Using PDOP to Estimate Kalman Filter's Measurement Noise Covariance for GPS Positioning

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Abstract. GPS is widely used nowadays but with the absolute positioning technology which is used commonly, the positioning accuracy is not high enough. Many studies use Kalman filter to improve the accuracy of the GPS positioning. As we know, we have to determine the measurement noise covariance when modeling the Kalman filter, most of the studies do not pay attention on this and just consider it as a constant. In this paper, we propose a novel method to use the PDOP which can be obtained from the GPS data easily to estimate the measurement noise covariance of the Kalman filter.

Keywords: GPS, Kalma filter, PDOP

1. Introduction

Since we can get the available satellite positioning parameters freely, the GPS system has been widely used in many different fields, like positioning, tracking systems, distance survey etc. Currently, the precision of GPS positioning can be dropped to sub-centimeter by using DGPS [1].

But there are still some problems of DGPS e.g., cycle slips, long initialization time, etc. DGPS based on the carrier phase technology is not so suitable in real-time kinematic distance survey, especially in high speed kinematic distance survey area e.g., survey two cars' distance in the highway. In addition, the DGPS technology needs a base station and devices such as RTK-GPS which are quite expensive. The common GPS devices are based on the absolute positioning technology (Pseudoranging) which just use the data sent by the satellites. But the accuracy is not so high because of many reasons, for instance multipath, clock biases, ionosphere errors, etc. Document [2] introduced some methods to improve the accuracy of absolute positioning. But they all focus on the GPS calculation model, not the data processing after receiving .

There are many studies on optimizing the GPS data to get more accurate position such as Paper[3] use inertial navigation system(INS) to improve the accuracy of the vehicle's position, Paper[4] use Kalman filter to do the data processing. Since the Kalman filter can estimate a "correct" position, it is widely used not only in the absolute positioning technology, but also in the DGPS technology. As we know, there are a few parameters have to be tuned in the Kalman filter such as the process noise covariance Q and the measurement noise covariance R [5]. But Most of these studies focus on building the dynamic model of Kalman filter and just consider the noise covariance as a constant. Paper [6] proposes a Sage adaptive filtering method to build the Kalman filter. But the method is quite complicated and do not use the real-time environment's information to estimate the noise.

As we know, the GPS receiver can get much real-time information, so we can use this information to improve the noise estimation. The most popular format of GPS information is NMEA-0183, from which we can get much information such as PDOP, HDOP and so on [7]. This information is relative to the accuracy of the position.

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In this paper we discuss how to use the DOP especially the PDOP to estimate the measurement noise covariance to model our Kalman filter. In Section 2, we introduce Kalman filter briefly. In Section 3, we introduce Dilution of Precision (DOP). In Section 4, we propose a method to use DOP in Kalman filter and show the experiment result we did. In Section 5, we summarize and draw conclusions.

2. Kalman filter

We know there are many errors included in the GPS's Pseudorang such as Ephemeris errors, GPS receiver clock errors, multipath errors .etc. Document [2] has introduced some method to deal with these errors and most of them can be modeled. But with Kalman filter, we can decrease these errors' effect efficiently.

The Kalman filter addresses the general problem of trying to estimate the state $x \in R^n$ of a discrete-time controlled process that is governed by the linear stochastic difference equation [5]:

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \tag{1}$$

The $n \times n$ matrix A relates the state at the previous time step $k-1$ to the state at the current step k, in the absence of either a driving function or process noise. The $n \times l$ matrix B relates the optional control input $u \in R^l$ to the state x.

We define the measurement $z \in R^m$, which is as follow:

$$z_k = Hx_k + v_k \tag{2}$$

The $m \times n$ matrix H in the measurement equation relates the state to the measurement z_k.

The random variables w_k and v_k represent the process and measurement noise (respectively). They are assumed to be independent (of each other), white, and with normal probability distributions:

$$p(w) \sim N(0, Q) \tag{3}$$
$$p(v) \sim N(0, R) \tag{4}$$

Q is the process noise covariance and R is the measurement noise covariance. Here is the Discrete Kalman filter Algorithm:

1) The Discrete Kalman filter time update equations are as follow:

$$x_k(k \mid k-1) = Ax_{k-1}(k-1 \mid k-1) + Bu_{k-1} \tag{5}$$
$$P_k(k \mid k-1) = AP_{k-1}(k-1 \mid k-1)A^T + Q \tag{6}$$

2) The Discrete Kalman filter measurement update equations are as follow:

$$K_k = P_k(k \mid k-1)H^T(HP_k(k \mid k-1)H^T + R)^{-1} \tag{7}$$
$$x_k(k \mid k) = x_k(k \mid k-1) + K_k(z_k - Hx_k(k \mid k-1)) \tag{8}$$
$$P_k(k \mid k) = (I - K_k H)P_k(k \mid k-1) \tag{9}$$

Here, $x_k(k \mid k-1)$ and $x_k(k \mid k)$ are the priori state estimate and posteriori state estimate respectively. $P_k(k \mid k-1)$ and $P_k(k \mid k)$ are the priori estimate error covariance and posteriori estimate error covariance respectively. K_k is the gain or blending factor that minimizes the a posteriori error covariance.

In order to use the Kalman filter to process the GPS data, we have to know the process noise covariance Q and the measurement noise covariance R. In practice, the Q and R matrices might change with each time step or measurement and hard to model and calculate.

In this paper, we assume the process noise covariance Q is constant, and we discuss how to use the PDOP to estimate the R.

3. Dilution of Precision (DOP)

The idea of Geometric DOP is to state how errors in the measurement will affect the final state estimation. This can be defined as [8]:

$$GDOP = \Delta OutputLocation / \Delta MeasuredData \tag{10}$$

$\Delta MeasuredData$ is the error on a measurement resulting, $\Delta OutputLocation$ is the differential between the location got from the receiver and the satellites. The GDOP can be illustrated in Fig.1 and Fig.2 as follow:
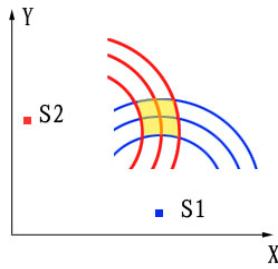

Fig.1 the position uncertainty is small
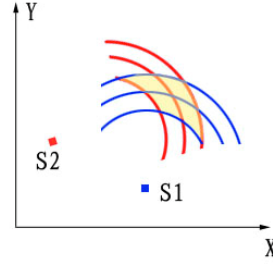(Low dilution of precision)

Fig.2 the position uncertainty is considerably larger
(High dilution of precision)

As can see above, when we use the Pseudorang to calculate the position, there are many errors included in the pseudorang. Two satellites can form an error area (the yellow area in the figure). If the angle of these two satellites is good enough, the error area will be small, but if not, the error area will be large. So we use DOP to describe how the satellites' positions affect the positioning precision. There are 5 parameters to describe the DOP:

$$PDOP = \sqrt{\sigma_E^2 + \sigma_N^2 + \sigma_U^2} / \sigma \tag{11}$$

$$HDOP = \sqrt{\sigma_E^2 + \sigma_N^2} / \sigma \tag{12}$$

$$VDOP = \sqrt{\sigma_U^2} / \sigma \tag{13}$$

$$TDOP = \sqrt{\sigma_T^2} / \sigma \tag{14}$$

$$GDOP = \sqrt{\sigma_E^2 + \sigma_N^2 + \sigma_U^2 + \sigma_T^2} / \sigma \tag{15}$$

Here $\sigma_E^2, \sigma_N^2, \sigma_U^2$ are the variances of the east, north and up components of the receiver position estimate respectively, and σ_T^2 is the variance of the receiver clock offset estimate. σ is the overall standard deviation in range[9]. VDOP is a measurement of the accuracy in standard deviation in vertical height; HDOP is a measurement of the accuracy in 2-D horizontal position; PDOP is a measurement of the accuracy in 3-D position; including the receiver clock offset estimate in PDOP we can get GDOP. Because σ_T^2 is difficult to measure so we did not consider it in this paper. We just use PDOP to estimate the measurement noise covariance.

The most popular GPS data format is NMEA-0183 which includes PDOP, HDOP, VDOP and so on [10]. With a low cost GPS receiver, we can receive the NMEA data easily.

4. Propose method and Experiment

As we can see from above, the PDOP represent the errors' variances and in the Kalman filter we need the measurement noise covariance R. It was proposed to use the PDOP to estimate the R. The PDOP will change timely based on different user's environments and satellites' positions, so it's suitable to estimate the measurement noise covariance. In this paper, we only used PDOP to replace R in the Kalman filter.

Our experiment was done in the playground with GPS receiver AKN-1M. Because we cannot know our real position since the position result from the receiver included errors, we use two receivers and fix their distance. In the first experiment, the fixed distance is 4 m and in the second experiment, it is 10 m. Both of the two receivers are static. We will do the experiment of the dynamic situation in the future. Our experiment can be described as follow:
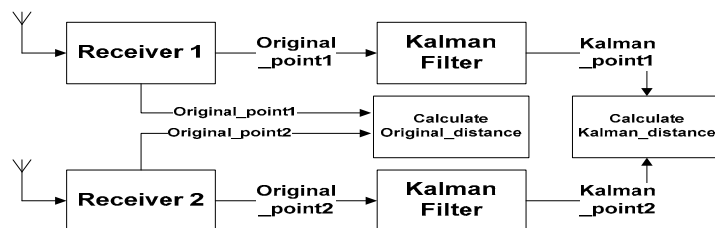

Fig.3 Flow chart of our experiment

We get the GPS data with these two receivers in the same time and calculate these two points' positions (we call them **Original_points**). The **Original_points** will be modified by our Kalman filter whose measurement noise covariance is the PDOP value. We call the modified points **Kalman_points**. In this way, we can calculate the distance between every two **Original_points** and every two **Kalman_points**. Then we can get the **Original_distance** and **Kalman_distance**. If there are no errors, these two distances will be the same and will be a constant. If not, these two distances will be different from the fixed distance. Here is the result of our experiment:

a) is the Original_points   b) is the Kalman_points

Fig.4 the fixed distance between two receivers is 4m

a) is the Original_points   b) is the Kalman_points

Fig.5 the fixed distance between two receivers is 10m

Figure 4 a) and figure 5 a) are the original points which were got from the GPS receivers; Figure 4 b) and figure 5 b) are the Kalman points which were got after Kalman filter. The fixed distance between two receivers in figure 4 is 4 m and in figure 5 is 10 m.

a) fixed distance is 4m   b) fixed distance is 10m

Fig.6 the red line is the Original_distance, the blue line is the Kalman_distance, the pink line is the fixed distance

Figure 6 a) and b) shows the calculation result of **Original_distance**(the red line)and **Kalman_distance** (the blue line). The pink line is the fixed distance between two GPS receivers which is 4m in a) and 10 m in b). From figure 6, we can see that the **Kalman_distance** is more close to the fixed distance than the **Original_distance**.

Tab.1 is the Mean and Variance of the Original_distance and Kalman_distance

Fixed distance=4m	Mean(m)	Variance
Original_distance	4.0397	0.0401
Kalman_distance	4.0034	0.0033
Fixed distance =10m	Mean(m)	Variance
Original_distance	9.9385	0.0373
Kalman_distance	10.001	0.0062

From Table 1, we can see that the mean of **Kalman_distance** is more close to the fixed distance and the variance of **Kalman_distance** is much smaller than the **Original_diatance**'s. From the experiment above, we can see that the result of our Kalman filter, which uses the PDOP as the measurement noise covariance, is much better than the one not using it.

5. Conclusion

In this paper, we propose a method which uses the PDOP to estimate the Kalman filter's measurement noise covariance. The PDOP is very easy to get from the GPS data. And from experiments, we showed that the position result becomes better by using our Kalman filter. In the future, we will finish the dynamic Kalman filter model and test our method in it.

6. Acknowledgements

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