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Order Quantity Optimization Problem with Limited Budget and Free Return

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Abstract. In this paper we formulate the order quantity optimization problem for a retailer facing correlated demands. The budget of the retailer is limited, but on the other hand free return policy effectively increases her budget for purchasing more products. This order quantity optimization problem is NP-hard, and we propose both heuristic and metaheuristic methods for solving it. Computational results demonstrate that simulated annealing method significantly improves the solution quality at a cost of more computation time.

Keywords: Order quantity, Inventory control, Heuristic optimization, Metaheuristics, Simulated annealing

1. Introduction

Classical inventory control problem focuses on deciding the optimal order quantity for a single commodity [11]. A well-known paper by [4] developed a so-called $s - S$ policy for a class of multi-period replenishment problem. In this paper, we consider the problem where the retailer sells multiple products and wants to determine how many units to order for each product. We assume that the retailer has limited budget and can only afford to buy a limited quantify of each product. In addition, we assume free return policy (see [5], [8] and [3]), that is, the retailer is allowed to return products to the supplier for cash or credit that will be used to pay for other products that she wants. There is a limit on the free return policy, and the credit the retailer can receive from free return is capped above. The retail demands for the products are correlated, and if the retailer orders a non-zero amount of a product, she incurs a fixed order cost for it, as well as a variable cost that depends on the order quantity.

In the following sections, we will formulate the order quantity optimization problem characterized by correlated demands, limited budget and free return policy. We will then develop a construction heuristic to solve the problem, and propose a metaheuristic method to improve the solution quality. We will present computational results, and conclude the paper with potential future work.

2. Problem Formulation

We consider the order quantity optimization problem for a retailer who sells multiple products. Let N be the number of products. In each period the retailer produces a forecast on the demand of each product. She also has forecasts, either derived from historical demand information or simply purchased from a third-party information provider, on the correlation of the demands. Further, if the retailer decides to order a non-zero quantity of a product, she incurs a fixed order cost for that product and a variable order cost that depends on how many unites are to be ordered. Since we allow free return, ordering a negative amount of a product simply means that the retailer returns the product for credits or cash. The retailer has limited budget, B, and is not allowed to return products exceeding a threshold value. For simplicity, we assume that this threshold is the same as her budget, B.

Mathematically, let ai denote the expected revenue the retailer will gain for stocking a single unit of product i, and let sij denote the covariance of the revenues for product i and j. ki denotes the fixed order cost for product i, and c(|xi|) is the variable cost if the retailer orders xi units of product i. Then, the retailer maximizes her risk-adjusted utility function, f(x), as follows.

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$$\max_{x_1,\ldots,x_N} \sum_{i=1}^{N} a_i x_i - r \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j s_{ij} - \sum_{i=1}^{N} k_i \mathbf{1}\{x_i \neq 0\} - \sum_{i=1}^{N} c(|x_i|) \quad (1)$$

$$\text{s.t.} \quad \sum_{i=1} a_i x_i \leq B$$

$$-\sum_{i=1}^{N} a_i x_i \leq B \quad (2)$$

$$x_i \text{ is an integer for } i = 1, \ldots, N, \quad (3)$$

where 1{ • } is the indicator function and r is the risk-adjustment coefficient. Constraints (1) and (2) limit the net amount of order the retailer can place. The problem is clearly NP-hard, since the quadratic knapsack problem [10] is a special case of the model presented above. To solve the problem, we will first construct a solution using a hill-climbing based approach. A meta-heuristic approach is then developed to improve the solution quality.

3 Construction Method

Constructing a good solution for the order quantity optimization with correlated demands is not a simple task due to the correlation between the demands. For instance, the well-known dynamic programming approach for knapsack problems [9] does not work here, due to lack of sub-problem optimality (i.e. Bellman's principle of optimality [2]). Instead, we develop and implement a hill-climbing based construction method as follows.

To construct a solution of reasonably good quality, we start with the naive solution x = (0; : : : ; 0), that is, the retailer doesn't order or return any product. We then iteratively improve the solution quality by changing xi by δi, with δi ∈ {−1; 1}; we call this operator 1-Opt. More specifically, in each iteration we go through i ∈ {1; : : : ;N} and check whether changing xi to xi + δi will improve current solution or not. If it does, we modify current solution by setting xi = xi+δi, and repeat the process. We stop until no further improvement can be made.

To search a larger neighborhood, we also implement a variant of the construction method with 2-Opt that allows simultaneously changing x_i and x_j by δi and δj respectively, where i/= j and δi; δj ∈ {−2;−1; 1; 2}. In each iteration of this construction method, we modify the order quantity of two products aiming to improve the risk-adjusted utility for the retailer while maintaining the feasibility of the solution. Again, we repeat the step until there is no further improvement.

4 Simulated Annealing

Simulated annealing (SA) algorithm is used to further improve the solution quality. SA is an extension to Hill-climbing algorithm. Given a new solution x^{new} that is better than or equal to the current solution $x^{current,}$ SA always accepts x^{new}. In addition, a move that leads to a solution worse than the current solution is accepted with probability $P_{accept} = e\frac{\Delta}{T}$, where Δ = f($x^{new}$) − f($x^{current}$) is the change in the retailer's utility function f(x), and T is a control parameter. Simulated annealing algorithm for our order quantity optimization problem is described in Algorithm 1 (see [6], [7] and [1] for more references on SA).

In our computational experiment, the initial solution is either x = (0; : : : ; 0) or the solution produced by the construction heuristic, and the initial temperature T is set as 60. The new temperature is calculated by the formula T′ = λT, where T is the new temperature and λ = 0:995 is the cooling rate. The stop criterion in Algorithm 1 is to stop the iterations when $x^{current}$ is not updated in the inner loop. The inner loop criterion is to perform the loop *L* times. The value of *L* is set as 30000.

Algorithm 1 Simulated Annealing for Order Quantity Optimization Problem

```
Get an initial solution x^current and an initial temperature T.
while Stop criterion not satisfied do
  while Inner loop criterion not satisfied do
    Select a neighbor x^new of x^current by 1-Opt or 2-Opt
    Let Δ = f(x^new) − f(x^current).
    if Δ ≥ 0 then
      Set x^current = x^new.
    else
      Set x^current = x^new with probability e^(Δ/T).
    end if
  end while
  Reduce temperature T.
end while
```

Table 1: Statistics of Solutions by Construction Heuristic and Simulation Annealing with 1- Opt

	mean	standard deviation	signal to noise ratio
Construction	54536.16	13034.38	4.18
SA	59339.35	13032.05	4.55

5 Computational Results

We implemented the construction method and SA algorithm in Java, and tested the methods with 100 randomly generated problem instances with N = 100 products. The results in Table 1 show that with 1-Opt only SA produces much better solutions than the construction heuristic. The average improvement is about 8.81%, at the cost of more computation time (however, on average the SA method took less than a minute on a Intel Core2Duo computer with 2 2.26GHz CPUs and 2G RAM). Lastly, when both 1-Opt and 2-Opt were used the gain of SA decreased to 0.80%, which demonstrated the power of a larger neighborhood move for the construction heuristic.

6 Conclusions and Future Work

We have formulated the order quantity optimization problem and solved it using construction heuristic and simulated annealing, and the computational results demonstrated that simulated annealing produced better results. For future work, we will explore the problems with asymmetric cost between orders and returns. In addition, other popular metaheuristic methods, such as Tabu Search and Genetic Algorithm, may produce solutions of better quality. Lastly, neighborhood moves that involve more than two products are worth exploring as well

7. References

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