

Evolutionary Algorithms Based Fixed Order Robust Controller Design and Robustness Performance Analysis

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Abstract. Evolutionary Algorithms (EAs) are stochastic search techniques that direct a population of solutions towards best possible results by using the natural principles. In recent years, these algorithms have grown to be an accepted optimization tool for many areas of scientific and engineering research, together with control system engineering design. Significant research exists concerning evolutionary algorithms to control system design and robustness performance analysis of controller. But, little work has been done with evolutionary optimization algorithms controls because of the problems related with robustness performance in early period of the evolution of controllers. Moreover, until recently the robustness performance of controllers based on evolutionary algorithms has not been researched in stipulate.

Keywords: Evolutionary algorithm, optimization, robustness, performance and controller

1. Introduction

A successfully designed control system should be always able to maintain stability and performance level in spite of uncertainties in the system. Due to its importance, however, the research on robust design has been going on all the time. A breakthrough came with the pioneering work by [1] on the theory, now known as the H_∞ control theory. A controller that performs suitably in the presence of plant parameter variation is said to be a robust. The aim in control system design is to design a controller, which connected to a system, provides a desired behaviour of the control system. Fixed order robust controllers have become interesting area of researchers because of simplicity and acceptable controller order [2].

Regardless of the existing elegant methods of robust control, engineers complain about the gap between theories and practice in control systems the design techniques cannot incorporate realistic constraints such is fixed structure [3]. Interesting ways solving the problems are heuristic algorithms. Mostly, the controllers used in industrial process are PI or PID controllers. Unfortunately, tuning of control parameters of these controllers for achieving both robustness and performance specifications is difficult. To overcome this problem, the approaches to design a robust control for structure specified controller were proposed in [4-5]. EAs have emerged as a contestant due to its flexibility effectiveness in variety of optimization applications.

PSO is general purpose optimizer that solves the wide range of optimization problems thus the PSO can be adapted to various categories of optimization. In IA, antigen represents the problem to be solved and an antibody set is generated where each number represents a candidate solution [6]. In IA n number of antibodies generated randomly [7].

In this paper, the performance and robust stability conditions of the system satisfying the H_∞ loop shaping are formulated as the cost function. EAs i.e., Particle Swarm Optimization (PSO) and Immune Algorithm (IA) are adopted to solve this problem and to achieve the performance of the designed controller.

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Simulation results validates that the robustness and performance of the proposed robust controller design approach.

Paper is organized as: section 2, presents brief overview of evolutionary algorithms, Loop shaping design is given in section 3, design procedure of the proposed schemes is discussed in section 4, Section 5 presents simulation results, robustness performance analysis is presented in section 6 and conclusion is placed in section 7.

2. Evolutionary Algorithms

Optimization has been essential component of many engineering technology fields, a number of approaches exist to get an optimal behaviour in a process of plant. EAs have emerged as a contestant due to its flexibility effectiveness in variety of optimization applications [2, 4, and 9]. PSO is population based optimization technique that has many advantages over other classical optimization procedures [6, 9]. A population of particles is initialized with random positions and velocities and a function is evaluated, using the particle's positional coordinates as input values. Positions and velocities are adjusted and the function evaluated with the new coordinates at each time step. When a particle discovers a pattern that is better than any it has found previously, it stores the coordinates in a vector. The difference between (the best point found by so far) and the individual's current position are stochastically added to the current velocity, causing the trajectory to oscillate around that point [8].

In IA, antigen represents the problem to be solved and an antibody set is generated where each number represents a candidate solution. Also an affinity is the fit of an antibody to the antigen. The role of antibody is to eliminate the antigen. Also an affinity is the fit of an antibody to the antigen. The role of antibody is to eliminate the antigen [9]. While affinity of all antibodies is known new population is generated through three steps: replacement, cloning and hyper mutation. In replacement step low antibodies are replaced those with highest affinity are selected by cloning and hyper mutation is applied where the mutation rate is inversely proportional to its affinity [10].

3. Loop Shaping

Two weighting functions W_1 and W_2 are specified to shape the original plant. The singular values of the shaped plant satisfy the closed-loop performance requirements. This organized procedure has its foundation in [11].

$$G_s = W_1 G_o W_2 \quad (1)$$

The weighting functions are chosen as

$$W_1 = \frac{0.80s + 4}{s + 0.001}, \quad W_2 = I \quad (2)$$

where I , is the identity matrix, the controller K_∞ is synthesized (Mat lab code) and final controller $K(s)$ is constructed by multiplying K_∞ with weighting functions as shown in Eq.(3)

$$K(s)_{final} = W_1 K_\infty W_2 \quad (3)$$

4. Fixed Order Robust Controller Design

In the proposed technique, PSO and IA are used to minimize the cost function. The transfer function of the identified plant model is given in Eq. (4)

$$G(s) = \frac{551.1e^{-0.12s}}{s^2 + 43.26s + 536.9} \quad (4)$$

The structure $K(p)$ of controller is specified before starting the optimization process. A set of controller parameters p is evaluated to minimize CF. In this paper, EAs (PSO and IA) are adopted to find the optimal values of controller parameters p^* in stabilizing controller $K(p)$ such that $\|T_{wz}\|_\infty$ is minimized.

By using eq. (3) controller $K(p)$ can be written as:

$$K(p) = W_1 K_\infty W_2 \quad (5)$$

It is assumed that W_1 and W_2 are invertible, therefore,

$$K_\infty = W_1^{-1} K(p) W_2^{-1} \quad (6)$$

A controller K_∞ which stabilize the original closed loop system and minimizes gamma.

$$\gamma = \inf_k \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I + G_s K_\infty)^{-1} M^{-1} \right\|_\infty \quad (7)$$

By substituting Eq. (6) in Eq. (7), the H_∞ -norms of the transfer matrix from disturbances to states, which has to be, minimized i.e. cost function is written as:

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ W_1^{-1} K(p) \end{bmatrix} (I + G_s W_1^{-1} K(p) (I \quad G_s)) \right\|_\infty \quad (8)$$

The specified controller structure is expressed in Eq. (9)

$$K(p) = k_p + \frac{k_i}{s} \quad (9)$$

4.1. Proposed scheme using PSO

Step-1 initializes several sets of population parameters p as population of particles, where p is considered as a vector of controller parameters.

Step-2 Specify the controller structure, evaluate CF of each particle using Eq. (8)

Step-3 at each generation the velocity of each particle and position of the next is calculated.

Step-4 if current iteration is less maximum iterations then stop, go to step 3.

4.2. Proposed Scheme by Using IA

Step-1 Generate initial sets of parameters p as population of antibodies

Step-2 Specify the controller structure $K(p)$ where p is considered for each string of antibodies, evaluate CF of each antibody using Eq. (8).

Step-3 Best antibody in the present problem is chosen as an antigen, which has minimum CF.

5. Simulation Results

The fixed order robust controller design by using EAs has been simulated to predict performance of the proposed approach. By using the LSDP and Mat lab coding the K_∞ controller is obtained as:

$$K_\infty(s) = \frac{413.5s + 2205}{s^3 + 43.36s^2 + 5369s + 0.5369} \quad (10)$$

By using the H_∞ LSDP the final controller is obtained as:

$$K(s) = \frac{310.1s^2 + 3308s + 8221}{s^4 + 43.26s^3 + 537s^2 + 1.07s + 5.36 \times 10^{-2}} \quad (17)$$

The controller obtained by conventional techniques Eq. (17) is of 4th order and its structure is complex and difficult to implement practically. The specific controller structure is expressed in Eq. (18).

$$K(p) = k_p + \frac{k_i}{s} \quad (18)$$

The simulation was carried out using representation of particles. The size of initial population is 10 particles. Algorithm converged in 4th iterations, and gave optimal CF value of 1.474. Fig. 2 shows the plot of convergence of cost function versus iterations of PSO. The optimal solution of controller parameters was obtained which has satisfied stability margin of 0.680. The computed optimal values of controller parameters are shown in Eq. (19)

$$K(p)^* = 0.4629 + \frac{0.0415}{s} \tag{19}$$

The step response of the control system with optimized controller parameters using PSO is shown in Fig.1; the step response presents rise time 1.25 sec., about 2% overshoot and the settling time is about 1.23 sec.

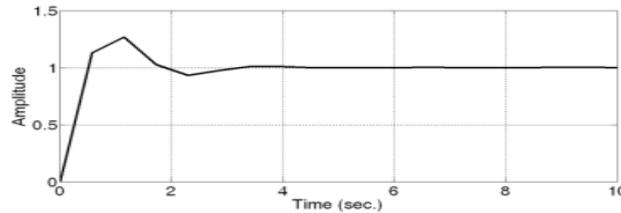


Fig. 1: Step response obtained with PSO

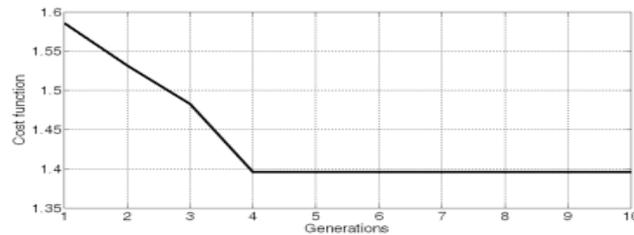


Fig. 2: Convergence of CF Vs.iterations of PSO

The simulation was carried out using IA with representation of antibodies. The size of initial population was 10 antibodies, colonial affinity was calculated and single bit mutation was used. IA converged on the 3rd iteration and gave the optimal CF of 1.309. Fig. 3 shows the plot of convergence of cost function versus iterations of IA, which has satisfied stability margin of 0.763. Obtained optimal values of controller parameters are shown in Eq. (20).

$$K(p)^* = 0.9991 + \frac{0.584}{s} \tag{20}$$

The step response of the control system with optimized controller parameters by using IA is shown in fig.4; the step response presents rise time 1.06 sec., 2% overshoot and the settling time is about 2 sec. the results obtained clearly shows the effectiveness of proposed scheme.

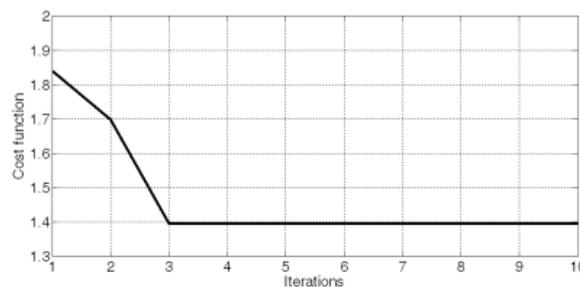


Fig.3: Convergence of CF Vs. iterations of IA

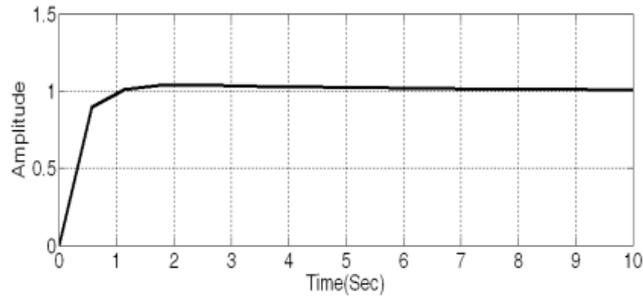


Fig. 4: Step response obtained by IA

6. Robustness Performance Analysis

In order to validate the suitability and robustness performance of designed controllers, some parameters of the nominal plant in Eq. (4) were varied as follows:

$$G_{\Delta}(s) = \frac{551.1e^{-0.12s}}{(8s^2 + 43.26s + 536.9)} \quad (21)$$

The designed optimal controller Eq. (19) is tested on perturbed plant Eq. (20). The result shown in fig. 5, demonstrates that the designed controller using IA have reasonably good performance and robustness.

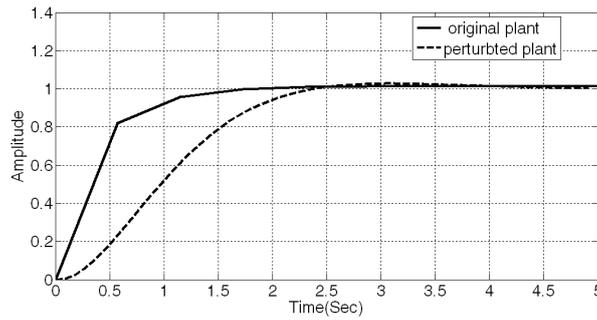


Fig.5: Shows robust check of IA controller

Tab.1: Comparison between optimized parameters

Parameters	PSO	IA	H_{∞}
K_p	0.4629	0.9991	-
K_i	0.415	0.584	-
CF	1.474	1.309	1.496

The CF value and the parameters of the controller optimized using PSO and IA were compared. Results shown in Tab.1, indicates that EAs gave much better solutions than conventional H_{∞} . Moreover, the best CF value is obtained from IA as compared to PSO

Tab.2: Comparison between PSO and IA

Parameters	PSO	IA
S_t in sec.	2.87	2.0
R_t in sec.	1.26	1.06
%Overshoot	2.5%	2%

Comparisons shown in Tab.2, PSO has higher settling time than IA. So tuning PI controller for plant using IA is more optimal than PSO. The controller optimized with IA has provided much better response than controller optimized with PSO.

7. Conclusion

An appropriated performance that satisfying the time domain specifications and robustness is evaluated by PSO and IA, the responses from the designed controllers from proposed EA schemes were compared. This proposed technique is an alternative method which directly considers the performance specifications and robustness in the design and in which the structure of controller is not restricted to PID. The controller $K(p)$ can be replaced by any fixed-structure controller and the proposed algorithms can still be applied functionally.

8. References

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