

# Design of All-pass Digital Filters Using Second-Order Cone Programming

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**Abstract.** This paper presents a new method for designing an all-pass (AP) digital filter such that its phase response fits a given desired phase response in the minimax error sense. The AP filter design utilizes a new optimization method called second-order cone programming (SOCP) to find the optimal AP filter coefficients. As compared with the existing linear programming (LP) design, the SOCP-based design yields a more accurate fitting. An example is given to illustrate the performance of the SOCP-based AP filter design approach.

**Keywords:** digital filter, signal processing, all-pass filter, second-order cone programming (SOCP).

## 1. Introduction

In digital communications, a linear-phase communication channel is required for equalizing the phase response of the communication channel such that the waveform of the transmitted signal remains unchanged after the signal transmission. Generally speaking, there are two types of phase compensation circuits; one is the finite-impulse-response (FIR) type [1], and another is the infinite-impulse-response (IIR) type. For the FIR case, since both the magnitude and phase responses must be taken into account, the approximation problem is highly non-linear. On the other hand, the all-pass (AP) IIR type only needs to approximate the phase response because its magnitude response is always unity, which makes the phase-compensation circuit design easier [2]. Since the magnitude response of an AP digital filter is always one, it has also been successfully utilized in designing variable fractional-delay (VFD) digital filters [3]-[6]. Basically, since the phase response of an AP digital filter is also highly non-linear with respect to its coefficients, an approximation way needs to be found for linearizing the non-linear design problem as a linear one. In [2], the AP phase-compensation circuit design is approximately linearized as a linear programming (LP) problem, and the stability problem is also discussed.

The main objective of this paper is to propose a new method for designing an AP phase-compensation circuit (digital filter). We show that the AP digital filter design can be simplified as a second-order cone programming (SOCP) problem whose constraints are no longer linear with respect to the filter coefficients. Once the SOCP design is formulated, we can utilize various SOCP-solver (software) to obtain the optimal solution of the formulated SOCP design. In this paper, we apply the well-known efficient software *SeDuMi* to solve the SOCP problem [7], which implements a primal-dual interior-point algorithm called centering-predictor-corrector method. After the SOCP design formulation, we will use a design example to show the design performance and compare the SOCP design with the existing LP design. Simulation results show that the proposed SOCP design yields more accurate results (smaller design errors) than the LP design.

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## 2. SOCP-Based AP Filter Design

$$\begin{aligned} H(z) &= \frac{a_N + a_{N-1}z^{-1} + a_{N-2}z^{-2} + \dots + z^{-N}}{1 + a_1z^{-1} + a_2z^{-2} + \dots + a_Nz^{-N}} \\ &= z^{-N} \cdot \frac{D(z^{-1})}{D(z)} \end{aligned} \quad (1)$$

In this paper, we assume that a given desired (ideal) phase response  $\theta_d(\omega)$  is to be approximated in the whole frequency range  $\omega \in [0, \pi]$ , where  $\omega$  is the normalized angular frequency. To approximate  $\theta_d(\omega)$ , we use an AP digital filter with transfer function

The frequency response  $H(\omega)$  can also be rewritten as

$$H(\omega) = e^{-jN\omega} \cdot \frac{D^*(\omega)}{D(\omega)}. \quad (2)$$

Assume that the desired phase response to be approximated using the AP digital filter is  $\theta_d(\omega)$ . Thus, the desired frequency response is

$$H_d(\omega) = e^{j\theta_d(\omega)}$$

and the frequency response (FR) error can be computed as

$$\begin{aligned} \hat{e}_H(\omega) &= H(\omega) - H_d(\omega) \\ &= \frac{\hat{e}_R(\omega) + j\hat{e}_I(\omega)}{D(\omega)} \\ &= \frac{\hat{e}_H(\omega)}{D(\omega)} \end{aligned} \quad (3)$$

$$\begin{aligned} \hat{e}_R(\omega) &= \cos(N\omega) - \cos\theta_d(\omega) - \sum_{n=1}^N a_n [\cos\theta_d(\omega) - \cos(N-n)\omega] \\ \hat{e}_I(\omega) &= -\sin(N\omega) - \sin\theta_d(\omega) - \sum_{n=1}^N a_n [\sin\theta_d(\omega) + \sin(N-n)\omega]. \end{aligned} \quad (4)$$

Ignoring the denominator, we can solve the SOCP problem:

$$\begin{aligned} \text{minimize} \quad & \epsilon \\ \text{subject to} \quad & \sqrt{\hat{e}_R^2(\omega) + \hat{e}_I^2(\omega)} \leq \epsilon. \end{aligned} \quad (5)$$

The SOCP design (5) can be solved by using the well-known software SeDuMi [7].

## 3. Design Example

The desired phase response  $\theta_d(\omega)$  is given as

$$\theta_d(\omega) = \begin{cases} -12\omega & \omega \in [0, \pi/2] \\ -8\omega - 2\pi & \omega \in [\pi/2, \pi]. \end{cases} \quad (6)$$

To approximate this phase specification, we set the AP filter order  $N = 10$ . Also, 1001 discrete points of the frequency  $\omega \in [0, \pi]$  is used in solving the SOCP problem (5).

Fig. 1 illustrates the actual phase response  $\theta(\omega)$  along with the desired phase response  $\theta_d(\omega)$ , and Fig. 2 shows the phase errors. It can be seen that the actual phase response fits the desired one very accurately. Table 1 lists the design errors from the proposed SOCP-based minimax design. It can be seen that the SOCP design yield smaller design errors than the existing LP design.

Table 1: Design Errors

Method	$e_{\theta 2}$	$e_{\theta max}$	$e_{H2}$	$e_{Hmax}$
SOCP	0.2989%	0.1323	5.8368%	-17.57 dB
LP [2]	0.3721%	0.1779	7.2651 %	-15.01 dB

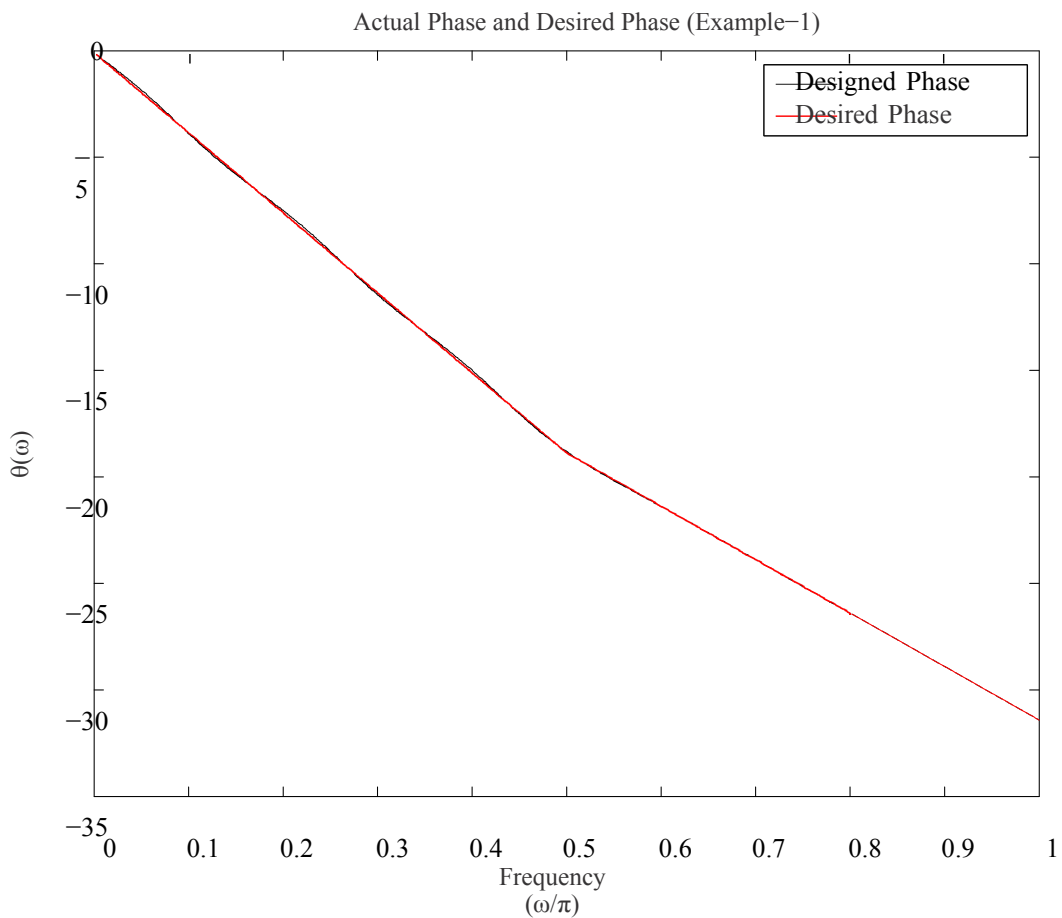


Fig. 1: SOCP-based phase response.

## 4. References

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