2012 International Conference on System Modeling and Optimization (ICSMO 2012)
IPCSIT vol. 23 (2012) © (2012) IACSIT Press, Singapore

Local search heuristics for the generalized vehicle routing problem

PetricA C. Pop [1] and Andrei Horvat-Marc [1]

[1] North University of Baia Mare, Romania

Abstract. The generalized vehicle routing problem (GVRP) is an extension of the well-known vehicle routing problem (VRP), where the set of customers is partitioned into a given number of subsets (clusters) and the aim is to find a collection of routes starting and ending at the depot of minimum cost, such that each cluster should be visited exactly once, the entering and leaving nodes of each cluster is the same and the sum of all the demands of any route does not exceed the capacity of the vehicle. In this paper, we consider theoretical aspects of several neighborhoods: basic adaptations of simple VRP and TSP neighborhoods and some new ones.

Keywords: Heuristics, Local search, Neighborhood, Vehicle Routing Problem, Generalized Vehicle Routing Problem

1. Introduction

Problems associated with determining optimal routes for vehicles from one or several depots to a given set of locations/customers are known as Vehicle Routing Problems (VRPs). These problems have many practical applications in the field of distribution, collection and logistics. A wide body of literature exists on the problem (for an extensive bibliography, see for example Laporte and Osman [5]).

Given a set of vehicles, a set of locations containing as well the depot location and the distance between each pair of locations the VRP consists in finding the minimum cost tour for each vehicle such that all locations are visited, and each vehicle returns to the depot. Because the simplicity of the VRP, most attractive to may researchers has been the variations of the VRP, built on the basic VRP with extra features:

the Capacitated VRP;

the VRP with Time Windows;

the VRP with Multiple Depots;

the Multi-Commodity VRP;

the Generalized Vehicle Routing Problem.

We will confine in this paper on the last mentioned variation of the classical VRP, the generalized vehicle routing problem (GVRP) introduced by Ghiani and Improta [3]. The GVRP is the problem of of designing optimal delivery or collection routes, subject to capacity restrictions, from a given depot to a number of predefined, mutually exclusive and exhaustive clusters.

The aim of this paper is to define, describe and analyse some known and some new neighbourhoods of the GVRP.

2. Definition and Complexity of the Gvrp

Let $G = (V,A)$ be a directed graph with $V=\{0,1,2,\ldots,n\}$ as the set of vertices and $A=\{(i,j) \mid i,j\in V, i\neq j\}$. We assume that a nonnegative cost denoted by c_{ij} associated with each arc $(i,j)\in A$. The set of vertices is partitione into k+1 mutually exclusive nonempty subsets V_p, $p\in\{0,1,\ldots k\}$, called clusters, such that

$V=V_0 \cup V_1 \cup \ldots \cup V_k$ and $V_l \cap V_p = \varnothing$ for all $l,p \in \{0,1,\ldots k\}$ and $l \neq p$. The cluster V_0 has only one vertex 0, which represents the depot, and remaining n nodes belonging to the remaining k clusters represent geographically dispersed customers. Each customer has a certain amount of demand and the total demand of each cluster can be satis_ed via any of its nodes. There exist m identical vehicles, each with a capacity Q.

The GVRP consists in finding the minimum total cost routes of starting and ending at the depot, such that each cluster should be visited by exactly once, the entering and leaving nodes of each cluster is the same and the sum of all the demands of any tour (route) does not exceed the capacity of the vehicle Q.

The GVRP involves two related decisions:

choosing a node subset $S \subseteq V$ such that $|S \cap Vp|=1$, for all $p=1,\ldots,k$

finding the minimum cost collection of routes satisfying the demands and capacity restrictions in the subgraph of G induced by S.

An illustrative scheme of the GVRP and a feasible tour is shown in the next figure.

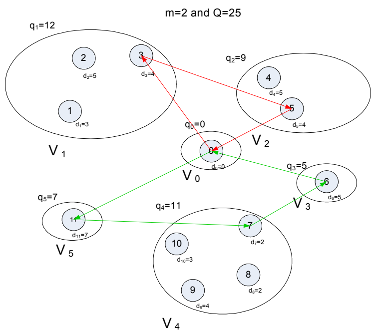

Fig. 1 A feasible solution to the Generalized Vehicle Routing Problem

The GVRP is NP-hard because it includes the generalized traveling salesman problems as a special case when $m=1$ and $Q=\infty$.

Because of the complexity of the GVRP, researchers have focused on efficient transformations of the problem into simpler combinatorial optimization problems: Ghiani and Improta described in [3] a transformation of the GVRP into the capacitated arc routinf problem, Baldacci et al. [1] proved that the reverse transformation is valid and Pop et al. [10] described an efficient transformation of the GVRP into the classical VRP.

Integer linear programming formulations of the GVRP have been provided by Kara and Bektas [2] and Pop et al. [7].

In order to obtain good solutions for large GVRP instances, one should consider heuristic and metaheuristic approaches. Some metaheuristics have been already proposed in the literature: an ant colony algorithm [9], a genetic algorithm approach [8], an adaptive large neighborhood search [2] and an incremental tabu search heuristic [6].

3. Local Search

Local search (LS) is a general method for designing heuristic algorithms for optimization problems. The basic idea of the LS is to start with some feasible solution x of the problem, to evaluate it (i.e. calculate the correspondinding objective function $c(x)$) and then evaluate $c(y)$ for some feasible solution y which is a neighbor of x. If a neighbor y with $c(y)<c(x)$ is found then select y and repeat the same procedure. If no such neighbor is found then the process stops: a local optimum was found.

The characteristics of the LS algorithms are determined once we specify what it means for feasible solutions to be neighbors.

The following tradeoff arises in LS approaches: when we consider larger neighborhoods there are fewer local optima and better solutions are likely to be obtained, but on the other hand, the larger the neighborhood is, more feasible solutions have to be examined at each iteration and the algorithm gets slower.

4. Local Search Heuristics for the GVRP

We present in this section 7 neighborhoods used in our local search approach. Two of them operate within the routes, called intra-route neighborhoods, and the rest operate between the routes and are called inter-route neighborhoods.

The two intra-route neighborhoods denoted in the figure two-point move and three-opt move are used for improving single vehicle routes. As we can see there are different ways to reconnect the paths that yield a different route. Among all pairs of edges whose two-point move, respectively three-opt move decreases the length we chose the one that gives the shortest route. These procedures are iterated until no improvements are found.

The five inter-route neighborhoods move customers or arcs from one route to another. In the one-point move we simply move a customer from one route to another one. In the twp-opt move we remove two edges from two routes and replace with another two new edges results another two routes. In the or-opt move, we remove a string of two, three or four vertices from one route and insert the string into a new route. In the three-point move, we swap the position of a pair of adjacent nodes with the position of a thir vertex. Finally in the cross-exchange move, we remove four edges from two different routes and replace them with four new edges.

It is important to mention that concerning the described neighborhoods, we investigate all the possible connections of the exchanged vertices within the clusters in order to get improved routes (i.e. the nodes belonging to the marked clusters after the exchange may be different).

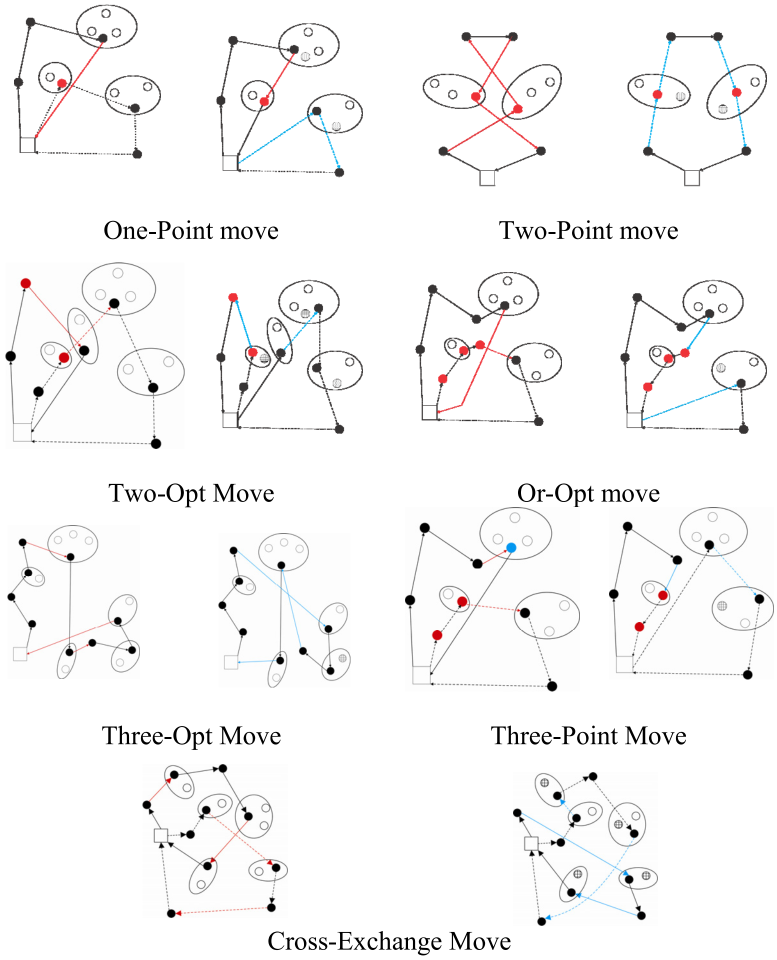

Fig. 2. Local search operators in GVRP

5. Acnowledgements

This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS - UEFISCDI, project number PN-II-RU-TE-2011-3-0113.

6. References

[1] BALDACCI R.; BARTOLINI E., LAPORTE G. Some applications of the generalized vehicle routing problem. Journal of the Operational Research Society [J], 2010, Vol. 61 (7), pp. 1072-1077.

[2] BEKTAS T.; ERDOGAN G.; ROPKE S. Formulations and Branch-and-Cut Algorithms for the Generalized Vehicle Routing Problem. Transportation Science [J], 2011, Vol. 45 (3), pp. 299-316.

[3] GHIANI, G; IMPROTA G. An efficient transformation of the generalized vehicle routing problem. European Journal of Operational Research [J], 2000, Vol. 122 (1), pp. 11-17.

[4] KARA Imdat; BEKTAS Tolga. Integer linear programming formulation of the generalized vehicle routing problem. EURO/INFORMS Joint International Meeting, Istanbul, July 06-10, Turkiye 2003.

[5] LAPORTE G.; OSMAN I.H. Routing problems: A bibliography. Annals of Operations Research [J], 1995, Vol. 61, pp. 227-262.

[6] MOCCIA L.; CORDEAU J-F.; LAPORTE G. An incremental neighbourhood tabu search heuristic for the generalized vehicle routing problem with time windows. Technical Report, https://www.cirrelt.ca/DocumentsTravail/CIRRELT-2010-12.pdf , 2010.

[7] POP Petrica; KARA Imdat; HORVAT-MARC Andrei. New mathematical models of the generalized vehicle routing problem and extensions. Applied Mathematical Modelling [J], 2012, Vol. 36, pp. 97-107.

[8] POP Petrica; MATEI Oliviu; POP SITAR Corina; CHIRA Camelia. A genetic algorithm for solving the generalized vehicle routing problem. In Proceedings of HAIS 2010, Lecture Notes in Artificial Intelligence, 2010, Vol. 6077, pp. 119-126.

[9] POP Petrica; PINTEA Camelia; ZELINA Ioana; DUMITRESCU Dan. Solving the Generalized Vehicle Routing Problem with an ACS-based Algorithm. American Institute of Physics, 2009, Vol. 1117, pp. 157-162.

[10] POP Petrica; POP SITAR Corina. A new efficient transformation of the generalized vehicle routing problem into the classical vehicle routing problem. Yugoslav Journal of Operations Research [J], 2011, Vol. 21, No. 2.