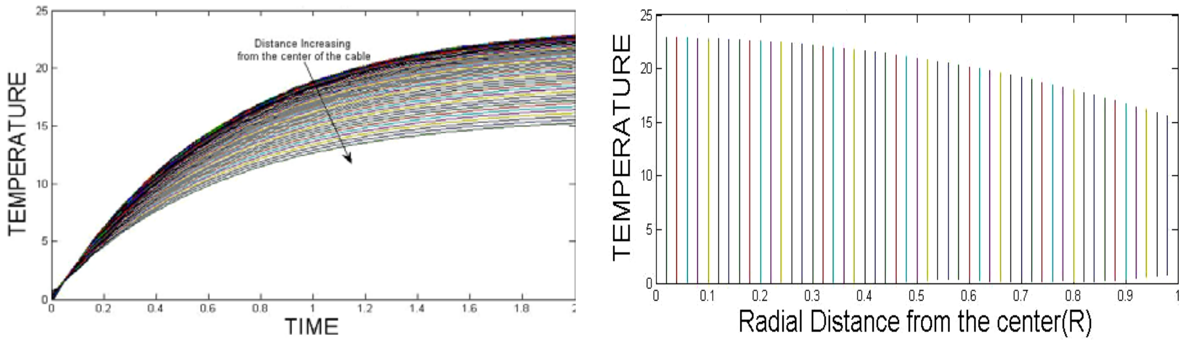

Fig 1 : AWG #6 Copper cable (variation of temperature with time) and (variation of temperature with radial distance)

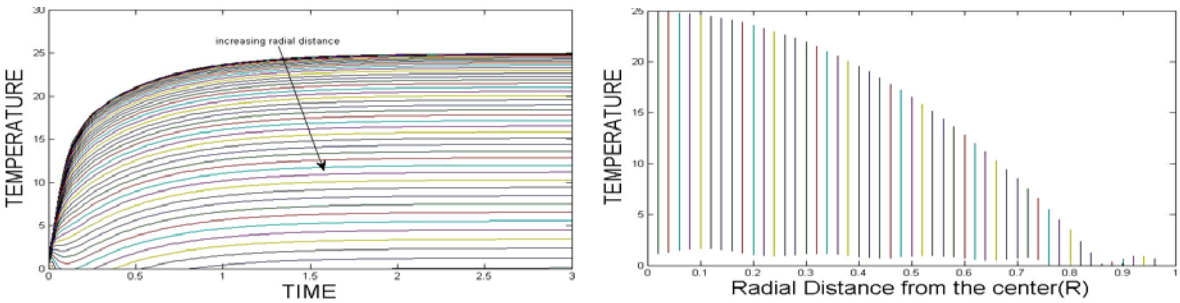

Fig 2 : AWG #1 Copper cable (variation of temperature with time) and (variation of temperature with radial distance)

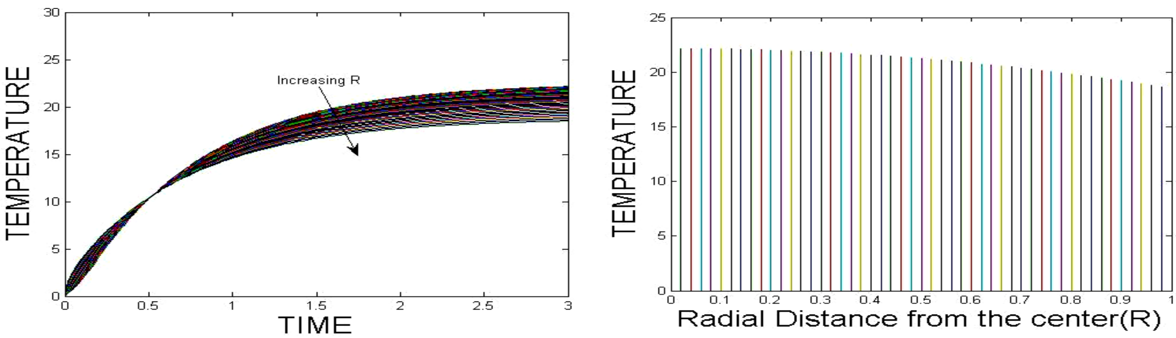

Fig 3 : AWG #10 Copper cable (variation of temperature with time) and (variation of temperature with radial distance)

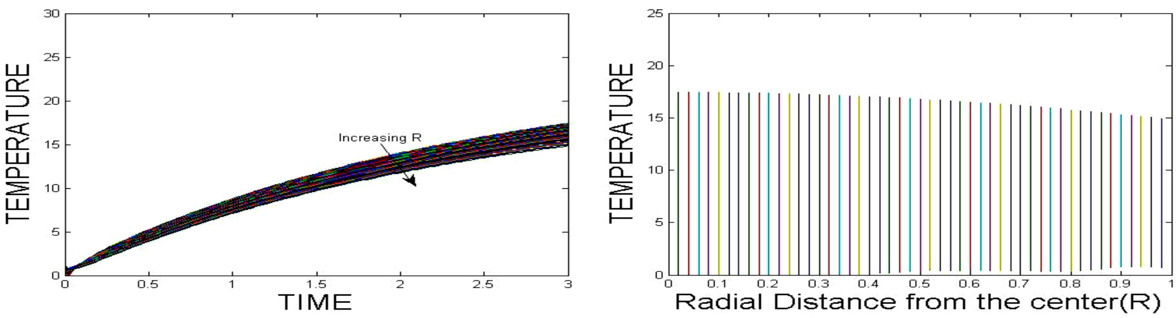

Fig 4 : AWG #6 Aluminium cable (variation of temperature with time) and (variation of temperature with radial distance)

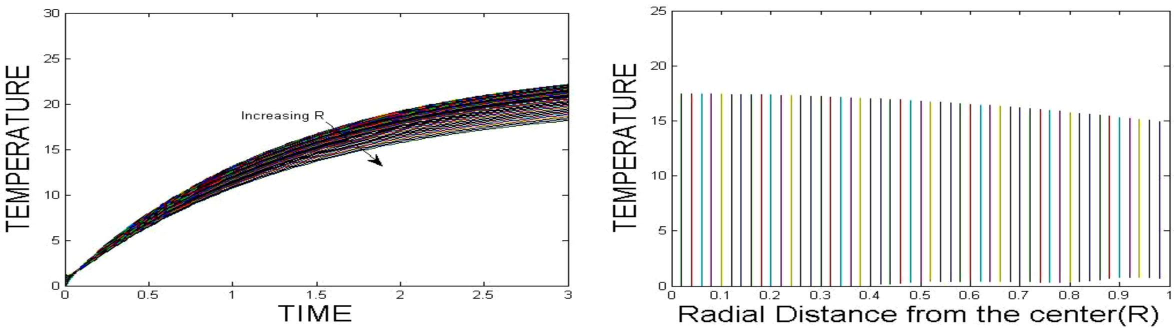

Fig 5 : AWG #1 Aluminium cable (variation of temperature with time) and (variation of temperature with radial distance)

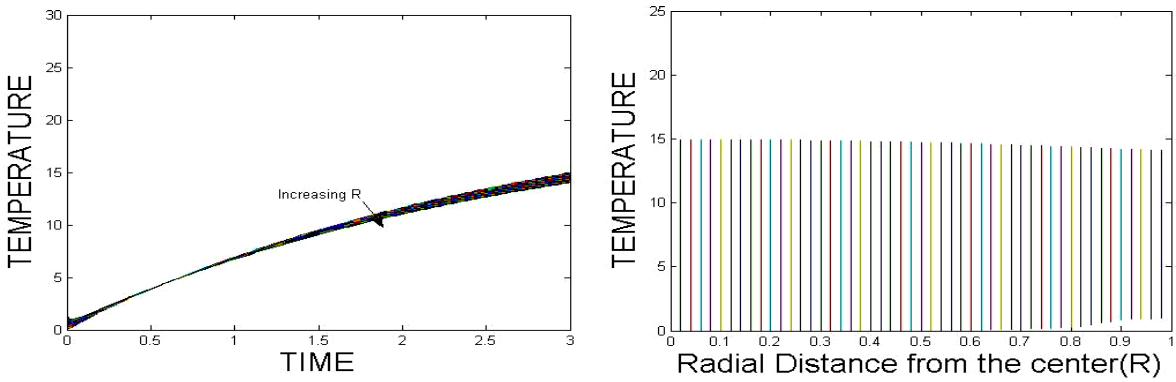

Fig 6 : AWG #10 Aluminium cable (variation of temperature with time) and (variation of temperature with radial distance)

6. Conclusion

The main purpose of this study is to have an idea about the temperature variation within an underground power cable for two commonly used materials. Initially the temperature within the cable starts to increase from the initial value of zero and as time increases it settles down to the steady state value. The value of temperature starts decreasing as we move towards the boundary from the center as expected because of the heat radiated out at the surface. The study shows the fall in temperature is the highest for the least number of AWG cable (for both Aluminium and Copper). As we go for the higher number of AWG cable the more flattened temperature profile is observed. However, if we compare between Aluminium and Copper cable performance for the heat radiation, it is seen that Copper shows far better performance than Aluminium in this regard. For AWG # 10 cable Copper still gives a reduction in temperature from the center to the surface whereas the Aluminium shows almost flattened result for the study period. From the above point of view although it seems that Copper cable should only be used instead of Aluminium but in practice this is not the case. Because, the selection of cable material not only depends on the heat radiation capacity of the cable but also on various other features like, insulation type and thickness, voltage rating, moisture resistance, maximum allowable pulling tension, minimum bending radius, maximum length that can be out on a reel, etc. However, the purpose of this study is to compare the performance of these two cables from the heat radiation point of view and that is done by the help of partial differential equation.

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