QPP Interleaver Based Turbo-code For DVB-RCS Standard

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Abstract. We propose a double-binary turbo-code (DBTC), which incorporates Quadratic Polynomial Permutation (QPP) type interleaver, as an alternative to the DBTC used, in the DVB-RCS (Digital Video Broadcasting – Return Channel via Satellite) standard. The proposed DBTC outperforms the current turbo-code (TC) used in standard with exactly the same complexity (both DBTCs, the standard and the proposed one, have, as component codes, convolutional codes with 8 states). The paper presents in detail the parameters for the proposed DBTC and its performance.

Keywords: Channel coding, convolutional code, quadratic polynomial permutation, interleaver, turbo-code

1. Introduction

The error correction is an indispensable operation to the current communication systems. It is especially used in wireless communication systems, where transmission errors are consistent. The TCs [1] have a special place between the error-correcting codes. They have imposed their performances in most of the applications in communications: 3GPP–Group Radio Access Network, [2], Digital Video Broadcasting–Return Channel via Satellite DVB RCS [3], deep space [4], wireless metropolitan network standard IEEE 802.16 (WiMAX), [5], etc. Standard specifications (listed above) for designing a TC include details about the component code, interleaver type, interleaving length[1] but also about the structure of the data block and implicitly the trellis termination.

This paper intends the DVB-RCS standard. More specifically, the paper presents a DBTC, similar in complexity to that of standard, but with superior bit/frame error rate (BER/FER) versus signal to noise rate performance. More details (regarding [3]) about the DBTC currently used in DVB-RCS can be found in [6].

The TC proposed in this paper was built on a quasi-exhaustive search on the sets of recursive and systematic, double-binary convolutional codes[2] and QPP interleaver type. The paper's structure is as follows. The next section describes the family in which we sought DBTC component codes, a selection criterion and a selected code. Section 3 describes QPP interleavers. Section 4 is intended to describe the comparative performance of the standard and the proposed DBTCs, and in Section 5 we present conclusions and further research topics.

2. Eight State DBTCS

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[1] Length that fixed the data block size;

[2] Family from which the component code from the actual DVB-RCS standard takes part;

The controller and observer canonical forms, **[7]**, for the encoder from convolutional component code used in the standard DVB-RCS are presented in Fig. 1. The generator matrix and the transfer function have the expressions:

$$G_{DVB}(D) = \begin{bmatrix} 1 & 0 & g_2(D)/g_0(D) \\ 0 & 1 & g_1(D)/g_0(D) \end{bmatrix}, \tag{1}$$

$$x_0(D) = \frac{g_2(D)}{g_0(D)} \cdot x_2(D) + \frac{g_1(D)}{g_0(D)} \cdot x_1(D), \tag{2}$$

where $g_0(D) = 1 + D + D^3$, $g_1(D) = 1 + D + D^2 + D^3$, $g_2(D) = 1 + D^2 + D^3$, and $x_i(D) = \sum_{j=0}^{N-1} x_{i,j} \cdot D^j$, $0 \le i \le 2$, represent the *D* transforms of the information sequences and of the redundancy sequence respectively, and *N* is the interleaver length. For the sake of simplicity we will use from now on the denotation (in decimal): $G_{DVB} = [13\ 15\ 11]$. The code word has the form $y(D) = x(D) \cdot G_{DVB}(D) = [x_2(D)\quad x_1(D)\quad x_0(D)]$.

Remark: the output relationship in the form (2) requires a trellis closing at the zero state. However in the DVB-RCS standard the tail-biting closing technique is used **[8]**. In this case, a term that should take into consideration the initial state of the encoder should be added in (2). This term has not been added, since his absence does not affect the purpose here.

The set of all duo-binary convolutional family members (DBCC) with 8 states is obtained by changing the values of the polynomial coefficients g_i. A search on this set, in terms of identifying some DBCCs to equip TCs was presented in **[9]**. For assessing the quality of a code, the convergence of the corresponding decoding process criterion was used in **[9]**. Thus, if the TC is equipped with a criterion for stopping the iterations, **[10]**, its convergence, denoted in the following C_v, is defined as the ratio of the total number of iterations, *iter*(*Nb*) necessary for the decoding of *Nb* turbo-coded blocks and *Nb*.

When *Nb* is very big:

$$C_v = \lim_{Nb \to \infty} \frac{iter(Nb)}{Nb}. \tag{3}$$
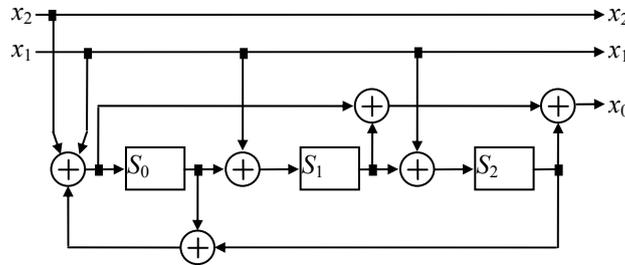
a) controller canonical form
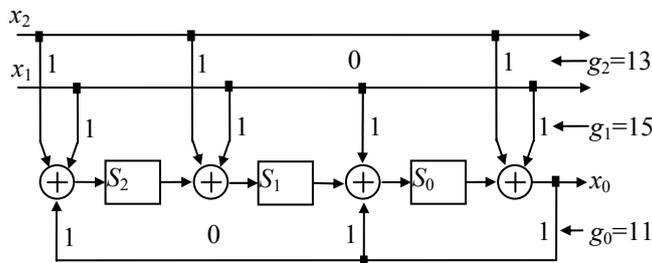
b) observer canonical form

Fig.1 The scheme of the component code of the TC specified in the standard DVB-RCS.

Certainly, for small SNR, C_v will tend to $iter_{max}$ (the maximum number of iterations performed by the TC decoder for the decoding of a block, which is a priori set). With the increasing of the SNR, the weight of the non-convergent blocks (which require a number of iterations equal with $iter_{max}$) decreases. In this case the dependence of C_v on the selection of TC increases.

A remark obtained through simulations is that C_v can be calculated for a small Nb. Another remark based on practice is that a good TC has a good convergence. Starting with these two remarks, the TC's functionality was simulated with each of the component codes using an S interleaver. The simulations were stopped if the convergence C_v was bigger than a given limit:

$$\text{stop the simulation if } C_v(Nb) > C_{v0} - Nb \cdot dC_v, \tag{4}$$

where dC_v represents the speed of threshold's decreasing. The given threshold $C_{v0} - Nb \cdot dC_v$ decreases with Nb to prevent the simulation for a specific code to last forever. In the search of the best component code we have used the following values of the parameters C_{v0} and dC_v: 4.5 and $15 \cdot 10^{-4}$ respectively. It was used an S interleaver with the length of 752 (so that a data block contains 1504 bits).

After the investigations on the DBCC set with 8 states, the most performing encoders found in [9] are defined by the matrices: [15 9 11], [15 9 13], [13 15 11] = G_{DVB} and [11 15 13]. Because the TCs obtained with the codes defined by the matrices [15 9 11] and [15 9 13] are equivalent in performance (see [11, 12]), we selected the [15 9 13] matrix for the construction of a performing DBTC.

3. The QPP Interleavers

The QPP interleavers, proposed in [13], have an interleaving relationship of the form:

$$\pi(j) = (f_0 + f_1 \cdot j + f_2 \cdot j^2) \bmod N, \quad \forall\ 0 \leq j < N, \tag{5}$$

where f_0 is optional.

One of the main merits of this class of interleavers is its propriety of „contention-free" described and argued in [14]. Other properties of this interleaver family and their performance in the binary TCs are described in [15]. The number of distinct interleavers belonging to the QPP family can be reduced to half of the total number of variants N^2, where N is even, considering the following result:

Proposition 1 If N is even and $\pi_1(j) = (f_1 \cdot j + f_2 \cdot j^2) \bmod N$, $\forall\ 0 \leq j < N$, with $0 < f_1 < N/2$ is an interleaver function, then $\pi_2(j) = (f_1 \cdot j + (N/2) \cdot j + f_3 \cdot j^2) \bmod N$, $\forall\ 0 \leq j < N$, with $f_3 = (f_2 + (N/2)) \bmod N$, is an interleaver function equal with π_1.

Proof

If $j = 2 \cdot k$, then we have gradually:

$\quad \pi_2(j) = \pi_2(2 \cdot k) =$

$(f_1 \cdot 2 \cdot k + (N/2) \cdot 2 \cdot k + f_2 \cdot 4 \cdot k^2 + (N/2) \cdot 4 \cdot k^2) \bmod N =$

$(f_1 \cdot 2 \cdot k + f_2 \cdot 4 \cdot k^2) \bmod N = \pi_1(2 \cdot k) = \pi_1(j).$

If $j = 2 \cdot k + 1$, then we have gradually:

$\quad \pi_2(j) = \pi_2(2 \cdot k + 1) =$

$(f_1 \cdot 2 \cdot k + f_1 + (N/2) \cdot 2 \cdot k + (N/2) + f_2 \cdot 4 \cdot k^2 + f_2 \cdot 4 \cdot k + f_2 + (N/2) \cdot 4 \cdot k^2 + (N/2) \cdot 4 \cdot k + (N/2)) \bmod N =$

$(f_1 \cdot 2 \cdot k + f_1 + f_2 \cdot 4 \cdot k^2 + f_2 \cdot 4 \cdot k + f_2) \bmod N =$

$\pi_1(2 \cdot k + 1) = \pi_1(j).$

For the previous selected matrix we made a search of a QPP interleaver. We set $f_1 = 0$ and for f_2 and f_3, we took values between 1 and $N/2 - 1$, respectively 1 and $N - 1$, in accord with Proposition 1. The values for N were selected from the DVB-RCS standard specifications, [3], reproduced in Table 1. Unfortunately, for 3

of the specified lengths (212, 220 şi 228) there are not QPP interleavers. For the other lengths, after the search realized, we selected one interleaver for each length. The selected interleavers together with the DBCC having the generating matrix [15 9 13] were used for the implementation of the DBTC, proposed in this paper. Details about interleavers and about the performance obtained are given in the next section.

4. The Performance of the Proposed DBTC Versus the DBTC From the DVB-RCS Standard

In Table 2 – line 2, for each length of the 12 defined in the DVB-RCS standard (line 1), are indicated the total number of QPP interleavers. Unfortunately, for 3 of the lengths (212, 220 and 228) there are not QPP interleavers. For the other 9 lengths, we selected one interleaver through a quasi-exhaustive search. The parameters of the selected QPP interleavers are indicated in the lines 2, 3 and 4 from Table 2. For these 9 interleavers, in Table 1 are indicated the data block dimension & the interleaving length, in conformity with the DVB-RCS standard specifications.

In Fig. 2 and 3 are presented the BER and FER performance of the DBCC defined by the matrix [15 9 13] compared with those which correspond to the TC from the standard. In both figures, the curves drawn with continuous line correspond to the standard DBTC and those in dashed line are obtained for the proposed DBTC.

Analysing the diagrams, we remark the high performances of the QPP interleavers according to those from the standard, for 6 of the 9 compared lengths. For the small lengths, of 48 and 64, the performance of the TC is practically similar to the standard TC. For a single length (856) the standard TC is higher in performance to the proposed one. This is due to the small number of the QPP interleavers from that length (424 interleavers). The coding gain provided by the proposed TC, evaluated at a FER of 10^{-6}, is situated around the value of 0.2 dB (for the average interleaving lengths, 424, 432 and 440) and about 0.4 dB for the higher lengths, 752, 848 and 864.

Tab. 1. Data block dimension & interleaving length; DVB-RCS standard specifications.

Data block size (bytes)	12	16	53	55	57	106	108	110	188	212	214	216
Interleaver length, N	48	64	212	220	228	424	432	440	752	848	856	864

Tab. 2. The parameters of the proposed QPP interleavers.

Interleaver length, N	48	64	212	220	228	424	432	440	752	848	856	864
No. of QPP interleavers	48	480	0	0	0	208	5040	160	1104	1248	424	20448
f_1	7	27	-	-	-	55	175	29	165	161	373	203
f_2	12	24	-	-	-	318	162	110	282	530	642	252
d_{min}	6	4	-	-	-	24	14	20	26	28	36	18
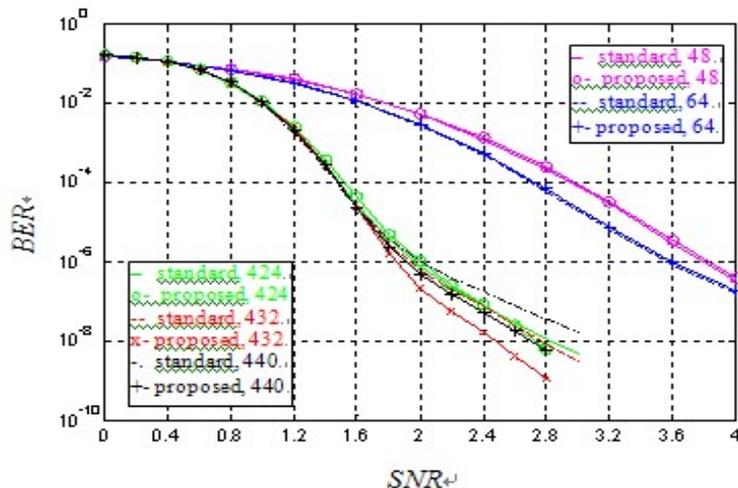
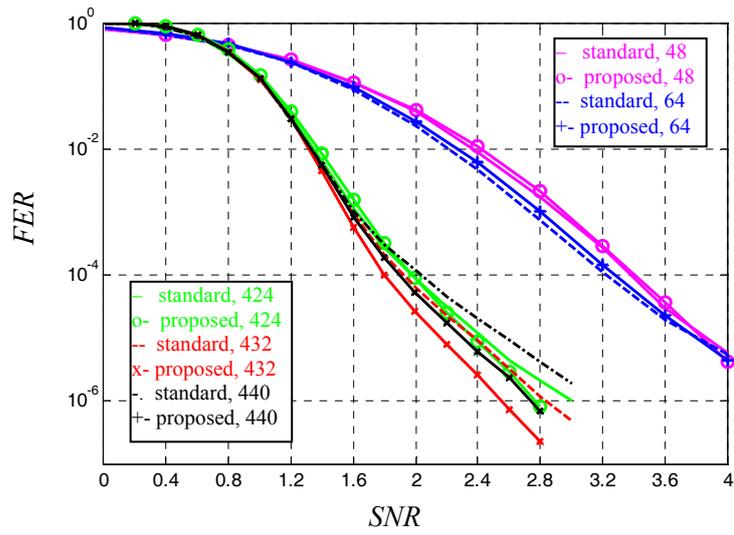Fig. 2 Proposed DBTC versus standard DBTC performance at N = 48, 64, 424, 432 and 440.

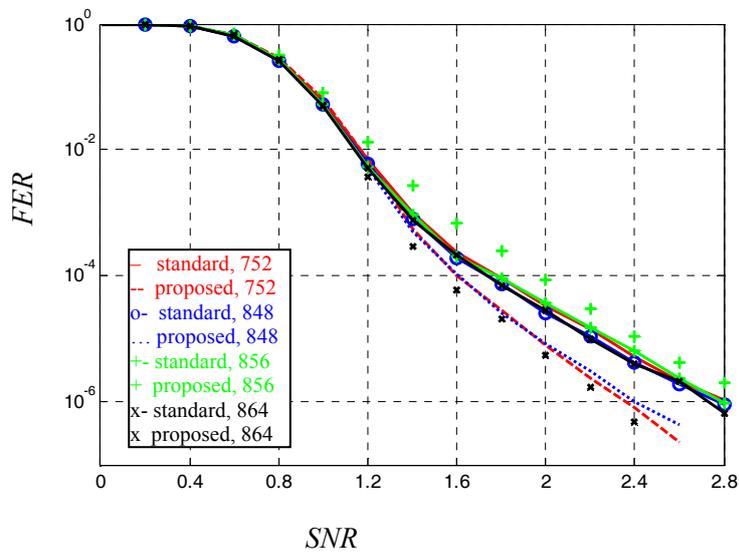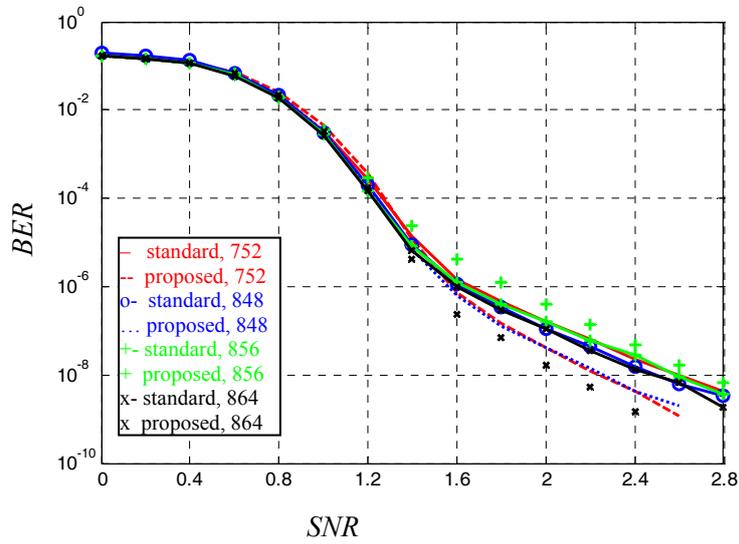

Fig. 3 Proposed DBTC versus standard DBTC at N = 752, 848, 856 and 864.

5. Conclusions

In this paper we proposed a DBTC for the DVB-RCS standard. The proposed DBTC incorporates the DBCC defined by the generator matrix [15 9 13] and the QPP type interleavers. The performance obtained with the proposed interleaver and the one from the standard are comparatively presented. For 6 of the lengths provided by the standard, the proposed DBTC exceeds the one from the standard in performance. At lengths of 48 and 64 the proposed DBTC has the same performance with the one in the standard. For the interleaving lengths of 856, the best selected interleavers do not offer superior performance for the proposed DBTC in comparison with the standard. Unfortunately, for 3 of the interleaving lengths, defined in the standard, there is not any interleaver in the QPP family. For these lengths, the possible solution could be to use other types of interleavers or to regroup the sequence of bits in blocks of different lengths where QPP interleavers exist. These solutions could be the subject of a future work.

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