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The Application and Research of Element-free Method in Numerical Calculation of 3D Electromagnetic Field

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Abstract. In recent years, as a new computation method, element-free method received huge number of praise as it has advantages of high precision in calculation, simple form and clear goal. Compared with the finite element method, the boundary value of complex boundary conditions can be solved by those nodes defined by characters of the function without unit information, thus it gets rid of the limitation of unit which makes the element-free method become the most effective one method in numerical calculation of three-dimensional electromagnetic field.

Keywords: element-free method, electromagnetic fields, numerical calculation

1. Introduction

All along, the traditional finite element method is the most effective one in the numerical calculation of electromagnetic fields. That is, by dispersing the whole electromagnetic into unit form and generating the Field function through the infinite approximation of interpolation gains the required value. In this method, the cell shape should be set in a certain range and the angle which is too big or too small is not allowed. When dealing with3D electromagnetic field value, the analysis will be very heavy. Although there are already corresponding grid generators assisted, it still takes up too much time. Compared with the finite element method, the element-free one has inherent advantages. Particularly in:

Firstly, it only needs the node information which doesn't have to make up a unit. Thus the requirement to data becomes simple. And the grid does not have to be split. In the whole calculation process, the pre-processing work of dealing with the data is greatly simplified as just adding or deleting nodes.

Secondly, the shape function, constructed by moving least squares, can achieve a high level of continuous derivative with its advantages of high precision in calculation and good smoothness. For those nodes with distant discrete location in 3D electromagnetic field, there's no need to think over the shape in their unit and you can have them calculated directly.

Thirdly, high open, which will be easy to be expanded and controlled.

2. The Theoretical Basis

2.1. MLS

.MLS's full name is the Moving Least Square. It is just the mathematical basis of element-free method. The following is just its working principle.

Integrating the data of several nodes constructs the corresponding mathematical function. Different from the general global least squares, MLS uses partial continuous approximation method to constantly fitting the relationship between the data to the request and the real value through a weighting function with a limited and monotonous work domain. And the function is constructed with the weighting function in continuous

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changing in the spatial coordinates of determined coefficient. So the structure of the function is more accurate.

In the domain Ω, the field function is assumed for the U(X) in which (x, y, z) T is used to represent variable X in the whole 3D space. Now, assumed that I have a field function with n nodes whose value as shown in Equation 1:

$$u^*(x_i) = u_i^*, \quad i = 1, \ 2, \ \ldots n \tag{1}$$

When the node's value is determined, we can construct the corresponding approximate function uh(x) according to the nodes. The equation of the function is that as shown in Equation 2:

$$u^h(x) = \sum_{j=1}^{m} p_j(x) a_j(x) = p^T(x) a(x) \tag{2}$$

In Equation 2,PT(x) is m wiki function which is a polynomial formula while a(x) is an m-dimensional matrix. And a(x) expressed as shown in Equation 3:

$$a(\mathrm{x})^{\mathrm{T}} = [a_1(\mathrm{x}) \ \ \mathrm{a}_2(\mathrm{x}) \ \ \ldots \ a_{\mathrm{m}}(\mathrm{x})] \tag{3}$$

In the above formula 2, a global approximation is used which has two aspects of drawback. One is that it is not conducive to simplifying the calculation, the other is it is not conducive to convergence. Here I use a partial approximation method to deal with the whole space in fragmentation process. Its function is as shown in Equation 4:

$$u^h(x, x_I) = \sum_{j=1}^{m} p_j(x_I) a_j(x) = p^{\mathrm{T}}(x_I) a(x) \tag{4}$$

In the above formula, the key of calculation is that how to determine the value of a(x).Here we must construct a weighted norm J(x) within a partial range and obtain the minimum value at the same time. J(x) function is as shown in Equation 5:

$$u(x, y) = \sum_{\substack{k=0 \\ n=2k+1}}^{\infty} \sin\frac{n\pi}{0.5} y \frac{\sinh\frac{n\pi}{0.5}(0.5 - x)}{\sinh\frac{0.5n\pi}{0.5}} = \sum_{\substack{k=0 \\ n=2k+1}}^{\infty} \sin 2n\pi y \frac{\sinh 2n\pi(0.5 - \pi)}{\sinh n\pi} \tag{5}$$

In the above formula, xi is just the N nodes in the affected domain. These nodes do not have to make up unit which is also the advantage of element-free method. And in which w(x-xI) is just the weighting function whose role is to control the effect of each node in affected domain to the whole electromagnetic field. If a node is not in the field to be calculated which has no effect in the whole field, the specific value of the weighting function will be 0.

Only the result of the derivation of J(x) is 0,the minimum value of J(x) is achieved. After a derivation a equation can be drawn as shown in Equation 6:

$$P^T W(x) P a(x) - P^T W(x) u^* = 0 \tag{6}$$

From this, we can draw values of a(x) accurately, namely:

$$a(x) = (P^T W(x) P)^{-1} P^T W(x) u^* \tag{7}$$

After the above operations, we can draw the final function is:

$$u^h(x) = p^T(x)(P^T W(x) P)^{-1} P^T W(x) u^* \tag{8}$$

2.2. The weight Function.

The weight function plays a crucial role in the calculation process of the entire function. It has decisive influence on the convergence rate of the calculation, computational complexity and smoothness of field function. I f the initial definition of the weight function is accurate, it can fit the function effectively to make the calculation and actual value infinitely close. There is no clearly definition or statement to determine and select the weight function at present. Experience occupies an important position. In general, the selection of the weight function is mainly based on the three principles as follows:

(1) The value of the weight function is as possible as positive, not negative;

(2) The weight function should be as possible as continuous and differentiable;

(3) The monotonicity of weighting function is decreasing .The larger of the distance from the nodes to the center fields, the smaller of the value of weight function, until there is no effect to the fields and eventually to zero.

3. The boundary value of the element-free method in the electromagnetic field

3.1. The boundary value in the electric fields.

The boundary value of the static electric field can be attributed to that its potential u satisfies Poisson equation, as follows:

$$\begin{cases} \nabla \bullet \beta \nabla u = -f & \text{in} \quad \Omega \\ u = u_0 & \text{on} \quad \Gamma_1 \\ \partial u / \partial n = \Psi & \text{on} \quad \Gamma_2 \end{cases} \qquad \text{(equation 1)}$$

Ω means field space, β means dielectric constant , Γ1 means first boundary of field space, Γ2 means second boundary of field space,Ψ means the derivative that potential of the second boundary against boundary function , f means charge density. The extreme value between the Poisson equation and the functional are the same. Functional as follows:

$$\begin{cases} I(u) = \int_\Omega \frac{1}{2} \beta \nabla u \bullet \nabla u d\Omega - \int_\Omega f u d\Omega - \int_{\Gamma_2} \psi ds - \min \ \text{in} \ \Omega \\ u = u_0 \qquad \text{on} \ \Gamma_1 \end{cases} \tag{9}$$

In the element-free method for Calculation processing, on the processing of the first boundary is not satisfied the interpolating condition. Therefore, it will not be able to keep the equation balanced by impose conditions. We must add binding terms in the functional .At present, there are mainly several methods for binding terms conditions.

3.2. Lagrange Multiplier Method.

Its major feature is to add new variables for the entire equation by interpolation function. By improving the boundary conditions, the result of the method is more accurate. But the disadvantages are also obvious. It will lead to the increase of the entire equation order due to adding the new variables. It will bring inestimable difficulties for the continuous derivative and calculation. Therefore, this method is suitable for smaller scope of field calculations

3.3. Penalty Function Method

The method is to convert the binding problems into unrestricted problems by adding objective functions to unconstrained functions to deleting the binding problems. The method solves the problem that lagrange multiplier method is suitable for smaller scope of field while eliminating the boundary interpolations. You

need have no consideration on the element shape. It has greater effectiveness for the larger complexity of geometry shape .The disadvantage is that it could not be accurately determined for penalty coefficient of penalty function .In general, values of penalty coefficient is 103 to104 . It is harder to solve for different functional forms. Therefore, there is no transferability to solve problems.

3.4. Modified Variational Atom Method

The method is improved on the basis of Lagrange multiplier method. It remains the zonal distribution features of factors by variational principle export the physical implication of Lagrange multiplier method. But, to some extent, it reduces the data accuracy of the original algorithm. It still have limitations in the practical calculation

3.5. Coupled Finite Element Method.

This method is the organic combination between finite element method and element-free method. In the external it uses the finite element method while in the internal uses the element-free method. The method makes use of the advantages of the finite element method and element-free method. It is the most widely used and the most functional method at present. But it also has disadvantages. How to decide the boundaries of the external and internal, which parts use the finite element method and which parts use the element-free method will need to be studied.

3.6. The boundary value in the electromagnetic field

After analyzing the boundary value, it will be very easy to value the magnetic field .Magnetic fields and electric fields are very similar. In magnetic fields, when you make the magnetic potential φ instead of the potential m you can get the governing equation of boundary value of the magnetic field. By using the magnetic vector potential A as the dependent variable .Specific equation is as follows:

$$\begin{cases} \nabla^2 \mathrm{A} = -\mu \mathrm{J} & \text{in} \quad \Omega \\ \mathrm{A} = \mathrm{A}_0 & \text{on} \quad \Gamma_1 \\ \dfrac{\partial A}{\partial n} = 0 & \text{on} \quad \Gamma_2 \end{cases} \qquad \text{(Equation 2)}$$

As the same as the electric fields, the solution between the equations and the pan value is the same .It can be converted to formula10, as follows:

$$\begin{cases} I(A) = \int_{\Omega} \left(\dfrac{1}{2\mu} \left| \nabla \times A \right|^2 - J \bullet A \right) d\Omega = \min \ \text{ in } \ \Omega \\ \mathrm{A}=A_0 \qquad\qquad \text{on } \ \Gamma_1 \end{cases} \qquad (10)$$

It is implemented by the Gaussian integral. Its basic thought is that the entire integration fields is divided into several integral sub-domains that can cover the entire field .For the inside integral sub-domains, you just consider the value of its contribution. For the outside, it is not considered.

Guass integration may get accurate results of arbitrary order and ignoring the relationship between integration fields and nodes. The disadvantage is that it costs much time for scanning the entire nodes of the whole fields one by one and judge the value of its contribution. It has higher complexity. Even for the local field, it also needs higher programming skills to calculate and the difficulty isn't small.

4. Influencing factors of the calculation precision

In order to judge the factors that the element-free method in the electromagnetic field values .We adopted the 3-D electrostatic fields as the model that we study, which used a rectangular section, long straight and hollow conductive slot. There is no distribution of physical charge inside the conductive slot. The potential value of the boundary is showed in Figure 1:

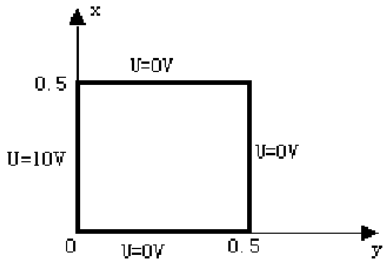

Figure 1 Conductive slot section

According to variable separation approach, the potentiometric inside the slot can be converted into formula as follows:

$$u(x,y)=\sum_{\substack{k=0\\n=2k+1}}^{\infty}\sin\frac{n\pi}{0.5}y\frac{\sinh\frac{n\pi}{0.5}(0.5-x)}{\sinh\frac{0.5n\pi}{0.5}}=\sum_{\substack{k=0\\n=2k+1}}^{\infty}\sin 2n\pi y\frac{\sinh 2n\pi(0.5-\pi)}{\sinh n\pi} \tag{11}$$

In the entire computation process, there are four parameters affect the results as follows:

4.1. The Weight Function.

It is the heart of the EFM. It determines the accuracy and the complexity of the electromagnetic field values. So you must consider specially for selecting it and determine its value and parameters according to the different application situation.

4.2. The Basic Function.

Basis function determines the accuracy of calculation. Under the same conditions of node density, weight function and edge influence, using different basis functions to calculate, you will get different result. The bigger of the basis function, the greater of the accuracy, but its efficiency is lower. So it is not the bigger of the basis function, the better of the result. In the condition of meeting the needs of the case, it is as much as possible to find a balance point. Generally speaking, it is much more to adopt the quadratic basis functions.

4.3. Influence Radius.

Element-free-method is taken the approach of an infinite method. The size of the radius directly determined the number of nodes within this range .If the radius is too small, the formation of local influence is too much and increasing the complexity of the calculation. If it is too large, you will not achieve the purpose of effective division. The continuity of the calculation could not be effectively guaranteed.

4.4. Distributed Density of Nodes.

The calculation is obviously affected by distributed density. When the distributed density of nodes is too large, the accuracy of calculation is high .On the opposite, if the distributed density of nodes is too small, the efficiency is improved, but the accuracy of calculation is lower or even distorted.

5. Summary

The major work of this paper is the research of the element-free-method in the Numerical Calculation of Electromagnetic Field .First, it introduced the theoretical basis of the element-free method and particularly analyze the moving least squares method .Second ,it described the boundary value problems of the element-free method in the Electromagnetic Field , including the electric fields and magnetic fields. Finally, according to practical examples to analyze the main factors that influence the element-free method in the Numerical Calculation .Due to the lack of space, it did not give specific numerical values to argue for the numerical problems of the three-dimensional electromagnetic field .The readers who are interested in this field can have further perfections on this basis.

6. References

[1] Zhou Weiyuan , Kou Xiaodong Element-free-method and its application in engineering Journal of Fluid Mechanics 1998.

[2] Hu Yunjin,Zhou Weiyuan, Kou Xiaodong Three dimensional element-free-method and its application Chinese Journal of Rock Mechanics and Engineering 2004.

[3] Liu Zhenhua Research and application of element-free method to electromagnetic field analysis The Electrical Engineering School in the North China Electric Power University 2002.

[4] Liu Suzhen, Yang Qingxin Element-free method applied to 2D electrical field numerical calculation Journal of Hebei University of Technology 1999.

[5] Zhou Xiaoping, Zhou Ruizhong A Study of Interpolating Function Applied in Element-Free Galerkin Method Journal of Fuzhou University (Natural Sciences Edtion) 2002.